

Spatial Precoder Design Using Fixed Parameters of MIMO Channels

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Abstract—In this paper, we introduced the novel idea of linear spatial precoding based on fixed and known parameters of a MIMO channel where antenna spacing and antenna placement at the transmitter and receiver arrays are considered as fixed parameters. This precoder reduces the effects of antenna spacing and antenna placement and improves the performance of space-time coded MIMO systems. Unlike the previous precoder designs found in the literature, this precoder does not require frequent feedback of channel state information (partial or full) from the receiver. Closed form solutions for the precoder is presented for systems with up to three receiver antennas. A generalized method is proposed for more than three receiver antennas. Simulation results show that at low SNRs, this precoder provides significant performance improvement over a non-precoded system for small antenna aperture sizes.

I. INTRODUCTION

Space-time coding has been proposed in recent years to achieve diversity and coding gain over MIMO fading channels [1, 2]. This does not require any knowledge of the spatial channel at the transmitter. The codes are designed, assuming that the channel gains between the transmitter and receiver antennas undergo uncorrelated independent flat fading. Such an assumption is valid only if the scattering environment is isotropic, i.e., scattering is uniformly distributed over the receiver and transmitter antenna arrays, and also only if the antennas in an array are well separated. Recent studies have shown that insufficient antenna spacing and scattering channel correlations reduce the performance of a space-time coded communication system. This has motivated the design of linear precoders for space-time coded multiple antenna systems with partial channel state information at the transmitter [3–6], where the receiver feeds back information about the channel to the transmitter in the form of channel correlation coefficients (covariance feedback). In order to be cost effective and optimal, these designs assumed that the channel remains stationary (channel statistics are invariant) for a large number of symbol periods and the transmitter is capable of acquiring robust partial channel state information. However, when the channel is non-stationary or it is stationary for a small number of symbol periods, receiver will have to frequently feedback the channel correlation statistics to the transmitter. As a result, the system becomes costly and the optimum precoder design, based on the previously possessed information, becomes outdated quickly. In some circumstances feeding back channel information is not possible. This has motivated us to design a precoder based on fixed and known parameters of the system, such as antenna

spacing and antenna geometry of a MIMO channel, where the transmitter does not require any feedback information from the receiver.

Designing such a precoder will reduce the effect of non-ideal antenna placement, which is a major contributor to the spatial channel correlation, on the MIMO system performance in stationary channels as well as non-stationary channels. The spatial channel model developed in [7] provides us a way to factor the MIMO channel into deterministic and random matrices, where the deterministic part depends only on the physical configuration of antennas at the transmitter and receiver arrays.

We derive the spatial precoder by minimizing the pairwise error probability (PEP) upper bound, derived previously in [8] for the spatial channel model in [7], subject to a transmit power constraint assuming scattering environment surrounding the transmitter and receiver regions is uncorrelated. For a system that uses orthogonal space-time block coding (O-STBC), we show that the optimum linear precoder for a MISO fading channel is essentially given by the classical “water-filling” strategy in information theory [9]. For a MIMO channel, the linear precoder is determined by a novel generalized water-filling scheme. Closed form solutions for the precoder is presented for systems with up to three receiver antennas. A generalized method is proposed for more than three receiver antennas. We demonstrate the precoding gain achieved from our linear spatial precoder by simulating the performance of several O-STBCs proposed in [2] for different spatial scenarios.

Notations: Throughout the paper, the following notations will be used: $[\cdot]^T$, $[\cdot]^*$ and $[\cdot]^\dagger$ denote the transpose, complex conjugate and conjugate transpose operations, respectively. The symbol \otimes denotes Matrix Kronecker product. The notation $E\{\cdot\}$ denotes the mathematical expectation, $\text{vec}(\mathbf{A})$ denotes the vectorization operator which stacks the columns of matrix \mathbf{A} and $\text{tr}\{\cdot\}$ denotes the matrix trace. The matrix \mathbf{I}_n is the $n \times n$ identity matrix and bold lower letters denote vectors.

II. SYSTEM MODEL

Consider a MIMO system consisting of n_T transmit antennas and n_R receive antennas. At time instant k , the space-time encoder at the transmitter takes a set of input data bits and produces a $n_T \times 1$ code vector $\mathbf{x}(k)$. Then, each code vector is multiplied by a $n_T \times n_T$ fixed linear precoder matrix \mathbf{F} before transmitting out from n_T transmit antennas. Assuming

quasi-static fading, the signals received at n_R receiver antennas during L symbol periods can be expressed in matrix form as

$$\mathbf{Y} = \mathbf{H}\mathbf{F}\mathbf{X} + \mathbf{N},$$

where \mathbf{X} is the space-time codeword matrix formed by L successive code vectors $\mathbf{x}(k)$, $\mathbf{X} = [\mathbf{x}(1), \mathbf{x}(1), \dots, \mathbf{x}(L)]$ with L the code length, \mathbf{N} is the $n_R \times L$ white Gaussian noise matrix in which elements are zero-mean independent Gaussian distributed random variables with variance $N_0/2$ per dimension and \mathbf{H} is the $n_R \times n_T$ channel matrix. The elements of \mathbf{H} are modeled as zero-mean complex Gaussian random variables (Rayleigh fading).

By taking into account physical aspects of scattering, the channel matrix \mathbf{H} can be decomposed into deterministic and random parts as [7]

$$\mathbf{H} = \mathbf{J}_R \mathbf{H}_S \mathbf{J}_T^\dagger, \quad (1)$$

where \mathbf{H}_S represents the random non-isotropic scattering environment surrounding the receiver and the transmitter regions, while \mathbf{J}_R and \mathbf{J}_T represent the effects of antenna configurations at the receiver and transmitter antenna arrays, respectively. The reader is referred to [7] for definitions of \mathbf{H}_S , \mathbf{J}_R and \mathbf{J}_T . Decomposition (1) plays a major role in this paper.

III. PROBLEM SETUP

Assume that perfect channel state information (CSI) is available to the receiver and maximum likelihood (ML) detection is employed at the receiver. Suppose codeword \mathbf{X}_k is transmitted, but the ML-decoder chooses codeword \mathbf{X}_ℓ , then the average PEP is upper bounded by [8]

$$P(\mathbf{X}_k \rightarrow \mathbf{X}_\ell) \leq \frac{1}{\det[\mathbf{I}_{n_T n_R} + \frac{\eta}{4} \mathbf{R}[\mathbf{I}_{n_R} \otimes \mathbf{X}_\Delta]]}, \quad (2)$$

where $\mathbf{X}_\Delta = \mathbf{F}(\mathbf{X}_k - \mathbf{X}_\ell)(\mathbf{X}_k - \mathbf{X}_\ell)^\dagger \mathbf{F}^\dagger$, $\mathbf{R} = E\{\mathbf{h}^\dagger \mathbf{h}\}$ with $\mathbf{h} = (\text{vec}(\mathbf{H}^T))^T$ a row vector and \mathbf{H} has the decomposition (1), and η is the average symbol energy-to-noise ratio (SNR) at each receiver antenna.

Suppose transmitter configuration matrix \mathbf{J}_T has the singular value decomposition (svd) $\mathbf{J}_T = \mathbf{U}_T \mathbf{\Lambda}_T \mathbf{V}_T^\dagger$ and receiver configuration matrix \mathbf{J}_R has the svd $\mathbf{J}_R = \mathbf{U}_R \mathbf{\Lambda}_R \mathbf{V}_R^\dagger$. Substituting svds of \mathbf{J}_T and \mathbf{J}_R in (1) and using the Kronecker product identity [10, page 180] $\text{vec}(\mathbf{A}\mathbf{X}\mathbf{B}) = (\mathbf{B}^T \otimes \mathbf{A}) \text{vec}(\mathbf{X})$, we obtain

$$\mathbf{h} = \mathbf{h}_{JS}(\mathbf{U}_R^T \otimes \mathbf{U}_T^\dagger), \quad (3)$$

where $\mathbf{h}_{JS} = (\text{vec}(\mathbf{H}_{JS}^T))^T$ and $\mathbf{H}_{JS} = \mathbf{\Lambda}_R \mathbf{V}_R^\dagger \mathbf{H}_S \mathbf{V}_T \mathbf{\Lambda}_T^\dagger$. Applying the same Kronecker product identity to $\text{vec}(\mathbf{H}_{JS}^T)$ yields $\mathbf{h}_{JS} = \mathbf{h}_S[(\mathbf{V}_R^* \mathbf{\Lambda}_R^T) \otimes (\mathbf{V}_T \mathbf{\Lambda}_T^\dagger)]$, where $\mathbf{h}_S = (\text{vec}(\mathbf{H}_S^T))^T$. Then the covariance matrix \mathbf{R} of the MIMO channel \mathbf{H} is given by

$$\mathbf{R} = (\mathbf{U}_R^* \otimes \mathbf{U}_T) \mathbf{R}_{JS} (\mathbf{U}_R^T \otimes \mathbf{U}_T^\dagger), \quad (4)$$

where $\mathbf{R}_{JS} = [(\mathbf{\Lambda}_R^* \mathbf{V}_R^T) \otimes (\mathbf{\Lambda}_T \mathbf{V}_T^\dagger)] \mathbf{R}_S [(\mathbf{V}_R^* \mathbf{\Lambda}_R^T) \otimes (\mathbf{V}_T \mathbf{\Lambda}_T^\dagger)]$ with $\mathbf{R}_S = E\{\mathbf{h}_S^\dagger \mathbf{h}_S\}$ the covariance matrix of the scattering environment.

In this work, our main consideration is to design a linear precoder which compensates for any detrimental effects of non-ideal antenna placement/configuration on the performance of space-time block codes. Here we assume that the scattering environment surrounding the transmitter and the receiver regions is "rich", i.e., $\mathbf{R}_S = \mathbf{I}$. This assumption yields the simplification

$$\mathbf{R}_{JS} = [(\mathbf{\Lambda}_R^* \mathbf{V}_R^T) \otimes (\mathbf{\Lambda}_T \mathbf{V}_T^\dagger)] [(\mathbf{V}_R^* \mathbf{\Lambda}_R^T) \otimes (\mathbf{V}_T \mathbf{\Lambda}_T^\dagger)] \quad (5a)$$

$$= (\mathbf{\Lambda}_R^* \mathbf{\Lambda}_R^T) \otimes (\mathbf{\Lambda}_T \mathbf{\Lambda}_T^\dagger), \quad (5b)$$

where (5b) follows from (5a) by matrix identity [10, page 180] $(\mathbf{A} \otimes \mathbf{C})(\mathbf{B} \otimes \mathbf{D}) = \mathbf{AB} \otimes \mathbf{CD}$, provided that the matrix products \mathbf{AB} and \mathbf{CD} exist, and unitary matrix properties $\mathbf{V}_R^\dagger \mathbf{V}_R = \mathbf{I}$ and $\mathbf{V}_T^\dagger \mathbf{V}_T = \mathbf{I}$. Substituting (5b) into \mathbf{R} and the result in (2) with the identity yields the average PEP upper-bound

$$P(\mathbf{X}_k \rightarrow \mathbf{X}_\ell) \leq \frac{1}{\det[\mathbf{I}_n + \frac{\eta}{4} [\mathbf{R} \otimes \mathbf{T}] [\mathbf{I}_{n_R} \otimes (\mathbf{U}_T^\dagger \mathbf{X}_\Delta \mathbf{U}_T)]]},$$

where $n = n_T n_R$, $\mathbf{R} = (\mathbf{\Lambda}_R \mathbf{\Lambda}_R^\dagger)^T$, $\mathbf{T} = \mathbf{\Lambda}_T \mathbf{\Lambda}_T^\dagger$ and we have used the identity $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$. Note that both \mathbf{R} and \mathbf{T} are diagonal matrices, where the diagonal of \mathbf{R} consist of squared singular values of \mathbf{J}_R (or eigen-values of $\mathbf{J}_R \mathbf{J}_R^\dagger$) and diagonal of \mathbf{T} consist of squared singular values of \mathbf{J}_T (or eigen-values of $\mathbf{J}_T \mathbf{J}_T^\dagger$). With O-STBCs, $(\mathbf{X}_k - \mathbf{X}_\ell)(\mathbf{X}_k - \mathbf{X}_\ell)^\dagger = \beta_{k,\ell} \mathbf{I}$, where $\beta_{k,\ell}$ is a positive constant associated with the O-STBC used. Using this property of O-STBCs we obtain the PEP upper-bound

$$P(\mathbf{X}_k \rightarrow \mathbf{X}_\ell) \leq \frac{1}{\det[\mathbf{I}_{n_T n_R} + [\mathbf{R} \otimes \mathbf{T}] [\mathbf{I}_{n_R} \otimes \mathbf{Q}]]}, \quad (6)$$

where

$$\mathbf{Q} = (\eta \beta_{k,\ell} / 4) \mathbf{U}_T^\dagger \mathbf{F} \mathbf{F}^\dagger \mathbf{U}_T.$$

We can restate our objective to be to find the optimum spatial precoder \mathbf{F} such that the PEP upper bound (6) is minimized under a transmit power constraint, for given transmitter and receiver antenna configurations in a rich scattering environment. Here, the linear spatial precoder $\mathbf{F} = \sqrt{4/(\beta_{k,\ell} \eta)} \mathbf{U}_T \mathbf{Q}^{\frac{1}{2}} \mathbf{U}_n^\dagger$, where \mathbf{U}_n is any unitary matrix, and \mathbf{F} must satisfy the power constraint $\text{tr}\{\mathbf{F} \mathbf{F}^\dagger\} = n_T$ or equivalently, \mathbf{Q} must satisfy the power constraint $\text{tr}\{\mathbf{Q}\} = n_T \eta \beta_{k,\ell} / 4$.

IV. OPTIMUM SPATIAL PRECODER DESIGN

The linear precoder \mathbf{F} is designed by minimizing the maximum of all PEP upper bounds subject to the power constraint $\text{tr}\{\mathbf{F} \mathbf{F}^\dagger\} = n_T$. The logarithm of the PEP upper-bound (6) is used as the objective function. Note that \mathbf{Q} in (6) is positive semi-definite as $\mathbf{Q} = \mathbf{B} \mathbf{B}^\dagger$, with $\mathbf{B} = \sqrt{(\eta \beta_{k,\ell}) / 4} \mathbf{U}_T^\dagger \mathbf{F}$,

then the positive semi-definite matrix \mathbf{Q} is obtained by solving the optimization problem:

$$\begin{aligned} \min \quad & -\log \det [\mathbf{I}_{n_T n_R} + (\mathbf{R} \otimes \mathbf{T})(\mathbf{I}_{n_R} \otimes \mathbf{Q})] \\ \text{subject to} \quad & \mathbf{Q} \succeq 0, \text{tr}\{\mathbf{Q}\} = \frac{n_T \eta \beta}{4}, \end{aligned} \quad (7)$$

where $\beta = \min_{k \neq \ell} \{\beta_{k,\ell}\}$ over all possible codewords. By applying Hadamard's inequality on $\det [\mathbf{I} + (\mathbf{R} \otimes \mathbf{T})(\mathbf{I} \otimes \mathbf{Q})]$ gives that this determinant is maximized when $(\mathbf{R} \otimes \mathbf{T})(\mathbf{I} \otimes \mathbf{Q})$ is diagonal [9]. Therefore \mathbf{Q} must be diagonal as \mathbf{T} and \mathbf{R} are both diagonal. Since $(\mathbf{R} \otimes \mathbf{T})(\mathbf{I} \otimes \mathbf{Q})$ is a positive semi-definite diagonal matrix with non-negative entries on its diagonal, $\mathbf{I} + (\mathbf{R} \otimes \mathbf{T})(\mathbf{I} \otimes \mathbf{Q})$ forms a positive definite matrix. As a result, the objective function of our optimization problem is convex [11, page 73]. Therefore the optimization problem (7) above is a convex minimization problem because the objective function and inequality constraints are convex and equality constraint is affine.

Let $q_i = [\mathbf{Q}]_{i,i}$, $t_i = [\mathbf{T}]_{i,i}$ and $r_j = [\mathbf{R}]_{j,j}$. Optimization problem (8) then reduces to finding $q_i > 0$ such that

$$\begin{aligned} \min \quad & -\sum_{j=1}^{n_R} \sum_{i=1}^{n_T} \log(1 + t_i q_i r_j) \\ \text{subject to} \quad & \mathbf{q} \succeq 0, \quad \mathbf{1}^T \mathbf{q} = \frac{n_T \eta \beta}{4} \end{aligned} \quad (8)$$

where $\mathbf{q} = [q_1, q_2, \dots, q_{n_T}]^T$ and $\mathbf{1}$ denotes the vector of all ones. Introducing Lagrange multipliers $\lambda \in \mathbb{R}^{n_T}$ for the inequality constraints $-\mathbf{q} \preceq 0$ and $v \in \mathbb{R}$ for the equality constraint $\mathbf{1}^T \mathbf{q} = n_T \eta \beta / 4$, we obtain the Karush-Kuhn-Tucker (K.K.T) conditions

$$\begin{aligned} \mathbf{q} \succeq 0, \quad \lambda \succeq 0, \quad \mathbf{1}^T \mathbf{q} &= \frac{n_T \eta \beta}{4} \\ \lambda_i q_i &= 0, \quad i = 1, 2, \dots, n_T \\ -\sum_{j=1}^{n_R} \frac{r_j t_i}{1 + r_j t_i q_i} - \lambda_i + v &= 0, \quad i = 1, 2, \dots, n_T. \end{aligned} \quad (9)$$

λ_i in (9) can be eliminated since it acts as a slack variable¹, giving new K.K.T conditions

$$\begin{aligned} \mathbf{q} \succeq 0, \quad \mathbf{1}^T \mathbf{q} &= \frac{n_T \eta \beta}{4} \\ q_i \left(v - \sum_{j=1}^{n_R} \frac{r_j t_i}{1 + r_j t_i q_i} \right) &= 0, \quad i = 1, \dots, n_T, \end{aligned} \quad (10a)$$

$$v \geq \sum_{j=1}^{n_R} \frac{r_j t_i}{1 + r_j t_i q_i}, \quad i = 1, \dots, n_T. \quad (10b)$$

For $n_R = 1$, the optimal solution to (10) is given by the classical "water-filling" solution found in information theory [9]. The optimal q_i for this case is given in Section IV-A. For $n_R > 1$, the main problem in finding the optimal q_i for given t_i and $r_j, j = 1, 2, \dots, n_R$ is the case that, there are

¹If $g(x) \leq v$ is a constraint inequality, then a variable λ with the property that $g(x) + \lambda = v$ is called a slack variable [11].

multiple terms that involve q_i on (10a). Therefore we can view our optimization problem (8) as a *generalized water-filling* problem. In fact the optimum q_i for this optimization problem is given by the solution to a polynomial obtained from (10a). In Sections IV-B and IV-C, we provide closed form expressions² for optimum q_i for $n_R = 2$ and 3 receiver antennas and a generalized method which gives optimum q_i for $n_R > 3$ is discussed in Section IV-D.

A. MISO Channel

Consider a MISO channel where we have n_T transmit antennas and a single receive antenna. The optimization problem involved in this case is similar to the water-filling problem in information theory, which has the optimal solution

$$q_i = \begin{cases} \frac{1}{v} - \frac{1}{t_i}, & v < t_i, \\ 0, & \text{otherwise}, \end{cases}$$

where the water-level $1/v$ is chosen to satisfy $\sum_{i=1}^{n_T} \max(0, 1/v - 1/t_i) = n_T \eta \beta / 4$.

B. $n_T \times 2$ MIMO Channel

We now consider the case of n_T transmit antennas and $n_R = 2$ receive antennas. The optimum q_i for this case is

$$q_i = \begin{cases} A + \sqrt{K}, & v < t_i(r_1 + r_2); \\ 0, & \text{otherwise}, \end{cases} \quad (11)$$

where v is chosen to satisfy $\sum_{i=1}^{n_T} \max(0, A + \sqrt{K}) = n_T \eta \beta / 4$ with $A = [2r_1 r_2 t_i^2 - v t_i(r_1 + r_2)] / 2v r_1 r_2 t_i^2$ and $K = [v^2 t_i^2 (r_1 - r_2)^2 + 4r_1^2 r_2^2 t_i^4] / 2v r_1 r_2 t_i^2$.

C. $n_T \times 3$ MIMO Channel

For the case of n_T transmit antennas and $n_R = 3$ receive antennas, the optimum q_i is given by

$$q_i = \begin{cases} -\frac{a_2}{3a_3} + S + T, & v < t_i(r_1 + r_2 + r_3); \\ 0, & \text{otherwise}, \end{cases} \quad (12)$$

where v is chosen to satisfy $\sum_{i=1}^{n_T} \max(0, -a_2/3a_3 + S + T) = n_T \eta \beta / 4$ with

$$\begin{aligned} S + T &= \left[R + \sqrt{Q^3 + R^2} \right]^{\frac{1}{3}} + \left[R - \sqrt{Q^3 + R^2} \right]^{\frac{1}{3}}, \\ Q &= \frac{3a_1 a_3 - a_2^2}{9a_3^2}, \quad R = \frac{9a_1 a_2 a_3 - 27a_0 a_3^2 - 2a_2^3}{54a_3^3}, \end{aligned}$$

$a_3 = v r_1 r_2 r_3 t_i^3$, $a_2 = v t_i^2 (r_1 r_2 + r_1 r_3 + r_2 r_3) - 3r_1 r_2 r_3 t_i^3$, $a_1 = v t_i (r_1 + r_2 + r_3) - 2t_i^2 (r_1 r_2 + r_1 r_3 + r_2 r_3)$ and $a_0 = v - t_i (r_1 + r_2 + r_3)$.

²Proofs of optimum q_i for $n_R = 2$ and 3 receiver antennas will be presented in a later publication.

TABLE I

TRANSMIT ANTENNA CONFIGURATION DETAILS.

Antenna Configuration	Tx aperture radius	Num. of modes	$\text{rank}(\mathbf{J}_T \mathbf{J}_T^\dagger)$
2-Tx	0.1λ	3	2
3-Tx UCA	0.115λ	3	3
3-Tx ULA	0.2λ	5	3
4-Tx UCA	0.142λ	5	4
4-Tx ULA	0.3λ	7	4

D. A Generalized Method

We now discuss a method which allows to find optimum solution to (8) for a system with n_T transmit and n_R receive antennas. The complementary slackness condition $\lambda_i q_i = 0$ for $i = 1, 2, \dots, n_T$ states that λ_i is zero unless the i -th inequality constraint is active at the optimum. Thus, from (10a) we have two cases: (i) $q_i = 0$ for $v > t_i \sum_{j=1}^{n_R} r_j$, (ii) $v = \sum_{j=1}^{n_R} r_j t_i / (1 + r_j t_i q_i)$ for $q_i > 0$ [11, page 243]. For the later case, the optimum q_i is found by evaluating the roots of n_R -th order polynomial in q_i , where the polynomial is obtained from $v = \sum_{j=1}^{n_R} r_j t_i / (1 + r_j t_i q_i)$. Since the objective function of the optimization problem (8) is convex for $q > 0$, there exist at least one positive root to the n_R -th order polynomial for $v < t_i \sum_{j=1}^{n_R} r_j$. In the case of multiple positive roots, the optimum q_i is the one which gives the minimum to the objective function of (8). In both cases, v is chosen to satisfy the power constraint $\mathbf{1}^T \mathbf{q} = n_T \eta \beta / 4$.

V. SIMULATION RESULTS

In practise, insufficient antenna spacing may cause individual antennas in an antenna array to be correlated, which leads to performance loss from orthogonal space-time block coded system. In this section, we will illustrate the performance improvements of O-STBCs using the spatial precoder \mathbf{F} derived in Section-IV. In particular, the performance is evaluated for small antenna separations and different antenna configurations in a rich scattering environment. In our simulations we use the rate 1 O-STBC code for $n_T = 2$ and rate 3/4 O-STBC code for $n_T = 3, 4$ [2]. The modulation scheme used is 4-PSK.

A. MISO Channels

First we illustrate the water-filling concept for $n_T = 2, 3$ and 4 transmit antennas, where the transmit antennas are placed in uniform circular array (UCA) and uniform linear array (ULA) configurations³ with 0.2λ minimum separation between two adjacent antenna elements. For each transmit antenna configuration we consider, Table-I lists the radius of the transmit aperture, number of effective communication modes⁴[7] in the transmit region and the rank of the transmit side spatial correlation matrix $\mathbf{J}_T \mathbf{J}_T^\dagger$. Note that, in each case, $\mathbf{J}_T \mathbf{J}_T^\dagger$ is non-singular since \mathbf{J}_T is full rank.

Fig. 1 shows the water levels for various SNRs. For a given SNR, the optimal power value q_i is the difference between

³This precoder can be applied to any arbitrary antenna configuration.

⁴The set of modes form a basis of functions for representing a multipath wave field.

water-level $1/v$ and base level $1/t_i$, whenever the difference is positive; it is zero otherwise. Note that, with this spatial precoder, the diversity order of the system is determined by the number of non-zero q_i 's. It is observed that at low SNRs, only one q_i is non-zero for $n_T = 2$ and 3-UCA cases. In these cases, all the available power is assigned to the highest eigen-mode of $\mathbf{J}_T \mathbf{J}_T^\dagger$ (or to the single dominant eigen-channel of \mathbf{H}) and the system is operating in eigen-beamforming mode. With other cases, Fig. 1(c), (d) and (e), systems are operating in between eigen-beam forming and full diversity for small SNRs as well as moderate SNRs. In these cases, the spatial precoder assigns more power to the higher eigen-modes of $\mathbf{J}_T \mathbf{J}_T^\dagger$ (or to dominant eigen-channels of \mathbf{H}) and less power to the weaker eigen-modes (or to less dominant eigen-channels of \mathbf{H}).

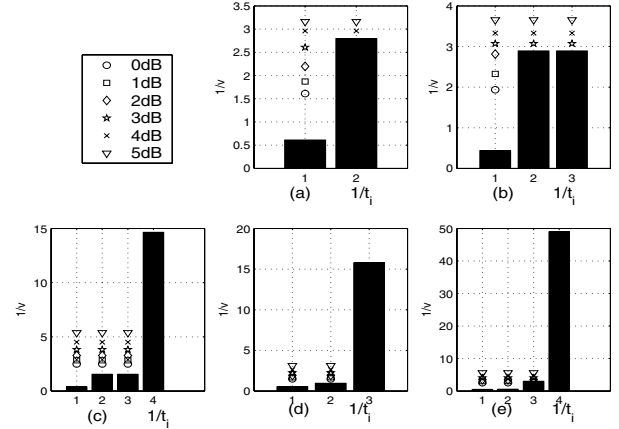


Fig. 1. Water level ($1/v$) for various SNRs for a MISO system. (a) $n_T = 2$, (b) $n_T = 3$ - UCA, (c) $n_T = 4$ - UCA, (d) $n_T = 3$ - ULA and (e) $n_T = 4$ - ULA for 0.2λ minimum separation between two adjacent transmit antennas.

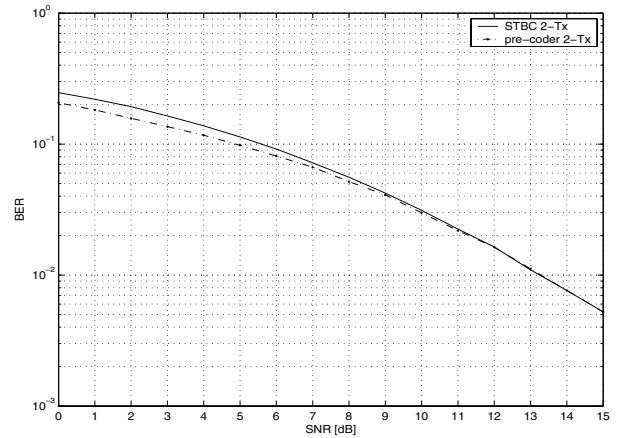


Fig. 2. Performance of spatial precoder with 2-Tx antennas and 1-Rx antenna for 0.2λ separation between two transmit antenna elements.

Fig. 2 illustrates the BER performance of the rate 1 O-STBC with and without spatial precoder for $n_T = 2$. It can be observed that at very low SNRs, we obtain a pre-coding gain of about 1.5dB. In fact, at very low SNRs, the optimum scheme is equivalent to eigen-beam forming. However, as the SNR increases, the precoder becomes redundant and the

optimum scheme approaches O-STBC, where it operates in full diversity.

BER performance results for 3-Tx UCA, ULA and 4-Tx UCA, ULA antenna configurations are shown in Fig. 3(a) and 3(b), respectively for rate 3/4 O-STBCs. For 3-Tx UCA, the results obtained are similar to the results of $n_T = 2$ case above. In this case, at low SNRs, the system operates in eigen beam-forming mode and at high SNRs, it is operating in full diversity mode as shown in Fig. 1(b). For the other three cases, it is observed that the optimum scheme provides a clear performance advantage over the O-STBC only system for all SNRs concerned. For example, at 0.01 bit-error-rate, we obtain a precoding gain of about 1dB. However, these systems operate in between eigen beam-forming and full diversity as the precoder assigns zero powers to some of the transmit diversity branches of the channel. As before, at higher SNRs, the system operates in full diversity and the optimum scheme approaches O-STBC.

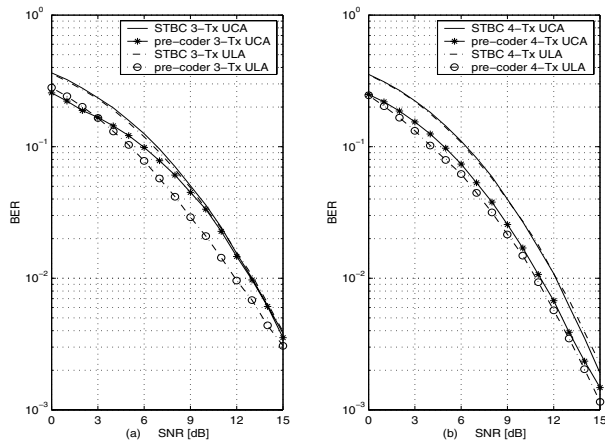


Fig. 3. Performance of spatial precoder with n_T -Tx antennas and 1-Rx antenna for 0.2λ minimum separation between two adjacent Tx antennas for UCA and ULA antenna configurations, (a) $n_T = 3$ and (b) $n_T = 4$.

B. MIMO Channels

We now examine the performance of the spatial precoder for multiple transmit and multiple receive antennas. For example, we consider $n_T = 2, 3$ transmit antennas and $n_R = 2$ receive antennas. In all cases, two receiver antennas are placed λ apart, which gives minimum effect on the performance due to antenna spacing. As before, the minimum separation between two adjacent transmit antennas is set to 0.2λ . Note that this situation reasonably models the uplink of a mobile communication system. For each case, the optimum q_i is calculated using (11). Fig. 4(a) illustrates the BER performance results for 2-transmit, 2-receive antennas for rate 1 O-STBC and Fig. 4(b) illustrates the BER performance results for 3-transmit, 2-receive antennas for rate 3/4 O-STBC. Performance results obtained here are similar to that of MISO cases above.

In practise, wireless channel experience scattering channel correlations both at the transmitter and the receiver antenna arrays. We observed (simulation results are not presented here) that the spatial precoder derived in this paper provides

significant precoding gain in the presence of transmit side scattering channel correlation.

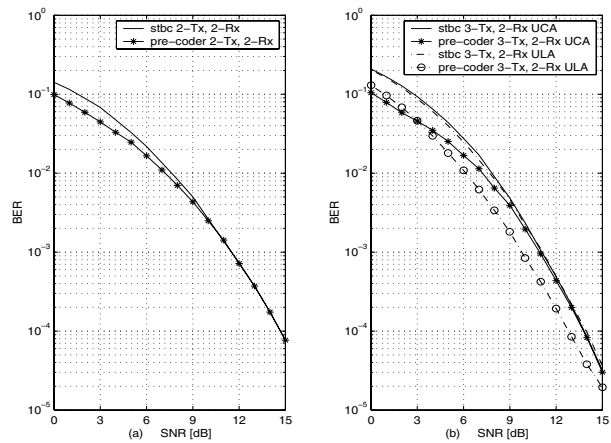


Fig. 4. Performance of spatial precoder with n_T -Tx antennas and 2-Rx antennas: Rx antenna separation λ and minimum Tx antenna separation 0.2λ , (a) $n_T = 2$ and (b) $n_T = 3$ for UCA and ULA antenna configurations.

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