Bounds on Mutual Information of Rayleigh Fading Channels with Gaussian Input

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***Abstract*— The mutual information of a discrete time Rayleigh fading channel is considered, where neither the transmitter nor the receiver has the knowledge of the channel state information. We specifically derive a lower bound for the mutual information of this channel when the input distribution is Gaussian. The bound is expressed in terms of the capacity of the corresponding non fading channel and the capacity when the perfect channel state information is known at the receiver.**

***Index Terms*— Differential Entropy, Channel Capacity, Mutual Information, Rayleigh Fading, Gaussian Distribution.**

I. INTRODUCTION

An independent and identically distributed (iid) Gaussian is the capacity achieving input distribution for additive white Gaussian noise (non-fading) channel, a Rayleigh fading channel when the Channel State Information (CSI) is perfectly known at the receiver, and when the CSI is known to both the transmitter and the receiver.

However, when CSI is not known by neither the transmitter nor the receiver, the capacity achieving distribution is not Gaussian [1]. Therefore, it is of practical interest to find the achievable information rate of Rayleigh fading channels for Gaussian distributed input.

Fading channels have been studied in depth and a multitude of literature is available on the upper and lower achievable rates over the wireless media; refer [2], [3] for a summary. The results presented under various channel models applying constraints for mathematical representations, and the availability of (CSI) at the transmitter and receiver.

The capacity of fading channels when the CSI is perfectly known at the receiver was investigated initially by Ericson [4], later by Lee [5], and Ozarow, Smamai and Wyner [6]. This capacity is calculated in an average sense due to the time varying nature of the signal to noise ratio (SNR). The fading channel with CSI at the receiver alone and at both the transmitter and the receiver was extensively studied in [7], [8].

The iid Rayleigh fading channel with no CSI was studied by Faycal [1], [9], where it was shown that the capacity achieving distribution is discrete with finite number of mass points with new emerging points as SNR increases. These mass point distribution tends to be uniform as SNR approaches infinity, deviating much form that of a Gaussian. The non coherent time selective Rayleigh fading channel has been further investigated by Yingbin and Venugopal [10] and derived upper and lower bounds on the capacity at high SNR. In this paper, we determine how the Gaussian input distribution can contribute in non coherent Rayleigh fading channel. We achieve this by drawing a lower bound on mutual information and identifying the maximum deviation of the actual capacity achieved with a discrete input in the presence of Gaussian input.

This paper is organized follows. Section II describes the system model for non coherent Rayleigh fading channel. Section III formulates the mutual information using the input and output probability distributions. The choice of Gaussian input is described in IV with detailed analysis of differential entropies at the channel output leading to the analytical lower bound of the mutual information. The numerical results are presented in brief in section V. The conclusions are given in section VI.

II. SYSTEM MODEL

Consider the Rayleigh fading channel,

$$y = ax + n \tag{1}$$

where y is the complex channel output, x is the complex channel input, a and n represent the fading and noise components associated with the channel. Note that the time index is omitted for simplicity. It is assumed that a and n are independent zero mean circular complex Gaussian random variables. Also assume that $\sigma_a^2/2$ and $\sigma_n^2/2$ are the equal variance of real and imaginary parts of the complex variables a and n respectively. The random variables a, x, and n are considered to be independent of each other. The input $x \,\epsilon\, X$ is average power limited: $\mathrm{E}[|X|^2] = \sigma_x^2 \leq P$. Neither the receiver nor the transmitter has the knowledge of channel state information.

III. THE MUTUAL INFORMATION

The Mutual information between the input and output of a Rayleigh fading channel can be expressed as [11]

$$I(X;Y) = \int_0^\infty \int_0^\infty p_{Y|X}(y|x)p_X(x)\log \frac{p_{Y|X}(y|x)}{\int_0^\infty p_{Y|V}(y|v)p_V(v)dv} dxdy \tag{2}$$

considering the probability distribution of the magnitudes of the input and output random variables X and Y. It should be noted here that since we only consider the distribution of magnitudes of the random variables, the integral in (2) is taken

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from 0 to ∞. The conditional probability density function $p_{Y|X}(y/x)$ [9] [11] is given by

$$p_{Y|X}(y|x) = \frac{2|y|}{\sigma_n^2 + \sigma_a^2|x|^2} e^{\frac{-|y|^2}{\sigma_n^2+\sigma_a^2|x|^2}}. \tag{3}$$

Assume the average mean squared power of both fading A, ($a \in A$) and noise N, ($n \in N$) are unity. This assumption is valid since the effective received power at the receiver is the combination of both σ_a^2 and σ_x^2 and the SNR is the ratio between the average received power and the average noise power. Therefore, the same output exists for various σ_a^2 and σ_n^2 on the appropriate selection of σ_x^2. With this assumption, (3) can be written as

$$p_{Y|X}(y|x) = \frac{2y}{1+x^2} e^{\frac{-y^2}{1+x^2}}. \tag{4}$$

Without loss of generality, the magnitude sign is removed in (4) and the same notation will be used throughout the rest of this paper.

It can be shown [11] that, (2) can be simplified to the following:

$$\begin{aligned} I(X;Y) = &-\int_0^\infty p_Y(y)\log_e p_Y(y)dy \\ &- \frac{1}{2}\int_0^\infty p_X(x)\log_e(1+x^2)dx + \log_e 2 - (1+\frac{\gamma}{2}). \end{aligned} \tag{5}$$

This was originally proven by Taricco [11] to calculate the channel capacity analytically using Lagrange optimization method applying an additional constraint. The result in here is expressed in nats and $\gamma = 0.5772...$ is the Euler's constant. We separate the terms in (5) as difference between the output entropy and it's conditional entropy on the input as, $I(X;Y) = h(Y) - h(Y|X)$. Therefore we represent

$$h(Y) = -\int_0^\infty p_Y(y)\log_e p_Y(y)dy, \tag{6}$$

and

$$h(Y|X) = \frac{1}{2}\int_0^\infty p_X(x)\log_e(1+x^2)dx - \log 2 + (1+\frac{\gamma}{2}). \tag{7}$$

IV. GAUSSIAN INPUT IN NON COHERENT RAYLEIGH FADING

Recall the channel model (1), and assume the input distribution is Gaussian. Then the distribution of both the real and imaginary parts of x are independent and Gaussian. Therefore, the distribution of the $|x|$ is Rayleigh with the probability density function [12]

$$p_X(x) = \frac{2x}{\sigma_x^2} e^{\frac{-x^2}{\sigma_x^2}},\ x \geq 0. \tag{8}$$

It is assumed that both the real and imaginary parts of input have equal variance ${\sigma_x}^2/2$. The magnitude sign is omitted in (8) as mentioned in the previous section.

A. Output Conditional Entropy

Having described the input distribution $p_X(x)$ for non coherent Gaussian input channel, let's look in to the output conditional entropy $h(Y|X)$ in (7). By substituting (8) in (7) we have

$$h(Y|X) = \int_0^\infty \left[\frac{x}{\sigma_x^2} e^{\frac{-x^2}{\sigma_x^2}} \log_e(1+x^2)\right] dx - \log_e 2 + (1+\frac{\gamma}{2}). \tag{9}$$

With the detailed proof provided in Appendix A, we can reduce (9) to

$$h(Y|X) = \frac{-e^{\frac{1}{\sigma_x^2}}}{2}\mathrm{Ei}\left(\frac{-1}{\sigma_x^2}\right) - \log_e 2 + (1+\frac{\gamma}{2}), \tag{10}$$

where the exponential integral $\mathrm{Ei}(x) = -\int_{-x}^\infty e^{-t}/t\,dt$. The channel capacity when the CSI is perfectly known at the receiver is [2], [4], [5],

$$C_{\text{rcsi}} = -e^{\frac{1}{snr}}\mathrm{Ei}\left(\frac{-1}{snr}\right), \tag{11}$$

where $snr = \sigma_x^2$ since $\sigma_n^2 = 1$. Therefore, $h(Y|X)$ in non coherent Rayleigh fading with Gaussian input can be expressed as

$$h(Y|X) = \frac{1}{2}C_{\text{rcsi}} - \log_e 2 + (1+\frac{\gamma}{2}). \tag{12}$$

B. Output Entropy

With $|X|$ Rayleigh, as provided in (4), the output probability density function $p_Y(y) = \int_0^\infty p_X(x)p_{Y|X}(y|x)dx$ can be written as

$$p_Y(y) = \int_0^\infty \frac{2x}{\sigma_x^2} e^{\frac{-x^2}{\sigma_x^2}} \frac{2y}{1+x^2} e^{\frac{-y^2}{1+x^2}} dx. \tag{13}$$

We substitute (13) in (6) to get

$$\begin{aligned} h(Y) = &-\int_0^\infty \left[\int_0^\infty \frac{2x}{\sigma_x^2} e^{\frac{-x^2}{\sigma_x^2}} \frac{2y}{1+x^2} e^{\frac{-y^2}{1+x^2}} dx\right] \\ &\times \log_e \left[\int_0^\infty \frac{2x}{\sigma_x^2} e^{\frac{-x^2}{\sigma_x^2}} \frac{2y}{1+x^2} e^{\frac{-y^2}{1+x^2}} dx\right] dy. \end{aligned} \tag{14}$$

To the best of our knowledge, this integral can not be evaluated analytically $\forall\ \sigma_x^2$. The mutual information calculated numerically using Hermit polynomials and Simpson Rules are plotted in Fig 1.

We will derive an analytical lower bound for (2) to observe the effect on the channel with Gaussian input.

C. Lower Bound on Mutual Information

We have the following result.

Proposition 4.3.1: The mutual information of an iid non coherent Rayleigh fading channel when the input distribution is complex Gaussian, is lower bounded by

$$I(X;Y) \geq \frac{1}{2}(C_{\text{cnf}} - C_{\text{rcsi}}) \tag{15}$$

where C_{cnf} and C_{rcsi} are the capacity of the non fading complex Gaussian channel and the capacity of the Rayleigh fading channel when the CSI is perfectly known at the

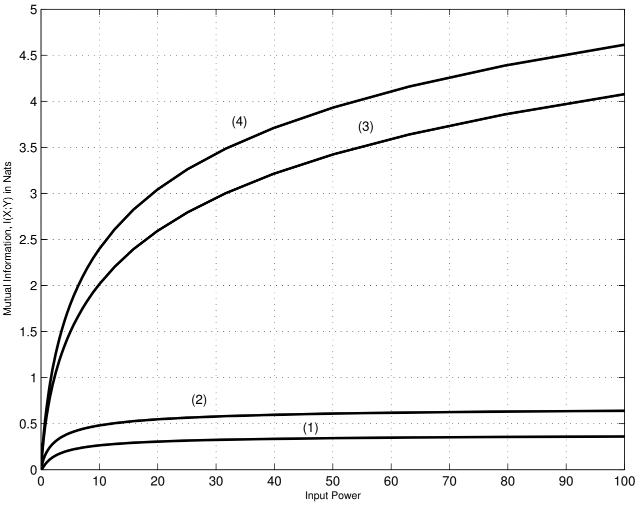

Fig. 1. The mutual information with Gaussian input is below the channel capacity achieved by a discrete input. (1) The mutual information with gaussian input. (2) The channel capacity with a discrete input. (3) The channel capacity, C_{rcsi} with CSI. (4) The non fading channel capacity, C_{cnf}.

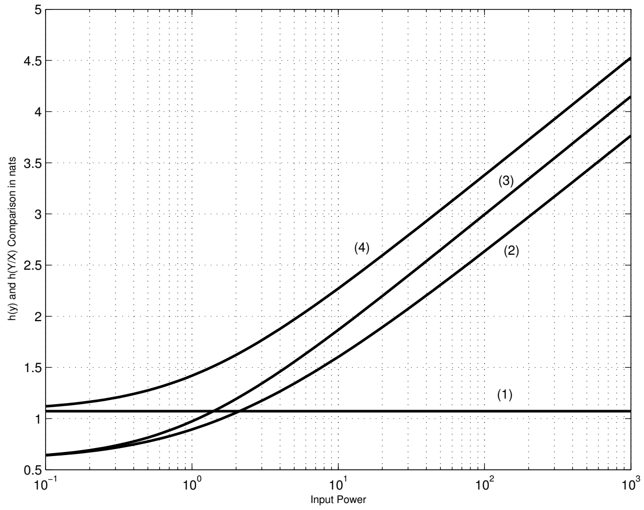

Fig. 2. Entropy Comparison with Fading and Non fading Models: (1) Output conditional entropy of non fading channel, $h_{\text{nf}}(Y)$. (2) Output conditional entropy of fading channel with Gaussian input. (3) Output entropy of fading channel, $h(Y)$. (4) Output entropy of non fading channel, $h(Y)_{\text{nf}}$. The $h(Y)_{\text{nf}}$ and $h(Y)$ treated in this paper has a similar shape and the difference is decreasing with SNR.

receiver. The equality holds when the average input power is zero.

Proof: We consider $I(X;Y)$, $h(Y)$, and $h(Y|X)$ when the input power (σ_x^2) is zero. Using (10), we get $h(Y|X)_{\sigma_x^2=0} = -\log_e 2 + (1+\frac{\gamma}{2})$. Since the mutual information is zero with no channel input, we can write

$$h(Y)_{\sigma_x^2=0} = h(Y|X)_{\sigma_x^2=0}. \tag{16}$$

The quantity $h(Y)$ in (14) is monotonically increasing with SNR, thus it has the minimum

$$h(Y)_{\text{min}} = -\log_e 2 + (1+\frac{\gamma}{2}). \tag{17}$$

Consider, a non fading channel whose capacity achieving distribution is Gaussian, where $h(Y|X)_{\text{nf}} = h(N) = \frac{1}{2}\log(\pi e \sigma_n^2)$ is constant over input power. The monotonic increase of $h(Y)_{\text{nf}} = \frac{1}{2}\log(\pi e(\sigma_n^2 + \sigma_x^2))$ with SNR results in significant increase in channel capacity.

However, in the presence of fading, $h(Y|X)$ is not a constant anymore and we investigate the effect on the mutual information of this Rayleigh fading channel by comparing output entropies in two cases. Here we only consider a one half of the full complex domain in the non fading model. Figure 2 portrays $h(Y)$ and $h(Y|X)$ of the two channel models, where

$$h(Y)_{\text{nf}} = \frac{1}{2}\log_e(\pi e(1+\sigma_x^2)) \tag{18}$$

and

$$h(Y|X)_{\text{nf}} = \frac{1}{2}\log_e(\pi e). \tag{19}$$

Note that the abbreviation "nf" refers the Gaussian channel with no fading present.

Since the Gaussian distributions are the entropy maximisers with the input power constraint,

$$h(Y)_{\text{nf}} > h(Y) \ \forall \ \sigma_x^2. \tag{20}$$

Lets define the difference in (20)

$$G = h(Y)_{\text{nf}} - h(Y), \tag{21}$$

and investigate the bounds when $\sigma_x^2 = 0$ and $\sigma_x^2 \to \infty$. The $G_{\sigma_x^2=0}$ can be written as

$$\begin{aligned} G_{\sigma_x^2=0} &= h(Y)_{nf,\sigma_x^2=0} - h(Y)_{\text{min}} \\ &= \frac{1}{2}\log_e(\pi e) + \log_e 2 - (1+\frac{\gamma}{2}). \end{aligned} \tag{22}$$

To calculate the difference when $\sigma_x^2 \to \infty$, we will use the upper bound

$$\lim_{\sigma_x^2 \to \infty} I(X;Y) \le \gamma \tag{23}$$

given in [13]. Using (12) and (14), we can write the mutual information of the channel as

$$I(X;Y) = h(Y) - \frac{1}{2}C_{\text{rcsi}} + \log_e 2 - (1+\frac{\gamma}{2}). \tag{24}$$

Substituting (23) and (24) in (21), we get,

$$\begin{aligned} G_{\sigma_x^2\to\infty} \ge{}& \frac{1}{2}\lim_{\sigma_x^2 \to \infty}\left[\log_e(\pi e(1+\sigma_x^2)) + e^{\frac{1}{\sigma_x^2}}\,\text{Ei}\left(\frac{-1}{\sigma_x^2}\right)\right] \\ &- \gamma + \log_e 2 - (1+\frac{\gamma}{2}) \\ ={}& L - \gamma + \log_e 2 - (1+\frac{\gamma}{2}), \end{aligned} \tag{25}$$

where $L = \frac{1}{2}(\gamma + \log_e(\pi e))$. Refer the Appendix B for the detailed proof. Therefore we can write (25) as,

$$G_{\sigma_x^2\to\infty} \ge \log_e(2\sqrt{\pi e}) - (1+\gamma). \tag{26}$$

Note that $G_{\sigma_x^2=0} > G_{\sigma_x^2\to\infty}$. Also the differential entropies defined here are monotonic and concave. Therefore we conclude that the maximum difference occurs at $\sigma_x^2 = 0$. This $G_{\text{max}} = G_{\sigma_x^2=0}$ can be used to lower bound $h(Y)$ in (14) and we get

$$h(Y) \ge h(Y)_{\text{nf}} - G_{\text{max}}. \tag{27}$$

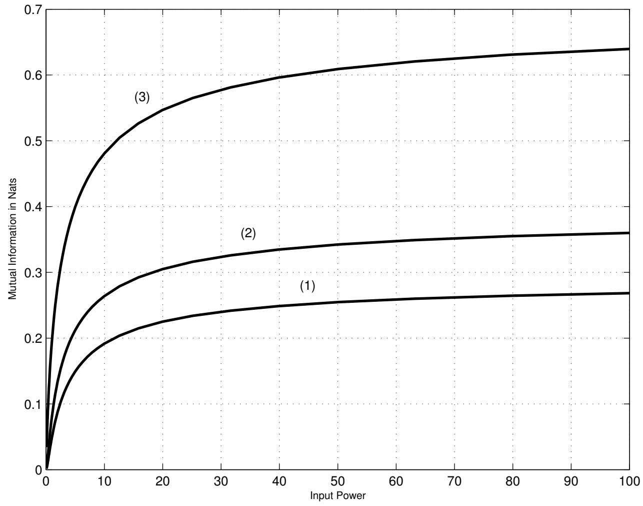

Fig. 3. Analysis of the lower bound in fading channel: (1) The lower bound with Gaussian input. (2) Numerical results of mutual information with Gaussian input. (3) The channel capacity achieved with a discrete input.

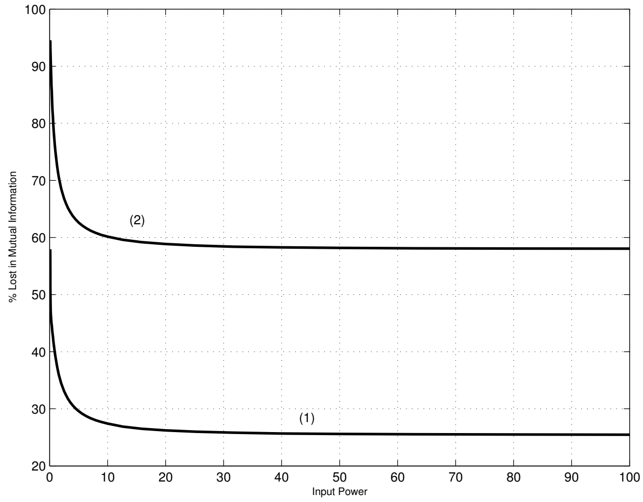

Fig. 4. Percentage lost using the lower bound: (1) With the mutual information found numerically for Gaussian input. (2) With the channel capacity.

Therefore, the mutual information in (24) can be lower bounded as

$$\begin{aligned} I(X;Y) &\geq h(Y)_{\text{nf}} - G_{\max} - \left[\frac{1}{2}C_{\text{rcsi}} - \log_e 2 + (1+\frac{\gamma}{2})\right] \\ &= h(Y)_{\text{nf}} - \frac{1}{2}\log_e(\pi e) - \frac{1}{2}C_{\text{rcsi}} \\ &= \frac{1}{2}\log_e\left(1+\sigma_x^2\right) - \frac{1}{2}C_{\text{rcsi}}, \end{aligned} \tag{28}$$

using (12), (18), and (22). With $C_{\text{cnf}} = \log_e\left(1+\sigma_x^2\right)$, we prove (15).

It should be noted here that (15) asymptotically converges to $\gamma/2$ since $\lim_{\sigma_x^2 \to \infty}(C_{\text{cnf}} - C_{\text{rcsi}}) = \gamma$ [13].

V. NUMERICAL RESULTS

We compare the new lower bound with the mutual information found numerically in section IV.

The lower bound in (15) is plotted against the input power in Fig 3 with the actual mutual information calculated numerically. Also, it has been compared with the capacity of this channel that is achieved by a discrete input [1]. The channel capacity is plotted for comparison only with two discrete mass points one located at the origin since the probability of other mass points are small at low SNR and even suited for a simple comparison at high SNR due to the percentage increase in capacity is low [9]. The percentage lost in mutual information with a Gaussian input on our lower bound is plotted in Fig 4. It is 30 $\%$ less than that of numerical values.

A. *Upper Bound*

Having found $h(Y|X)$ analytically in (10), it is easy to draw an upper bound since $h(Y)$ in (14) is maximised with a Gaussian distribution of $|Y|$. Therefore, we get

$$\begin{aligned} I(X;Y) &= h(Y) - \frac{1}{2}C_{\text{rcsi}} + \log_e 2 - (1+\frac{\gamma}{2}) \\ &\leq h_{\text{Gaussian}}(Y) - \frac{1}{2}C_{\text{rcsi}} + \log_e 2 - (1+\frac{\gamma}{2}) \\ &= \log_e(\pi e(1+\sigma_x^2)) - \frac{1}{2}C_{\text{rcsi}} + \log_e 2 - (1+\frac{\gamma}{2}). \end{aligned} \tag{29}$$

However, this upper bound is not tight since it is very high at low SNR. But it converges as SNR increases leading to a useful bound. Also this upper bound deviates a lot from the actual mutual information where the correct asymptotic value when SNR approaches infinity is given in (23). The asymptotic bound provided in [13] demonstrates that the mutual information corresponding to a Gaussian input is bounded in the SNR and the result does not depend heavily on the Rayleigh fading assumption.

VI. CONCLUSIONS

The mutual information of a non-coherent Rayleigh fading channel for a Gaussian input can be lower bounded as the difference between the capacities of non fading channel and the Rayleigh fading channel when the perfect channel state information is known at the receiver. Even the Gaussian input is not optimal, our result shows the minimum achievable information rate which can be used as the worse case scenario in non coherent Rayleigh fading channels. The lower bound found is never lower than 70 $\%$ of the actual.

VII. APPENDIX

A. *PROOF OF CONDITIONAL ENTROPY IN* (10)

We write (9) as

$$h(Y|X) = E_1 - \log_e 2 + (1+\frac{\gamma}{2}) \tag{30}$$

where

$$E_1 = \lim_{k_1 \to \infty} \int_0^{k_1} \frac{x}{k_2} e^{\frac{-x^2}{k_2}} \log_e(1+x^2)dx, \; x \geq 0 \tag{31}$$

and $k_2 = \sigma_x^2$.

Consider the integral part of (31). Using integration by parts, we get

$$\int_0^{k_1} \frac{x}{k_2} e^{\frac{-x^2}{k_2}} \log(1+x^2)dx = \left[-\frac{1}{2} e^{\frac{-x^2}{k_2}} \log\left(1+x^2\right)\right]_0^{k_1} + \int_0^{k_1} e^{\frac{-x^2}{k_2}} \frac{x}{(1+x^2)}. \quad (32)$$

Substituting $t = 1+x^2$, the second term of (32) can be written as

$$\int_0^{k_1} e^{\frac{-x^2}{k_2}} \frac{x}{(1+x^2)} = \frac{1}{2} e^{\frac{1}{k_2}} \int_1^{1+{k_1}^2} \frac{e^{\frac{-t}{k_2}}}{t} dt. \quad (33)$$

Substituting $u = t/k_2$ in the right hand side of (33) we get

$$\begin{aligned}\int_0^{k_1} e^{\frac{-x^2}{k_2}} \frac{x}{(1+x^2)} &= \frac{1}{2} e^{\frac{1}{k_2}} \int_{\frac{1}{k_2}}^{\frac{1+{k_1}^2}{k_2}} \frac{e^{-u}}{u} du \\ &= \frac{1}{2} e^{\frac{1}{k_2}} \left[\int_{\frac{1}{k_2}}^{\infty} \frac{e^{-u}}{u} du - \int_{\frac{1+{k_1}^2}{k_2}}^{\infty} \frac{e^{-u}}{u} du\right] \\ &= \frac{1}{2} e^{\frac{1}{k_2}} \left[\mathrm{Ei}\left[-(\frac{1+x^2}{k_2})\right]\right]_0^{k_1}. \quad (34)\end{aligned}$$

Using this identity in (32) we get

$$\int_0^{k_1} \frac{x}{k_2} e^{\frac{-x^2}{k_2}} \log(1+x^2)dx = \frac{1}{2} e^{\frac{1}{k_2}} \left[\mathrm{Ei}\left[-(\frac{1+x^2}{k_2})\right]\right]_0^{k_1} - \left[\frac{1}{2} e^{\frac{-x^2}{k_2}} \log\left(1+x^2\right)\right]_0^{k_1}. \quad (35)$$

Now we can write (31) as

$$E1 = \lim_{k_1 \to \infty} \frac{1}{2} \left[e^{\frac{1}{k_2}} \mathrm{Ei}\left[-(\frac{1+{k_1}^2}{k_2})\right] - e^{\frac{-{k_1}^2}{k_2}} \log_e\left(1+{k_1}^2\right)\right] - \frac{1}{2} e^{\frac{1}{k_2}} \mathrm{Ei}\left[-\frac{1}{k_2}\right]. \quad (36)$$

By applying La'Hospital's Rule, it can be shown that

$$\lim_{k_1 \to \infty} \frac{1}{2} e^{\frac{-{k_1}^2}{k_2}} \log_e\left(1+{k_1}^2\right) = 0. \quad (37)$$

Also note that $\mathrm{Ei}(-\infty) = 0$ [14], thus

$$E_1 = -\frac{1}{2} e^{\frac{1}{k_2}} \mathrm{Ei}\left[-\frac{1}{k_2}\right]. \quad (38)$$

By substituting (38) in (30) completes the proof.

B. PROOF OF THE ASYMPTOTIC ANALYSIS USED IN (25)

Let's define $\xi = \sigma_x^2$ and we write the asymptotic value in (25) as,

$$L = \frac{1}{2} \lim_{\xi \to \infty} \left[\log_e(\pi e(1+\xi)) + e^{\frac{1}{\xi}} \mathrm{Ei}\left(\frac{-1}{\xi}\right)\right] \quad (39)$$

where the exponential integral can be expressed as, [15]

$$\mathrm{Ei}(-x) = \gamma + e^{-x} \log x + \int_0^x e^{-t} \log t dt. \quad (40)$$

Using this identity we get,

$$\begin{aligned}L &= \frac{1}{2} \lim_{\xi \to \infty} \left[\log_e(\pi e(1+\xi)) + e^{\frac{1}{\xi}} \left(\gamma + e^{\frac{-1}{\xi}} \log \frac{1}{\xi}\right)\right] \\ &+ \frac{1}{2} \lim_{\xi \to \infty} e^{\frac{1}{\xi}} (\int_0^{e^{\frac{1}{\xi}}} e^{-t} \log t dt) \\ &= \frac{1}{2} \lim_{\xi \to \infty} \left[\log_e(\pi e(1+\frac{1}{\xi})) + \gamma e^{\frac{1}{\xi}}\right] + 0 \\ &= \frac{1}{2}(\gamma + \log_e(\pi e)) \quad (41)\end{aligned}$$

which competes the proof.

C. DIFFERENCE BETWEEN FADING AND NON FADING ENTROPIES

In order to lower bound our results, we have shown that the difference between $h(Y)_{nf}$ and $h(Y)$ in (14) is monotonically decreasing based on our asymptotic analysis with the property of concavity in differential entropies. We will try to further investigate the $G(\sigma_x^2)$ taking the derivative of it and verifying the negativity $\forall\ \sigma_x^2$. Lets assume $P = \sigma_x^2$ for simplicity, we get

$$G(P) = \frac{1}{2} \log[\pi e(\sigma_n^2 + P)] - h(Y). \quad (42)$$

Differentiating with respect to P we get

$$\frac{\partial G(P)}{\partial P} = \frac{1}{2(P+\sigma_n^2)} + \int_0^{\infty} \left[1 + \log\left[\int_0^{\infty} \frac{2xK}{P} e^{\frac{-x^2}{P}} dx\right]\right] (M_{P,K}) dy. \quad (43)$$

Where

$$(M_{P,K}) = \int_0^{\infty} \frac{2xK}{P^2} e^{\frac{-x^2}{P}} \left[\frac{x^2}{P} - 1\right] dx \quad (44)$$

and

$$K = \frac{2y}{(1+x^2)} e^{\frac{-y^2}{(1+x^2)}} \quad (45)$$

The derivative in (43) is not possible to solve analytically and only numerical evidence shows that $\frac{\partial G(P)}{\partial P} < 0\ \forall\ P$. Therefore, this results has been used in lower bounding the mutual information in (28).

VIII. ACKNOWLEDGEMENTS

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