Reliability Based Soft Transition Technique for Dual-Mode Blind Equalizers

Wee Gin Lim, Rodney A. Kennedy, and Thushara D. Abhayapala
Department of Telecommunication Engineering,
Australian National University
Canberra, ACT, 0200, Australia
[WeeGin.Lim, Rodney.Kennedy, Thushara.Abhayapala]@anu.edu.au

***Abstract*— In this paper, we propose a new technique that facilitates soft transition between a startup algorithm and a decision directed (DD) algorithm in blind adaptive equalizers. The algorithm-pair is combined using a reliability measure that is proportional to an estimate of the probability of the equalizer detecting a correct symbol. This measure takes into account both the equalized signal and its statistical distribution. The main feature of the technique is in the smooth and automatic switching from the startup algorithm to the DD algorithm and vice versa depending on the value of the reliability measure. This technique has been compared with the popular Benveniste-Goursat and Stop-And-Go algorithms and is shown to exhibit a faster rate of convergence and lower steady state error.**

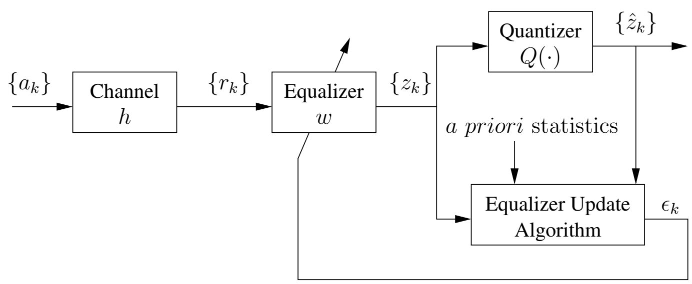

Fig. 1. A typical baseband equivalent channel and a linear equalizer that employs the dual-mode algorithm.

I. Introduction

In blind equalization, acquisition of the channel parameters is usually achieved by employing a robust startup algorithm such as the Sato algorithm [1] and the constant modulus (or Godard) algorithm [2]. When the convergence is achieved, the equalizer enters into the tracking mode and switches to a decision directed (DD) algorithm. In the absence of a training sequence, the "true" mean-squared error (MSE) cannot be computed. Therefore, it is difficult to determine an appropriate condition for switching to occur. Switching too early when there are many errors may result in the ill-convergence of the DD algorithm; switching too late may result in a slow rate of convergence. The problem is how to determine the reliability of the equalized output in the presence of intersymbol interference (ISI) and noise when only the received signal is available.

In the literature, several solutions to this problem have been proposed [3]–[7]. They presented combination techniques for the startup algorithm and the DD algorithm. The pioneers Benveniste *et al* [3] as well as the authors of [6], [7] suggested that the reliability of the equalized signal is related to the proximity of that signal to its nearest constellation point(s). Usually when the equalized signal is close to the constellation points, the DD algorithm will dominate. Conversely, when they are far away from the constellation points, the startup algorithm will dominate. Another approach by Picchi and Prati [4] suggested that it is reliable enough to update the DD algorithm when the signs of the error functions of both the Sato and the DD algorithms agree. Their idea is then extended in [5] to include a third algorithm so that the DD algorithm is only updated when the error functions of the Sato, the Godard and the DD algorithms all agree in sign. The above mentioned algorithms tend to have slow convergence. This is because the combination techniques of the algorithms depend only on the instantaneous equalized output or the signs of error functions, without (explicitly) taking into account the distribution of the *deconvolutional noise*, i.e., the sum of the residual ISI and the receiver noise.

In this paper, we propose a blind adaptive dual-mode algorithm that uses a reliability measure that is an *explicit* function of both the equalized output and its statistical distribution. In contrast to previous approaches, we derive this measure by applying Bayes theorem to obtain the probability of the equalizer correctly detecting a symbol. The resulting algorithm therefore depends on both the equalized output as well as the estimated variance of the deconvolutional noise. The system model is developed in Section 2. The development of the new algorithm is outlined in Section 3. Section 4 shows supportive simulation results.

II. System Model

Consider the combined channel-equalizer system depicted in Fig. 1. Let $\mathbf{h} \triangleq [h_0, h_1, \cdots, h_L]^T$ denote the coefficients of the channel filter of length $L+1$. The channel is assumed stationary, possibly non-minimum phase, but unknown. Let the source data sequence be $\mathbf{a}_k \triangleq [a_k, a_{k-1}, \cdots, a_{k-L}]^T$ with a time index k, drawn from the alphabet set

$$\mathbb{A} \triangleq \{d_1, d_2, \cdots, d_M\} = \{\pm 1, \pm 3, \cdots, \pm (M-1)\} \quad (1)$$

The two second authors are also with National ICT Australia (NICTA).

 0-7803-8533-0/04/$20.00 (c) 2004 IEEE

for M-ary pulse amplitude modulation (PAM) signaling. Then the input signal to the equalizer is

$$r_k = \mathbf{h}^T \mathbf{a}_k + n_k \tag{2}$$

where n_k is the additive noise. Also let $\mathbf{c}_k \triangleq [c_{-N,k}, \cdots, c_{0,k}, \cdots, c_{N,k}]^T$ be the $(2N+1)$ equalizer tap coefficients. We set

$$c_{p,0} = \begin{cases} 1 & p = 0 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

to have a center-tap initialization, which allows the causal development of approximate inverse filters for non-minimum phase systems. Then the equalizer output is given by

$$z_k = \sum_{n=-N}^{N} c_n r_{k-n} \tag{4}$$

$$= s_0 a_k + v_k \tag{5}$$

where

$$v_k = \sum_{j \neq 0, j=-N}^{N+L} s_j a_{k-j} + \sum_{n=-N}^{N} c_n n_{k-n} \tag{6}$$

is the so-called *deconvolutional noise* with a variance of σ_k^2, and $\{s_j\}, j = -N, \cdots, N+L$ is the set of coefficients of the combined channel-equalizer filter. Without loss of generality, the coefficient s_0 is defined as unity. We further assume:

($\mathcal{H}1$) the M-PAM source symbols are identically and independently distributed (i.i.d.);

($\mathcal{H}2$) v_k is white and Gaussian with zero mean and variance σ_k^2;

($\mathcal{H}3$) the impulse response of the combined channel-equalizer filter is absolutely summable, i.e., $\sum_{j=-\infty}^{\infty} |s_j| < \infty$.

While assumption ($\mathcal{H}2$) may not hold well for a wide range of channels, we expect it to hold, due to the central limit theorem [8, ch. 2], for our special case that considers the channel eye to be open. This assumption is further discussed in Section III-C.

III. Development of Algorithm

A. Stochastic Gradient Descent Blind Dual-Mode Algorithms

Blind adaptive equalization algorithms are often designed as stochastic gradient descent schemes to update the parameter vector by minimizing some cost functions that do not involve the use of the original input a_k but reflect the current level of ISI in the equalizer output. Define the mean cost function as

$$J(\mathbf{c}_k) \triangleq \frac{1}{2} \mathrm{E}\{\epsilon^2(z_k)\} \tag{7}$$

where $\epsilon^2(\cdot) : \mathbb{R} \to \mathbb{R}$ is a scalar cost function and z_k is the equalizer output. Denote the parameter vector of the equalizer $\mathbf{c}_k$ as its value at sample instant k and $\mathbf{r}_k = [r_{k-N}, \cdots, r_0, \cdots, r_{k+N+L}]^T$ as the regressor vector of the samples of the channel output. The stochastic gradient descent minimization algorithm is well-known to be

$$\mathbf{c}_{k+1} = \mathbf{c}_k - \mu \frac{\partial}{\partial \mathbf{c}_k} \frac{1}{2} \epsilon^2(z_k) \tag{8}$$

$$= \mathbf{c}_k - \mu \epsilon(z_k) \mathbf{r}_k^T. \tag{9}$$

Hence the blind algorithm can either be defined by the cost function or equivalently through $\epsilon(\cdot)$ which we call the *error function* since it replaces the prediction error in the LMS algorithm. Let $\epsilon_k^{\text{start}}$ and $\epsilon_k^{\text{steady}}$ be the error functions of the blind algorithms of the startup mode and the steady state mode, respectively. A dual-mode algorithm has an error function that combines these two types of error functions. Then a class of dual-mode algorithms can be expressed in the form

$$\epsilon_k = \beta_1 \epsilon_k^{\text{steady}} + \beta_2 \epsilon_k^{\text{start}} \tag{10}$$

where β_1 and β_2 are user defined functions that depend on signals available at the receiver.

B. Novel Dual-Mode Algorithm

We propose a dual-mode algorithm with an error function

$$\epsilon_k = \alpha_k \gamma \epsilon_k^{\text{steady}} + (1 - \alpha_k) \epsilon_k^{\text{start}} \tag{11}$$

which represents a convex combination between $\gamma \epsilon_k^{\text{steady}}$ and $\epsilon_k^{\text{start}}$ with convex parameter α_k, and γ is chosen to compensate for the differences in the variances of the respective error functions where it is sensible to assign $\gamma \triangleq \mathrm{E}\{|\epsilon_k^{\text{start}}|\}/\mathrm{E}\{|\epsilon_k^{\text{steady}}|\}$[1]. The formulation in (11) is largely conventional. The principle design issue is how to determine α_k as a function of signals available at the receiver. In what follows we identify α_k with a measure of reliability[2].

Let P_C be the probability of the output of the quantizer being correct given the output of the equalizer z_k. Then given α_k is a convex parameter we can relate the extreme values of α_k to P_C in the following way:

$$\begin{aligned} \alpha_k = 0, \quad &\text{when} \quad P_C \leq 1 - P_C \\ \alpha_k = 1, \quad &\text{when} \quad P_C = 1, \end{aligned} \tag{12}$$

The lower bound condition is satisfied when the probability of the equalizer detecting an incorrect symbol exceeds that of the correct symbol. Consequently, at high noise levels, $\alpha_k = 0$ almost for all values of z_k. At low noise levels, $\alpha_k = 0$ when z_k is at (or near to) the middle point of two adjacent alphabets. Empirical results often show that a bit-error-rate of less than 10% is often sufficient in allowing the decision directed algorithm to converge to its global minima. Therefore we also set $\alpha_k = 1$ whenever $\sigma_k^2 < \sigma_{\text{thr}}^2$ where σ_{thr}^2 is a suitable threshold.

Subsequently, we need to decide on the relationship between α_k and P_C whenever $0.5 \leq P_C < 1$ and $\sigma_k^2 \geq \sigma_{\text{thr}}^2$. We propose a simple linear relationship between α_k and P_C which is given by $\alpha_k = 2P_C - 1$. Other mappings are possible as long as α_k is a monotonically increasing function of P_C. Thus,

[1] γ is meant to be computed infrequently say at the end of each frame. When we consider the CMA and the LMS algorithm for example, we find that γ is approximately constant for a particular constellation size during the transitional period between startup and steady state, i.e., at the vicinity of an open eye condition. For the 4-PAM source, for example, it can be shown empirically that $\gamma \approx 14$.

[2] The definition of reliability is *the ability of a system or component to perform its required functions under stated conditions for a specified period of time* [9].

our proposed reliability measure can be compactly expressed as

$$\alpha_k = \begin{cases} 1 & \sigma_k^2 < \sigma_{\rm thr}^2 \\ 2P_C - 1 & 0.5 \le P_C < 1 \ \text{ and } \ \sigma_k^2 \ge \sigma_{\rm thr}^2 \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

The probability P_C can be interpreted as the posterior probability of the event of a correctly detected symbol given certain measurements. Such an event can be mathematically expressed as $\{A_{j^*} : \mathrm{Q}(z_k) = d_{j^*} = a_k\}$, where $\mathrm{Q}(\cdot)$ is the quantization operator and j^* is the index of the alphabet symbol that corresponds to a correct quantizer decision. All other incorrect events are $\{A_j : \mathrm{Q}(z_k) = d_j, \forall j \neq j^*\}$. Thus, we define P_C as the posterior conditional probability

$$P_C \triangleq P(A_{j^*}|z_k). \tag{14}$$

C. Computation of Reliability Measure α_k

To compute α_k we need to compute P_C and σ_k^2. The former can be calculated by applying Bayes theorem and the law of total probability:

$$P_C = P(A_{j^*}|z_k) = \frac{p(z_k|A_{j^*})P(A_{j^*})}{p(z_k)} \tag{15}$$

$$= \frac{p(z_k|A_{j^*})P(A_{j^*})}{p(z_k|A_{j^*})P(A_{j^*}) + \sum_{j \neq j^*} p(z_k|A_j)P(A_j)}, \tag{16}$$

where $p(\cdot)$ denotes the probability density function (pdf). From assumption $(\mathcal{H}1)$, we get $P(A_j) = 1/M, \forall j$. From assumption $(\mathcal{H}2)$, $p(z_k|A_j), \forall j$ is the pdf of a normalized Gaussian distribution, i.e.

$$p(z_k|A_j) = \frac{1}{\sigma_k\sqrt{2\pi}} \exp\left(\frac{-(z_k - d_j)^2}{(2\sigma_k^2)}\right). \tag{17}$$

Therefore, due to assumptions $(\mathcal{H}1)$ and $(\mathcal{H}2)$, P_C of (15) becomes a function of both z_k and σ_k^2 so that

$$P_C = \frac{\exp\left(\frac{-(z_k - \mathrm{Q}(z_k))^2}{2\sigma_k^2}\right)}{\sum_{j=1}^{M} \exp\left(\frac{-(z_k - d_j)^2}{2\sigma_k^2}\right)}. \tag{18}$$

To calculate the reliability measure α_k in (13) and P_C in (18), it is necessary to estimate the variance of the deconvolutional noise σ_k^2. However, estimation of this noise level is not a simple task because of the unknown channel. In our situation, we will rely on the deconvolutional noise being Gaussian due to the central limit theorem, i.e., the assumption of $(\mathcal{H}2)$. Recall that the conditions of the central limit theorem are such that there should be *many independent* terms in the impulse response of the deconvolutional noise which is the combined channel-equalizer response excluding the cursor s_0. In other words, the terms in $\{s_j\}, \forall j \neq 0$ should be many and independent of one another. When the channel eye is almost open, the impulse response $\{s_j\}, \forall j \neq 0$ should contain many small terms whose cross-correlation is small [8, ch. 2] if the equalizer has successfully minimized the cost function. It is actually appropriate to restrict the region of our consideration to the region when the channel eye is almost open as this is when switching usually occurs and the calculation of α_k is required.

There are two known methods to obtain σ_k^2, namely the decision directed MSE and the signal-to-noise ratio (SNR) moments estimator approaches.

1) SNR Moments Estimator: The variance of the deconvolutional noise can be obtained by solving simultaneously the equations of the second (M_2) and fourth (M_4) order moments of z_k [10]

$$M_2 \triangleq S + N \tag{19}$$

$$M_4 \triangleq k_a S^2 + 6SN + k_v N^2 \tag{20}$$

where S, N are the power scaling factors of the unit variance signal and noise respectively, and $k_a \triangleq \mathrm{E}\{|a_k|^4\}/\mathrm{E}\{|a_k|^2\}^2$ and $k_v \triangleq \mathrm{E}\{|v_k|^4\}/\mathrm{E}\{|v_k|^2\}^2$ are the kurtoses of the signal and the noise, respectively. Let the so-called excess kurtoses of a_k and v_k be $G_a \triangleq k_a - 3$ and $G_v \triangleq k_v - 3$. Further we assume the deconvolutional noise is Gaussian so that $G_v = 0$. Solving for S and N of (19) and (20) simultaneously, we get

$$\sigma_k^2 \triangleq N = M_2 \pm \sqrt{G_a^{-1}(M_4 - 3M_2^2)}. \tag{21}$$

The second and fourth order moments M_p, $p = 2, 4$, can be estimated recursively in time k by

$$\widehat{M}_p(k+1) = \rho \widehat{M}_p(k) + (1-\rho)|z_k|^p \tag{22}$$

where ρ is a forgetting factor that is close to 1. The initial values $\widehat{M}_2(0)$ and $\widehat{M}_4(0)$ are set to zero. Equation (21) needs to be computed only once in every frame[3]. The estimated variance is then substituted into P_C of (18) to acquire the reliability measure α_k.

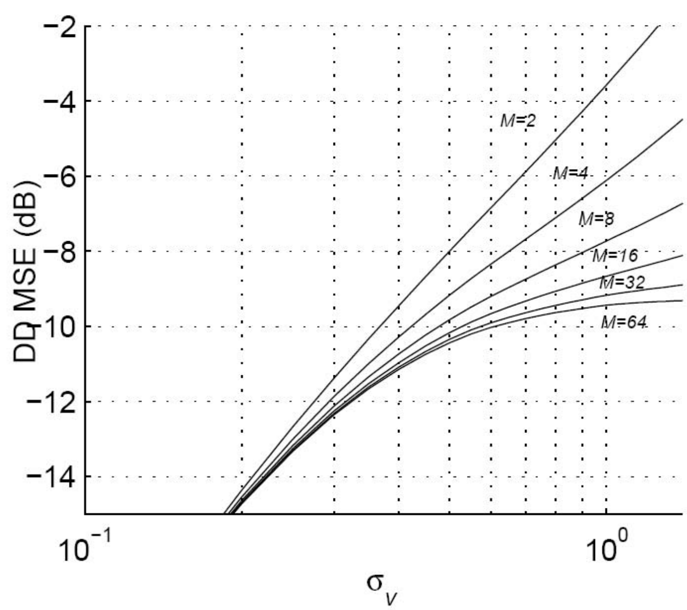

Fig. 2. Relationship between the DD MSE and σ_k for M-ary PAM with $M = 2, 4, \cdots, 64$. The range of values of σ_k is from 0.1 to 1.4 presented in the log scale.

2) Decision directed MSE (DD MSE) Method: Once the pdf of v_k is assumed Gaussian, there is a straightforward

[3] In our simulations using 4-PAM data, one frame consists of 100 symbols.

TABLE I
LOOKUP TABLE FOR DD MSE, $\kappa(\sigma_k)$ AND VARIANCE OF DECONVOLUTIONAL NOISE, σ_k^2 FOR M-PAM SIGNALS

$\sigma_{\mathbf{k}}$	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\sigma_{\mathbf{k}}^{\mathbf{2}}$	0.16	0.25	0.36	0.49	0.64	0.81	1.0
M-PAM	DD MSE, $\kappa(\sigma_k)$						
2-	0.157	0.232	0.311	0.392	0.476	0.565	0.663
4-	0.155	0.224	0.287	0.344	0.396	0.444	0.495
8-	0.154	0.219	0.275	0.32	0.354	0.383	0.411
16-	0.154	0.217	0.269	0.308	0.334	0.353	0.369
32-	0.154	0.216	0.266	0.302	0.324	0.338	0.348
64-	0.154	0.216	0.265	0.3	0.319	0.33	0.337

relationship between σ_k^2 and the DD MSE. This method involves a lookup table to be tabulated which can be computed offline. The value of σ_k is varied from say 0.4 to 1.0 at an interval of 0.1 and a large sample of z_k is generated for each σ_k. Let the DD MSE of these sample points be $\kappa(\sigma_k)$, where it is a function of σ_k. Then the DD MSE can be computed according to

$$\kappa(\sigma_k) = \frac{\sum_{i=1}^{L_s} |z_k - \mathrm{Q}(z_k)|^2}{L_s} \tag{23}$$

where L_s is the sample size used. The lookup table for M-PAM, $M = 2, 4, 8, 16, 32, 64$ for a range of σ_k values is tabulated in Table I.

One advantage of the DD MSE approach over the SNR moments estimator, apart from being less computationally intensive, is that it is a direct and unbiased estimator. In contrast the SNR moments estimator may suffer a large variance given higher order moments need to be estimated. On the other hand, the DD MSE approach is less robust for greater constellation sizes. For M-PAM, it may not be reliably used for constellation sizes larger than 16 as the relationship becomes increasingly nonlinear (see Fig. 2).

IV. SIMULATION RESULTS

With stationary channels, the equalizer achievements can be characterized in terms of convergence speed and steady-state error. As our performance measure, we have used the DD MSE which can be estimated recursively via

$$\mathrm{MSE}_{k+1}^{\mathrm{dd}} = \rho \mathrm{MSE}_k^{\mathrm{dd}} + (1-\rho)(z_k - \mathrm{Q}(z_k))^2 \tag{24}$$

where $\rho = 0.99$ is our forgetting factor. Results have been obtained via Monte Carlo simulations using 200 independent runs on two nonminimum phase channels: (one with a weaker coloring on the channel output $\mathbf{h}'$ [11], and another with a stronger coloring $\mathbf{h}''$ [12])

$$\begin{aligned}\mathbf{h}' &= [0.04, -0.05, 0.07, -0.21, -0.5, 0.72,\\ &\quad 0.36, 0, 0.21, 0.03, 0.07]^T\\ \mathbf{h}'' &= [0.8264, -0.1653, 0.8512, 0.1636, 0.81]^T.\end{aligned}$$

Simulations on both channels were carried out at an SNR of 25dB using 4-PAM signalling $\{\pm 1, \pm 3\}$. A total of 10^4 and 10^5 symbols were used in each simulation run when dealing with the channel $\mathbf{h}'$ and $\mathbf{h}''$, respectively. For the respective channels, we used a baud-rate equalizer with 20 taps and 40 taps initialized with a center tap strategy, employing a step size of 10^{-4} and 2.5×10^{-5}, respectively.

We compared the performance of the proposed dual-mode algorithm (new) with 4 other popular algorithms, namely the Benveniste-Goursat (BG) algorithm [3], the Stop-And-Go (SAG) algorithm [4], the dual-mode Godard algorithm (DMGA) [7] and the traditional "hard switching" (HS) algorithm. The HS algorithm will switch from the CMA to the LMS algorithm when $\mathrm{MSE}_k^{\mathrm{dd}} < 0.25$. The choice of startup and steady state algorithms in all cases is the CMA 2-2 and the DD LMS algorithm, respectively, except for the DMGA [7] which employs the decision adjusted modulus algorithm (DAMA) [13] at steady state. Their respective error functions are given below:

$$\epsilon_k^{\mathrm{CMA}} = z_k(|z_k|^2 - R_2) \tag{25}$$

$$\epsilon_k^{\mathrm{DD}} = z_k - \mathrm{Q}(z_k) \tag{26}$$

$$\epsilon_k^{\mathrm{DAMA}} = z_k(|z_k|^2 - \mathrm{Q}^2(z_k)) \tag{27}$$

where $\mathrm{Q}(\cdot)$ is the nearest neighbor quantizer.

For our simulations, the parameters of various algorithms of comparison are outlined below. In the notation of (10) we can express the error function in terms of the combination $\beta_k = [\beta_1, \beta_2]$. The name of the associated algorithm is superscripted on β_k. We have assigned $\beta_k^{\mathrm{BG}} = [4, |\epsilon_k^{\mathrm{DD}}|]$; $\beta_k^{\mathrm{SAG}} = [40, 0]$ for $\mathbf{h}'$ and $\beta_k^{\mathrm{SAG}} = [14, 0]$ for $\mathbf{h}''$ when $\mathrm{sgn}(\epsilon_k^{\mathrm{CMA}}) = \mathrm{sgn}(\epsilon_k^{\mathrm{DD}})$ and $\beta_k^{\mathrm{SAG}} = [0, 0]$ when $\mathrm{sgn}(\epsilon_k^{\mathrm{CMA}}) \neq \mathrm{sgn}(\epsilon_k^{\mathrm{DD}})$; $\beta_k^{\mathrm{DMGA}} = [1, 0]$, $\forall |z_k - \mathrm{Q}(z_k)| < 0.2$, and $\beta_k^{\mathrm{DMGA}} = [0, 1]$ otherwise; $\beta_k^{\mathrm{HS}} = [0, 1]$ when $\mathrm{MSE}_k^{\mathrm{dd}} > 0.25$, and $\beta_k^{\mathrm{HS}} = [14, 0]$ otherwise. As for our dual-mode algorithm, we used α_k from (13) and assigned $\gamma = 14$. The variance σ_k^2 is estimated from (17).

We simulated 200 runs for each dual-mode algorithm for both channels. The graphs of the DD MSE of the averaged runs are plotted in Fig. 3. Note that these graphs are obtained by averaging out only the MSE of the runs that have been successful in convergence. The summary of the results that includes the failure rate of convergence is tabulated in Table II. The failure rate is calculated by recording the number of runs where the MSE at the end of a particular run is higher than -13.15 dB $= 0.22$ then dividing by the total number of runs. The time to convergence has been normalized by the averaged number of symbols required by the fastest algorithm. From Table II, the hard switching algorithm is the most unreliable as it yields high failure rates. Both our algorithm and the DMGA are the smoothest in terms of low failure rate, but the DMGA yields higher steady state MSE and is also carrier phase blind. The general conclusion is that the proposed new algorithm is superior than others in terms of convergence speed and steady state error based on the results in Fig. 3 and Table II.

V. CONCLUSION

In this paper, we have constructed a novel dual-mode algorithm whose combination technique depends on a reliability measure of the equalized signals. This measure, unlike several traditional ones, reflects more accurately the probability of correctly detecting a symbol given the most current output and certain statistics regarding the distribution of the residual ISI

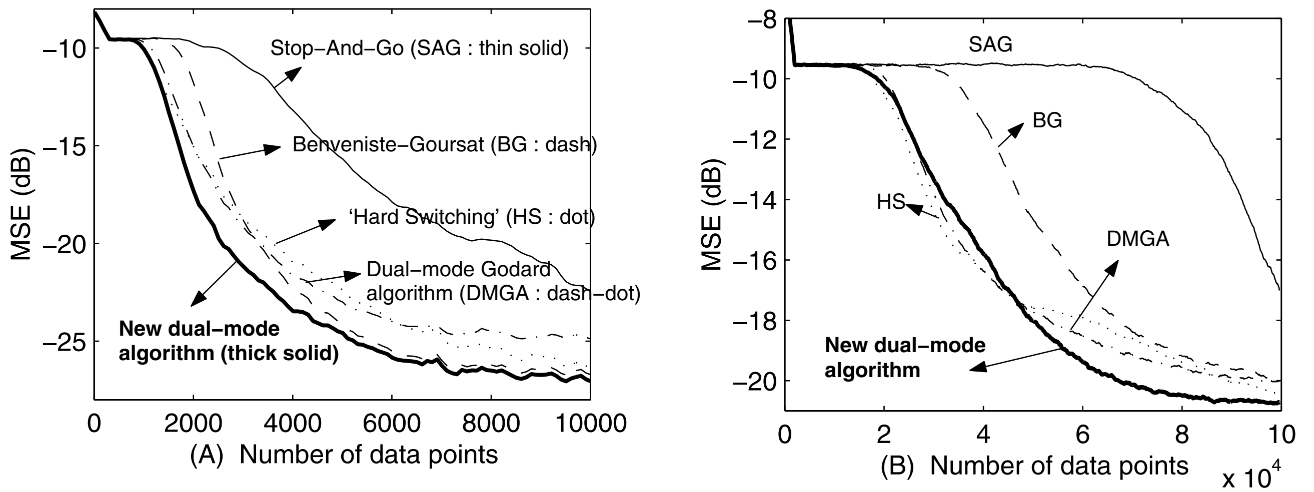

Fig. 3. A comparison of several dual-mode algorithms equalizing channels (A) $\mathbf{h}'$, and (B) $\mathbf{h}''$, both at an SNR of 25dB for 4-PAM signalling.

TABLE II

SUMMARY OF RESULTS IN FIG. 3 INCLUDING FAILURE RATES

Dual-mode Algorithm	$\mathbf{h}'$ Fail rate	$\mathbf{h}'$ Normalized time to reach -14dB	-18dB	$\mathbf{h}''$ Fail rate	$\mathbf{h}''$ Normalized time to reach -14dB	-18dB
New	0%	1	1	3.5%	1.11	1
BG	0%	1.45	1.35	16.5%	1.65	1.35
SAG	0%	2.70	2.88	55.0%	3.18	-
DMGA	0%	1.15	1.25	1.5%	1.06	1.07
HS	1.5%	1.13	1.31	53.5%	1	1.23
Time normalized by		1590	2110		28900	49700

and noise. The proposed algorithm exhibits faster and more reliable convergence corroborated by simulations performed under both lightly and severely distorted channels when compared to traditional algorithms such as the Benveniste-Goursat and the Stop-And-Go algorithms. The performance improvement over DMGA [7], however, is less discernible in our simulations. This new technique also eliminates any requirement for manual control of the parameters that govern the convergence speed and excess noise.

REFERENCES

[1] Y. Sato, "A method of self-recovering equalization for multi-level amplitude-modulations systems," *IEEE Trans. Commun.*, pp. 679–682, Jun 1975.

[2] D. N. Godard, "Self-recovering equalization and carrier tracking in two dimensional data communication system," *IEEE Trans. Commun.*, vol. 28, no. 11, pp. 1867–1875, Nov 1980.

[3] A. Benveniste and M. Goursat, "Blind equalizers," *IEEE Trans. Commun.*, vol. COM-32, no. 8, pp. 871–883, Aug 1984.

[4] G. Picchi and G. Prati, "Blind equalization and carrier recovery using a "Stop-and-Go" decision-directed algorithm," *IEEE Trans. Commun.*, vol. COM-35, no. 9, pp. 877–887, Sep 1987.

[5] D. Hatzinakos, "Stop-and-go sign algorithms for blind equalization," *Proc. SPIE, Adaptive Signal Processing*, vol. 1565, pp. 118–129, 1991.

[6] K. Hilal and P. Duhamel, "A blind equalizer allowing soft transition between the CMA and the DD algorithm for PSK modulated signals," *Proc. ICC, Geneva, Switzerland*, pp. 1144–1148, May 1993.

[7] V. Weerackody and S. A. Kassam, "Dual-mode type algorithms for blind equalization," *IEEE Trans. Commun.*, vol. 42, no. 1, pp. 22–28, Jun 1994.

[8] S. Haykin, *Blind Deconvolution*. Prentice Hall Information and System Sciences Series, 1994.

[9] *IEEE Standard Computer Dictionary: A Compilation of IEEE Standard Computer Glossaries.* New York, NY, 1990.

[10] D. R. Pauluzzi and N. C. Beaulieu, "A comparison of SNR estimation techniques for the AWGN channel," *IEEE Trans. Commun.*, vol. 48, no. 10, pp. 1681–1691, Oct 2000.

[11] J. G. Proakis, *Digital Communications*, 2nd ed. McGraw-Hill International Editions, 1989.

[12] J. Labat, O. Macchi, and C. Laot, "Adaptive decision feedback equalization: Can you skip the training period?" *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 921–930, Jul 1998.

[13] W. A. Sethares, G. A. Rey, and C. R. Johnson, Jr, "Approaches to blind equalization of signals with multiple modulus," *ICASSP-89*, vol. 2, pp. 972–975, 1989.

[14] J. J. Shynk, C. K. Chan, and R. P. Gooch, "Comparative performance study of several blind equalization algorithms," *SPIE 91, San Diego, CA (Optical Engineering)*, 1991.