A GENERALISED (M, N_R) MIMO RAYLEIGH CHANNEL MODEL FOR NON-ISOTROPIC SCATTERER DISTRIBUTIONS

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ABSTRACT

This paper extends a recently proposed space-time model for Rayleigh fading to include an arbitrary transmit antenna configuration of any shape and size transmitting simultaneously in a multiple-input multiple-output (MIMO) channel. The space-time correlation function and space-frequency cross spectrum function for a non-isotropic scatterer distribution around the receiver is derived for the arbitrary configuration as a further extension of a previous result of a multiple-input single-output Rayleigh wireless channel which used a ring of uniformly distributed scatterers model. Analysis based on achievable spectral efficiency for typical arbitrary transmit antenna configurations is given. The analysis demonstrates the utility of the correlation function.

INTRODUCTION

The combination of temporal and spatial diversity are two effective means by which communication quality and associated system performance can be significantly improved in a rich-scattering wireless environment. This has been demonstrated through proposals for space-time coded modulation over multiple-input multiple-output (MIMO) radio channels which are understood to be beneficial for high data-rate systems operating in rich-scattering wireless environments. Without designing highly detailed system models, which can be used for individual channel realisations, it is possible to obtain a more rigorous evaluation of the proposed coding schemes through simpler macroscopic system models. One such scheme, based on a traditional Clarkes/Jakes model [1], has been proposed in [2], where a rich isotropic distribution of scatterers is assumed around the mobile station (MS), the receiver. No major scatterers are located around the base station (BS), the transmitter. Space-time correlation and space-frequency cross spectrum functions corresponding to this distribution were derived. This was extended to the case of a (2, N_R), i.e. 2 transmit, N_R receive antennas, Rayleigh fading channel with a non isotropic distribution of scatterers in [3].

One major drawback of [2, 3] is that by consideration of time selectivity and/or frequency selectivity (with reasonable sized Doppler spread, and/or delay spread), limits the models to M =2 transmit antennas at a particular time instant. Thus, they are not applicable to many proposed mobile communication systems where application of space-time coding utilises $M > 2$ for continuous fast fading and/or frequency selective fading wireless scenarios. This paper will provide a useful generalisation of [3] that will mean the space-time model can be applied to any numbers of antennas at the BS in a macrocellular fast fading scenario. The application of the model can be for frequency selective and/or frequency non-selective wireless scenarios through derivation of appropriate space-time cross correlation, and space-frequency cross spectrum functions. Such functions can analyse space-time coding, or space-frequency coding, schemes. To demonstrate the utility of the proposed model a simple spectral efficiency analysis based on Shannon's capacity measure is given for the case of a (4, 4), fast flat Rayleigh fading wireless channel.

SPACE-TIME CROSS CORRELATION FORMULATION

Consider an arbitrary number of BS antennas, M, of any configuration, located at $y_1, y_2 \ldots, y_M$, with no major scatterers and only characterised by horizontal separation of an arbitrary number of wavelengths (λ), and N_R approximately co-located receive antennas at the MS. The discussion in this paper assumes a macrocellular radio channel. Fig. 1 illustrates the MIMO transmission model between the BS and MS for the case of $M = 4$.

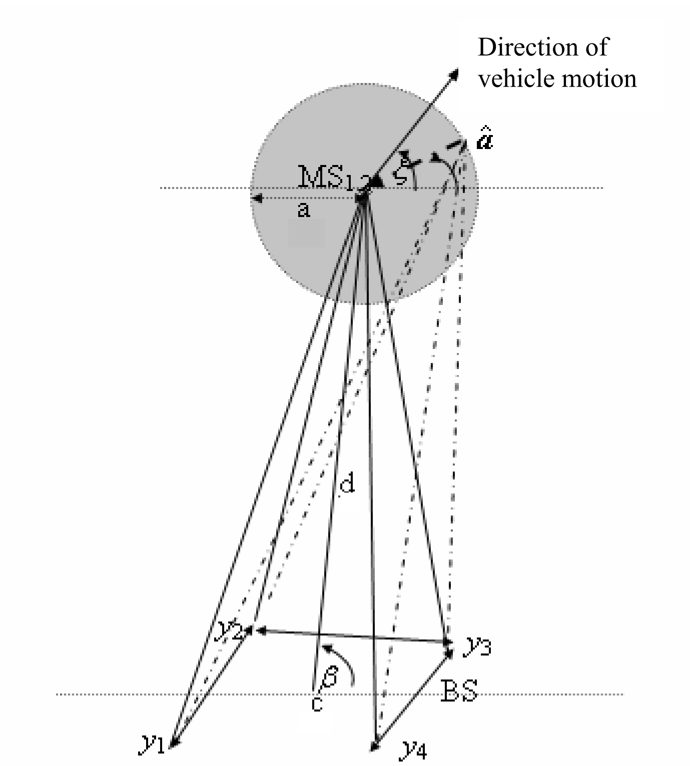

Fig. 1 (4, 2), $M = 4$, $N_R = 2$, MIMO Transmission model between base station (BS) and mobile station (MS).

The space-time cross correlation function can be defined as $\rho\left(y_{m'}, y_{m''}; \tau\right) \triangleq E c_{m'}\left(t\right) c_{m''}^{*}\left(d_{sp}, t-\tau\right) = \rho\left(d_{sp}, \tau\right)$, and in a manner similar to [2] we obtain,

$$\rho\left(d_{sp}, \tau\right) = \sigma^2 e^{\left(jkd_{sp}\cos(\beta-\gamma)\right)} \times \int_{\Omega} P\left(\hat{\boldsymbol{a}}\right)\left(e^{\left[j\cos\hat{\boldsymbol{a}}\left(2\pi f_D \tau \cos\xi + kz_{cs'}\right)\right]} \times e^{\left[j\sin\hat{\boldsymbol{a}}\left(2\pi f_D \tau \sin\xi - kz_{ss'}\right)\right]}\right) d\hat{\boldsymbol{a}} \tag{1}$$

where $d_{sp} = \left|y_{m''} - y_{m'}\right|$ is the magnitude separation of antennas m'' and m' at the BS, $k = 2\pi/\lambda$ is the wave number, $\hat{\boldsymbol{a}}$ is a unit vector pointing in the direction of wave propagation with reference to the MS, Ω corresponds to the unit circle over which integration is performed, $P\left(\hat{\boldsymbol{a}}\right)$ is the angular power density distribution function around the MS of scatterers and $j = \sqrt{-1}$, and γ is the angle of separation between antennas $y_{m''}$ and $y_{m'}$ with respect to y_1. Note that for uniformly distributed scatterers $P\left(\hat{\boldsymbol{a}}\right) = 1$ and for a finite number of scatterers the integration in (1) reduces to a summation as in [2]. Also with reference to (1), f_D is the maximal Doppler spread, ξ is the direction of movement of the MS with reference to the BS , σ is the variance of the channel gain, and $z_{cs}{}'$ and $z_{ss}{}'$, found using macrocellular far-field assumptions [2], are similar to those defined in [2, App. 1, eqn. (36,37)] as

$$z_{cs}{}' = c_s \sin\beta, \quad z_{ss}{}' = c_s \cos\beta \tag{2}$$

where $c_s = d_{sp}\sin(\beta-\gamma)\times a/d'$; a/d' represents the ratio of distance, a, of scatterers from the MS and the distance, d', of the MS to the centre of the antennas m'' and m', and β represents the angular position of the MS with respect to the BS. To further appreciate the material in this section the reader is referred to [2].

Similarly to [2], and using the 2-D modal expansion as in [4], if one lets $\hat{\boldsymbol{a}} = (1,\ \phi)$ the following formulation is obtained for the space-time correlation function

$$\rho\left(d_{sp},\tau\right) \equiv R_{c_m c_{m'}}\left(d_{sp},\tau\right) = \sigma^2 e^{jkd_{sp}\cos(\beta-\gamma)} . \int_0^{2\pi} \sum_{m=-\infty}^{\infty} \gamma_m e^{-jm\phi} e^{j\left(\cos\phi\left(2\pi f_D \tau \cos\xi + kz_{cs}'\right) + \sin\phi\left(2\pi f_D \tau \sin\xi - kz_{ss}'\right)\right)} d\phi \tag{3}$$

After an appropriate change of variables the following is obtained for the space-time correlation function,

$$R_{c_m c_{m'}}\left(\tau, d_{sp}\right) = \sigma^2\, \mathrm{e}^{jkd_{sp}\cos(\beta-\gamma)} .2\pi \sum_{m=-\infty}^{\infty} \gamma_m e^{jm\psi} J_m\left(z\right). \tag{4}$$

where $\psi = \tan^{-1}\left(b_1/a_1\right)$ and $z = 2\pi\sqrt{a_1^{\,2} + b_1^{\,2}}$; $b_1 = \left(f_D \tau \sin\xi - z_{ss}'/\lambda\right)$ and $a_1 = \left(f_D \tau \cos\xi + z_{cs}'/\lambda\right)$; and $J_m(\bullet)$ is the *m*th order Bessel function of the first kind.

Note that for an isotropic scatterer distribution,

$$\gamma_m = \begin{cases} 1/2\pi, & m = 0 \\ 0, & m \neq 0 \end{cases}, \text{ and } R_{c_{m'} c_{m'}}\left(d_{sp},\tau\right) = \sigma^2 e^{\left(jkd_{sp}\cos(\beta-\gamma)\right)} J_0\left(z\right) \tag{5}$$

, which for the case of $M = 2$, and thus $\beta = \gamma$, we obtain the same space-time correlation as defined in [4].

A 3-D plot of the magnitude of the cross-correlation function, $\left|R_{c_{m'} c_{m''}}\left(d_{sp},\tau\right)\right|$, is shown in Fig. 2 for $f_D = 0.01$, between diagonally opposite antennas for a uniform circular array (UCA), $M = 4$, with antenna spacing, $d_{sp} = 10\lambda$ (4 element UCA also shown in Fig. 2); $\xi = \pi/3$ and $\beta = \pi/6$ assuming a Laplacian distribution [5]. The angular spread, S_σ^2, of 10^{o} given, is found from the square root of the variance, [4]. $\left|R_{c_{m'} c_{m''}}\left(d_{sp},\tau\right)\right|$ can also be plotted for all other common angular power distributions for BS antenna configurations of arbitrary shape and size.

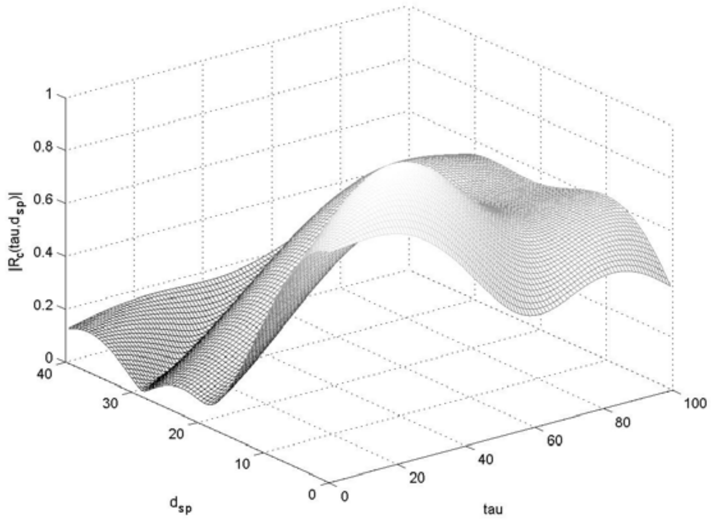

Fig. 2 3-D plot of magnitude of cross-correlation function, $|R_c(d_{sp}, \tau)|$, for $f_D = 0.01$, MS moving direction with respect to BS, $\xi = \pi/3$, and MS position with respect to BS, $\beta = \pi/6$, assuming a Laplacian distribution with an angular spread of 10^{o}.

SPACE-FREQUENCY CROSS SPECTRUM FORMULATION

Following from (5), and in a manner similar to the derivation of the space-frequency cross spectrum given in [4, eqn (3), p. 1176], a space-frequency cross spectrum can be found for a non-isotropic distribution of scatterers. We first observe that z can be reformulated as

$$z = \sqrt{\left(2\pi\left[f_D\tau + c_s \sin\left(\beta-\xi\right)\right]\right)^2 + \left(2\pi c_s \cos\left(\beta-\xi\right)\right)^2}\ . \tag{6}$$

The space-frequency cross spectrum is defined as $S_{c_{m'}c_{m''}}\left(d_{sp}, f\right) \triangleq \mathcal{F}\left\{R_{c_{m'}c_{m''}}\left(\tau, d_{sp}\right)\right\}$ where $\mathcal{F}\{\bullet\}$ is the Fourier transform with respect to τ. Thus

$$S_{c_{m'}c_{m''}}\left(d_{sp}, f\right) = \sigma^2\, \mathrm{e}^{\left(jkd_{sp}\cos\left(\beta-\gamma\right)\right)}.2\pi\sum_{m=-\infty}^{\infty}\left(\gamma_m.\mathcal{F}\left\{e^{im\psi}J_m\left(z\right)\right\}\right). \tag{7}$$

Thus following from some manipulation of (7), and (6), $S_{c_1c_2}\left(d_{sp}, f\right)$, can be expressed as

$$\begin{aligned} S_{c_1c_2}\left(d_{sp}, f\right) &= \sigma^2\, \mathrm{e}^{\left(jkd_{sp}\cos\left(\beta-\gamma\right)\right)}.2\pi\sum_{m=-\infty}^{\infty}\Big\{\gamma_m \\ &\times\sum_{n=-\infty}^{\infty} j^n\left(J_{m+n}\left(2\pi c_s\cos\left(\beta-\xi\right)\right)\sum_{m'=-\infty}^{\infty}\left[\mathcal{F}\left\{J_n\left(2\pi f_D\tau\right)\right\}J_{n-m'}\left(2\pi c_s\sin\left(\beta-\xi\right)\right)\right]\right)\Big\} \end{aligned} \tag{8}$$

where $\mathcal{F}\{\bullet\}$ is as defined as

$$\begin{aligned} \mathcal{F}\left\{J_n\left(2\pi f_D\tau\right)\right\} &= \pm\left(\pi f_D\sqrt{1-\left(f/f_D\right)^2}\right)^{-1}\cos\left(n\sin^{-1}\left(f/f_D\right)\right),\ f < f_D \\ &= \mp f_D{}^n f^{-n}\sin\left(n\pi/2\right)\left(\pi f_D\sqrt{1-\left(f_D/f\right)^2}\right)^{-1}\left(1+\sqrt{1-\left(f/f_D\right)^2}\right)^{-n},\ f > f_D \end{aligned} \tag{9}$$

where f_D is now the maximal Doppler spread. The values $\pm x_1/\mp x_2$ for $n = -\infty\ldots\infty$, and $n \neq 0$, depend on whether $n > 0$ and/or $|n|$ even, in which case the transform is $+x_1/-x_2$; otherwise, if $n < 0$ and n is odd, one has $-x_1/+x_2$.

It can also be readily observed that from (8), if we are summing over $n = -\infty\ldots\infty$, then the term $\mp x_2$ in equation (9) can be disregarded since x_2 is only non-zero for odd n, $n = -\infty\ldots\infty$, due to the $\sin\left(n\pi/2\right)$ term in x_2 and $j^n\sin\left(n\pi/2\right) + j^{-n}\sin\left(-n\pi/2\right) = 0$. Consequently (8) gives a closed form expression for the space-frequency cross spectrum with a non-isotropic distribution of scatterers.

SPECTRAL EFFICIENCY ANALYSIS FOR A FREQUENCY FLAT FADING (4, 4) MIMO CHANNEL

This section investigates the spectral efficiency of a (4, 4), i.e. $M = 4$, $N_R = 4$ co-located receive antennas, MIMO wireless channel, for Rayleigh frequency flat fading. Two separate non-isotropic scatterer distributions are considered, a $\cos^{2p}\phi$ distribution and a Laplacian distribution. The general channel transfer matrix, $\mathbf{H}_{M,N_R}\left(\tau, d_{sp}\right)$ is found as, similarly to [11],

$$\mathbf{H}_{M,N_R}\left(\tau, d_{sp}\right) = \frac{\hat{R}_M\left(\tau, d_{sp}\right)^{1/2}}{\sigma}A_{M,N_R}\ , \quad \text{where} \tag{10}$$

$$\hat{R}_M\left(\tau, d_{sp}\right) = \begin{bmatrix} R_c(\tau) & R_{c_1c_2}\left(\tau, d_{sp}\right) & \cdots & R_{c_1c_M}\left(\tau, d_{sp}\right) \\ R_{c_2c_1}\left(\tau, d_{sp}\right) & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & R_{c_{M-1}c_M}\left(\tau, d_{sp}\right) \\ R_{c_Mc_1}\left(\tau, d_{sp}\right) & \cdots & R_{c_Mc_{M-1}}\left(\tau, d_{sp}\right) & R_c(\tau) \end{bmatrix}. \tag{11}$$

The term $\hat{R}(\bullet)^{1/2}$ represents the space-time correlation matrix square root, A_{M,N_R} is an $M \times N_R$ matrix of zero mean, unit variance, complex normal i.i.d. variables. The term $R_c(\tau)$ represent the autocorrelation function for the particular non-isotropic distribution of scatterers.

Assuming $N_R = M$, for a rich-scattering wireless environment, we obtain the following for the Shannon capacity, C, of the M data subchannels [13] (if it is assumed that equal power is assigned to each subchannel),

$$C = \sum_{k=1}^{M} \log_2\left(1 + \lambda_k \frac{P_{BS}/M}{\sigma_n^2}\right) \tag{12}$$

where P_{BS} is the power assigned to the kth subchannel, σ_n^2 is the noise power and λ_k is the kth eigenvalue of the M data subchannels, found from the eigenvalues of $H(\tau)H(\tau)^\dagger$ where $(\bullet)^\dagger$ represents the matrix adjoint, or complex conjugate transpose.

Fig. 3 and Fig. 4 shows the cumulative distribution function of achievable spectral efficiency calculated according to (13) for a (4, 4) channel, found via Monte Carlo simulations after $\tau = M = 4$ time samples for a maximal Doppler spread, $f_D = 0.01$ for non-isotropic scatterer distributions. For the Laplacian distribution, [5], an angular spread of 10° is assumed and for the $\cos^{2p}\phi$ distribution, [6], an equivalent half-power beamwidth (HPBW) of the BS antennas of 131° is assumed. A Uniform Circular Array (UCA) is modelled at the BS where β and ξ are taken from uniform distributions on $(-\pi, \pi]$, for a equivalent antenna spacing d_{sp}, of 1λ, 5λ, 10λ and 20λ for an SNR, $P_{BS}/\sigma_n^2 = 20$ dB. Fig. 3 shows the case of $N_R = 2$ as a dashed lines to compare with the results for the Laplacian distribution, and Fig. 4 shows the comparison of Laplacian distribution and $\cos^{2p}\phi$ distributions.

Fig. 3 shows expected performance improvement for $N_R = 4$ over $N_R = 2$ receive antennas. There is also demonstrable improvement in spectral efficiency with increasing BS aperture. Fig. 4 shows that there is significant improvement in spectral efficiency for the particular Laplacian scatterer distribution over the $\cos^{2p}\phi$ distribution for smaller BS apertures. But for the larger BS apertures, i.e. for a UCA with antenna spacing, d_{sp} of 10λ or 20λ, the $\cos^{2p}\phi$ distribution shows improvement in achievable spectral efficiency over the Laplacian distribution considered.

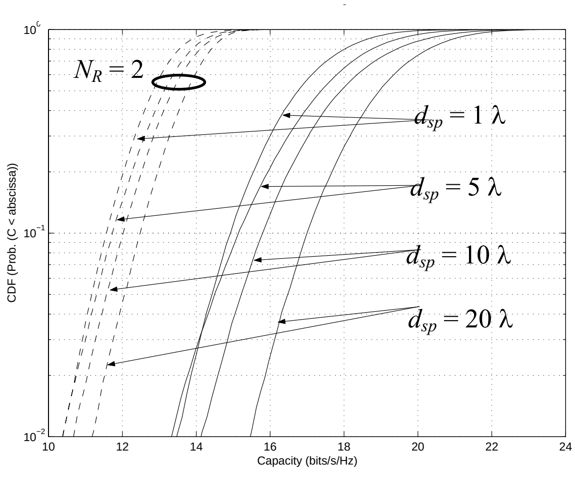

Fig. 3. Plot of cumulative distribution function (CDF) v. Capacity (bits/s/Hz) for a (4, 4) channel after $\tau = 4$ time samples, $f_D = 0.01$, found via Monte Carlo simulations for a UCA at the BS with various antenna spacing, d_{sp}. A Laplacian distribution, [5], is assumed with an angular spread of 10°, β and ξ are taken from uniform distributions on $(-\pi, \pi]$. The results for $N_R = 2$ are shown as dashed lines.

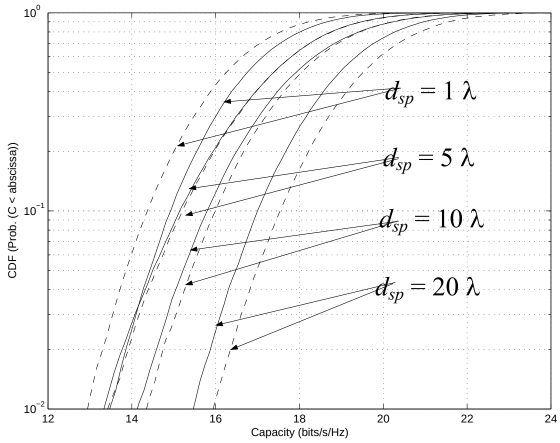

Fig. 4. Plots of CDF v. Capacity (bits/s/Hz) for a (4, 4) channel after $\tau = 4$ time samples, $f_D = 0.01$, found via Monte Carlo simulations for a UCA with various antenna spacing, d_{sp}. The results for a Laplacian distribution, [5], with an angular spread of 10° is shown as solid lines, and those for a $\cos^{2p}\phi$ distribution, [5], equivalent HPBW = 131°, are shown as dashed lines; β and ξ are taken from uniform distributions on $(-\pi, \pi]$.

CONCLUDING REMARKS

Closed-form solutions have been found for the space-time cross correlation and space-frequency cross spectrum which can be applied to continuous fading (M, N_R) MIMO channels for both frequency flat and frequency selective fading wireless scenarios with non-isotropic scatterer distributions. The correlation function has been successfully applied to a Rayleigh continuous flat fading channel to obtain a measure of performance based on achievable spectral efficiency both a Laplacian distribution, and a $\cos^{2p}\phi$ non-isotropic scatterer distribution.

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