Generalised space-time model for Rayleigh fading channels with non-isotropic scatterer distribution

D.B. Smith and T.D. Abhayapala

A generalised space-time model based on Clarkes/Jakes model is extended to the case of a non-isotropic distribution of scatterers. The respective space-time correlation function and space-frequency cross spectrum around the receiver are derived. A 3-D space-time plot of the correlation function is shown for a specific scatterer distribution based on a typical mobile radio scenario.

Introduction: The exploitation of temporal and spatial diversity can significantly improve communication quality and associated system performance in a rich-scattering wireless environment. This has been demonstrated through proposals for space-time coded modulations over multiple-input multiple-output (MIMO) radio channels which are understood to be very beneficial for high data-rate systems, both coherent [1], and non-coherent [2]. For the purposes of macroscopic system design, better system models are required to obtain a more rigorous evaluation of the proposed coding schemes. One such scheme, based on a traditional Clarkes/Jakes model [3], has been proposed in [4], where a rich isotropic distribution of scatterers is assumed around the mobile station (MS), with no major scatterers located around the base station (BS), and corresponding space-time cross correlation and space-frequency cross spectrum functions are derived. In this Letter we generalise [4] by incorporating a non-isotropic distribution of scatterers around the MS.

In the context of this Letter, the correlation and spectrum functions will be derived for a non-isotropic distribution of scatterers, assuming multiple transmit antennas at the BS, for a corresponding Rayleigh multiple-input single-output (MISO) or MIMO radio channel. The closed form expressions that will be developed for the space-time and space-frequency cross spectrum functions can be easily applied to analysis of frequency flat and frequency selective fading MISO/MIMO radio channels, respectively.

Space-time cross-correlation formulation: Consider two BS antennas, located at y_1 and y_2, with no major scatterers and only characterised by horizontal separation of an arbitrary number of wavelengths (λ), and N_R approximately co-located receive antennas. For the sake of discussion contained in this Letter, $N_R = 2$ will be considered, in a macrocellular wireless channel. The space-time cross-correlation function can be defined as, in a manner similar to [4], and following from [5],

$$\begin{aligned}\rho(y_1, y_2; \tau) &= \rho(d_{sp}, \tau)\\ &= \sigma^2 \exp(jkd_{sp}) \int_\Omega P(\hat{\boldsymbol{a}})(\exp[j\cos\hat{\boldsymbol{a}}(2\pi f_D \tau \cos\xi + kz'_c)]\\ &\quad \times \exp[j\sin\hat{\boldsymbol{a}}(2\pi f_D \tau \sin\xi - kz'_s)])\, d\hat{\boldsymbol{a}} \end{aligned} \tag{1}$$

where $d_{sp} = y_1 - y_2$ is the BS separation, $k = 2\pi/\lambda$ is the wave number, $\hat{\boldsymbol{a}}$ is a unit vector pointing in the direction of wave propagation, Ω corresponds to the unit circle over which integration is performed, $P(\hat{\boldsymbol{a}})$ is the angular power density distribution function around the MS of scatterers and $j = \sqrt{-1}$. Also with reference to (1), f_D is the maximal Doppler spread, ξ is the direction of movement of the MS and the corresponding reference frame containing the scatterers, σ is the variance of the channel, and z'_c and z'_s, found using macrocellular far-field assumptions [4], are as defined in [4, App. 1, eqn. (36, 37)],

$$z'_c = c_1 \sin\beta, \quad z'_s = c_1 \cos\beta \tag{2}$$

where $c_1 = d_{sp} \sin\beta \times a/d$; a/d represents the ratio of distance, a, of scatterers from the MS and the distance, d, of the MS to the centre of the transmit antenna configuration, and β represents the angular position of the MS with respect to the BS.

Similarly to [5], and using the 2-D modal expansion in [6], if one lets $\hat{\boldsymbol{a}} = (1, \phi)$ the following formulation is obtained for the space-time correlation function

$$\begin{aligned}\rho(d_{sp}, \tau) \equiv R_{c_1c_2}(d_{sp}, \tau) &= \sigma^2 e^{jkd_{sp}}\\ &\times \int_0^{2\pi} \sum_{m=-\infty}^{\infty} \gamma_m e^{-jm\phi} e^{j(\cos\phi(2\pi f_D \tau\cos\xi + kz'_c) + \sin\phi(2\pi f_D\tau\sin\xi - kz'_s))}\, d\phi\end{aligned} \tag{3}$$

where as in [5], $\gamma_m = \int_0^{2\pi} P(\varphi) e^{-jm\varphi}\, d\varphi$, and $P(\varphi)$ corresponds to $P(\hat{\boldsymbol{a}})$ in (1). Then by making an appropriate change of variables let $\theta = \phi + \psi$,

$$\begin{aligned}\int_0^{2\pi} e^{j(-m\phi + z\sin(\phi+\psi))} &= \int_\psi^{2\pi+\psi} e^{j(-m(\theta-\psi)+z\sin\theta)}\, d\theta\\ &= e^{jm\psi} \int_\psi^{2\pi+\psi} e^{j(-m\theta + z\sin\theta)}\, d\theta = e^{jm\psi} 2\pi \cdot J_m(z)\end{aligned} \tag{4}$$

where $\psi = \tan^{-1}(b_1/a_1)$ and $z = 2\pi\sqrt{(a_1^2 + b_1^2)}$; $b_1 = (f_D \tau \sin\xi - z'_s/\lambda)$ and $a_1 = (f_D\tau\cos\xi + z'_c/\lambda)$; and $J_m(\cdot)$ is the mth order Bessel function of the first kind.

Then the following is obtained for the space-time correlation function,

$$R_{c_1c_2}(\tau, d_{sp}) = \sigma^2 \exp(jkd_{sp}) \cdot 2\pi \sum_{m=-\infty}^{\infty} \gamma_m e^{jm\psi} J_m(z) \tag{5}$$

Now for an isotropic scatterer distribution,

$$\gamma_m = \begin{cases} \dfrac{1}{2\pi}, & m = 0\\ 0, & m \neq 0 \end{cases}, \quad \text{and} \quad R_{c_1c_2}(d_{sp}, \tau) = \sigma^2 \exp(jkd_{sp}) J_0(z) \tag{6}$$

which gives the space-time cross-correlation as defined in [4].

A 3-D plot of the magnitude of the cross-correlation function, $|R_{c_1c_2}(d_{sp}, \tau)|$, is shown in Fig. 1 for $f_D = 0.01$, $\xi = 7\pi/6$ and $\beta = \pi/4$ assuming a Laplacian distribution [7], which gives γ_m as, [5],

$$\gamma_m = e^{-jm\beta} \frac{(1 - (-1)^{\lceil m/2 \rceil} \zeta F_m)}{(1 + \sigma^2 m^2/2)(1 - \zeta)} \tag{7}$$

where $\zeta = e^{-\pi/(\sqrt{2}\sigma)}$; $F_m = 1$ for m even; and $F_m = m\sigma/\sqrt{2}$ for m odd. The angular spread, S_σ^2 of 10° given, is found from the square root of the variance, [5]. $|R_{c_1c_2}(d_{sp}, \tau)|$ can also be plotted for all other common angular power distributions.

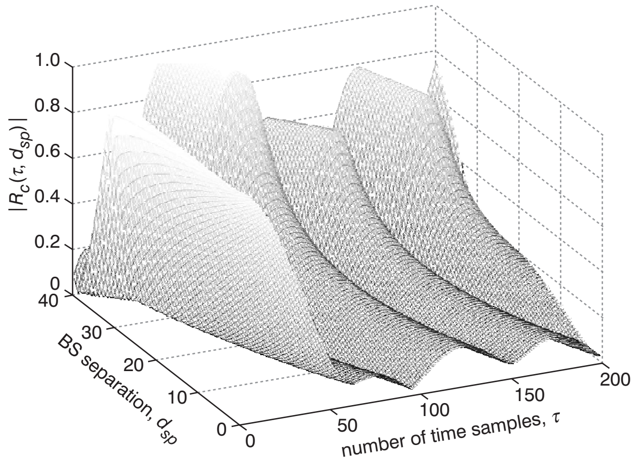

Fig. 1 *3-D plot of magnitude of cross-correlation function, $|R_c(d_{sp}, \tau)|$, for $f_D = 0.01$, MS moving direction with respect to BS, $\xi = 7\pi/6$ and MS position with respect to BS, $\beta = \pi/4$, assuming Laplacian distribution with angular spread of 10°*

Space-frequency cross spectrum formulation: Following from (5), and in a manner similar to the derivation of the space-frequency cross spectrum given in [4, eqn. (3), p. 1176], a space-frequency cross spectrum can be found for a non-isotropic distribution of scatterers. We first observe that z can be reformulated as

$$z = \sqrt{(2\pi[f_D\tau + c_1\sin(\beta - \xi)])^2 + (2\pi c_1 \cos(\beta - \xi))^2} \tag{8}$$

The space-frequency cross spectrum is defined as $S_{c_1c_2}(d_{sp}, f) \triangleq \mathcal{F}\{R_{c_1c_2}(\tau, d_{sp})\}$ where $\mathcal{F}\{\cdot\}$ is the Fourier transform with respect to τ. Thus

$$S_{c_1c_2}(d_{sp}, f) = \sigma^2 \exp(jkd_{sp}) \cdot 2\pi \sum_{m=-\infty}^{\infty} (\gamma_m \cdot \mathcal{F}\{e^{im\psi} J_m(z)\}) \quad (9)$$

For further simplification, using (5), and a result in [8], and the addition theorem for Bessel functions, one can express $e^{im\psi}J_m(z)$ as

$$\begin{aligned} e^{im\psi}J_m(z) &= \sum_{n=-\infty}^{\infty} j^n J_n(2\pi(f_D\tau + c_1\sin(\beta-\xi)))J_{m+n}(c_1\cos(\beta-\xi)) \\ &= \sum_{n=-\infty}^{\infty} j^n J_{m+n}(c_1\cos(\beta-\xi)) \sum_{m'=-\infty}^{\infty} J_n(2\pi f_D\tau) \\ &\quad \times J_{n-m'}(2\pi c_1 \sin(\beta-\xi)) \end{aligned} \quad (10)$$

Thus since the space-frequency cross spectrum is defined with respect to τ we need only to find $\mathcal{F}\{J_n(2\pi f_D\tau)\}$ for $n=-\infty\cdots\infty$. Thus using a result from [9, p. 66] and [10, p. 197] we have

$$\begin{aligned} \{\mathcal{F}J_n(2\pi f_D\tau)\} &= \pm\left(\pi f_D\sqrt{1-\left(\frac{f}{f_D}\right)^2}\right)^{-1} \cos\left(n\sin^{-1}\left(\frac{f}{f_D}\right)\right), \quad f<f_D \\ &= \mp f_D^n f^{-n} \sin\left(\frac{n\pi}{2}\right)\left(\pi f_D\sqrt{1-\left(\frac{f_D}{f}\right)^2}\right)^{-1} \\ &\quad \times \left(1+\sqrt{1-\left(\frac{f}{f_D}\right)^2}\right)^{-n}, \quad f>f_D \end{aligned} \quad (11)$$

where f_D is now the maximal Doppler spread. The values $\pm x_1/\mp x_2$ for $n=-\infty\cdots\infty$, and $n\neq 0$, depend on whether $n>0$ and/or $|n|$ is even, in which case the transform is $+x_1/-x_2$; otherwise, if $n<0$ and n is odd, one has $-x_1/+x_2$. Thus following from (9) and (10), $S_{c_1c_2}(d_{sp}, f)$, can be expressed as

$$\begin{aligned} &S_{c_1c_2}(d_{sp}, f) \\ &= \sigma^2\exp(jkd_{sp}) \cdot 2\pi \sum_{m=-\infty}^{\infty}\left\{\gamma_m \sum_{k=-\infty}^{\infty} j^n \left(J_{m+n}(2\pi c_1\cos(\beta-\xi))\right.\right. \\ &\quad \times \left.\left.\sum_{m'=-\infty}^{\infty}[\mathcal{F}\{J_n(2\pi f_D\tau)\}J_{n-m'}(2\pi c_1\sin(\beta-\xi))]\right)\right\} \end{aligned} \quad (12)$$

where $\mathcal{F}\{\cdot\}$ is as defined in (8).

It can also be readily observed that from (12), if we are summing over $n=-\infty\cdots\infty$, then the term $\mp x_2$ in (11) can be disregarded since x_2 is only nonzero for odd n, $n=-\infty\cdots\infty$, due to the $\sin(n\pi/2)$ term in x_2 and $j^n\sin(n\pi/2)+j^{-n}\sin(-n\pi/2)=0$. Thus (12) gives a closed form expression for the space-frequency cross spectrum with a non-isotropic distribution of scatterers.

Conclusion: Closed-form solutions were found for the space-time cross-correlation and the space-frequency cross spectrum, both of which can be applied to continuous Rayleigh fading MIMO channels for both frequency flat and frequency selective fading wireless scenarios, with a non-isotropic scatterer distribution. The method was demonstrated for a Laplacian distribution, but it is equally applicable for other common distribution functions in the assessment of performance with various BS antenna spacings.

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D.B. Smith (*Faculty of Engineering, Information and Communications Technology (ICT) Group, Level 24, Building 1, University of Technology, 1 Broadway, Sydney (UTS), NSW 2007, Australia*)

T.D. Abhayapala (*Department of Telecommunications Engineering, Research School of Information Sciences and Engineering (RSISE), The Australian National University (ANU), Canberra, ACT 0200, Australia*)

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