SPATIAL DECOMPOSITION OF MIMO WIRELESS CHANNELS

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ABSTRACT

In this paper a novel decomposition of spatial channels is developed to provide insight into spatial aspects of multiple antenna communication systems. The underlying physics of the free space propagation is used to model the channel in scatterer free regions around the transmitter and the receiver, and the rest of the complex scattering media is represented by a parametric model. The channel matrix is separated into a product of known and random matrices where the known portion shows the effects of the physical configuration of antenna elements. We use the model to show the intrinsic degrees of freedom in a multi-antenna system. Potential applications of the model are briefly discussed.

1. INTRODUCTION

Multiple antenna systems has shown significant improvement for communication over the wireless channel compared to the traditional single antenna systems. The main idea of these systems is to exploit the spatial aspects of multipath propagation to design spatial diversity receivers or transmitters [1] and spatial multiplexing systems [2]. Maximal exploitation of spatially separated multiple antennas in wireless communication needs realistic modelling of the multi-input multi-output (MIMO) channel models that include spatial aspects of signal propagation. Most of the existing channel models [3] assumed Rayleigh/Rice/Nakagami fading envelopes for the received signal and/or independent fading between each of the transmit-receiver antenna pairs. These models have poor physical significance because the antenna configuration is not modelled, hence they do not provide any insight into spatial aspects of MIMO channels and are not an ideal framework to build space-time communication systems.

Recent work related to better understanding of MIMO systems are reported in [4, 5, 6, 7, 8, 9]. The spatial channel model proposed in [8] is more general and realistic than the usual independent models, however it is based on uniform linear array antennas and discrete set of scatterers around the transmitter and the receiver arrays. this limits its generality. Another similar model appears in [9], where a general physical model is used to derive a virtual channel representation comprising of sets of virtual departing angles, arrival angles and scatterers. Again this model is only applicable to uniform linear arrays. Both of the above models have been used in good effect to study the limitations of MIMO systems.

The contribution of this work is the decomposition of a general spatial channel model which includes physical parameters of antenna configurations and a tractable parameterization of complex scattering environment, to overcome the above mentioned deficiencies in the traditional models. We separate the physical spatial channel into three regions of interest: (i) scatterer free region around the transmitter antennas, (ii) scatterer free region around the receiver antennas, and (iii) complex scattering media which is the complement of the union of regions in (i) and (ii). We use the underlying physics of the free-space propagation to model the channel in regions (i) and (ii), and the complex scattering media (iii) is represented by a parametric model. With this separation of the physical channel, we are able to decompose the channel matrix of a MIMO system into a product of three matrices, where two of them are fixed and known for a given antenna orientations and the other representing the parameters of the scattering media (iii). We also show that the combined spatial channel has an intrinsic spatial dimensionality governed by the minimum radii of balls which contains the transmitter and receiver antenna regions respectively.

2. SPATIAL CHANNEL MODEL

Consider a MIMO system with Q transmit antennas located at positions x_q, $q = 1, 2, \ldots, Q$ from a transmitter origin and P receiver antennas located at positions z_p, $p = 1, 2, \ldots, P$ from a receiver origin, within balls of radius r_T and r_R respectively, as shown in Fig.1. We assume that

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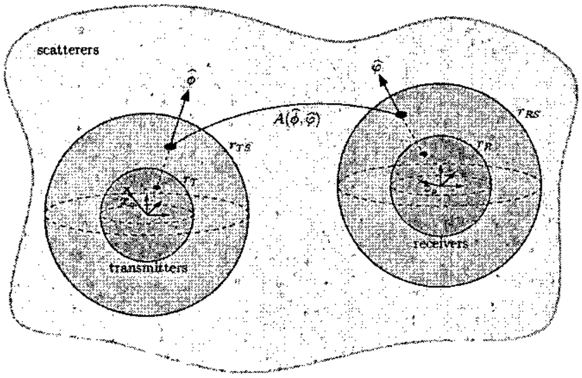

Figure 1: A general scattering model for a flat fading MIMO system. r_{T} and r_{R} are the radii of spheres which enclose the transmitter and the receiver arrays. $A(\widehat{\phi}, \widehat{\varphi})$ represents the gain of the complex scattering environment for signals leaving the transmitter scattering free region from direction $\widehat{\phi}$ and arriving at the receiver scattering free region from direction $\widehat{\varphi}$.

scatters are distributed outside balls of radii r_{TS} ($> r_{\mathrm{T}}$) and r_{RS} ($> r_{\mathrm{R}}$). Thus, the wireless channel has three spatial regions, namely, scattering free balls encompassing transmit antennas and receiver antennas, and the rest of the space assumed to be a complex scattering media. We assume that the surface of transmitter/receiver scattering free ball is in the farfield of the transmitter/receiver origin.

In this paper, we only consider flat fading channel environment where propagation delay is always less than the symbol period. Our attention is aimed to understand fading due to spatial effects rather than temporal effects.

Let $\boldsymbol{u} = [u_1, u_2, \ldots, u_Q]$ be the vector of baseband transmitted signals from the Q transmitters during a signaling interval, and let $A(\widehat{\phi}, \widehat{\varphi})$ be the complex gain of a signal leaving from the transmitter-scattering-free ball at an angle $\widehat{\phi}$ and entering the receiver-scattering-free ball at an angle $\widehat{\varphi}$, then the baseband received signal during a signaling interval by the pth receiver antenna located at $\boldsymbol{z}_p$ from the receiver origin is given by

$$v_p = \int\int \sum_{q=1}^{Q} u_q A(\widehat{\phi}, \widehat{\varphi}) e^{ik\boldsymbol{x}_q\cdot\widehat{\phi}} e^{-ik\boldsymbol{z}_p\cdot\widehat{\varphi}} d\widehat{\phi}\, d\widehat{\varphi} + n_p \tag{1}$$

where n_p is the Additive White Gaussian Noise (AWGN) at the pth receiver antenna, and the integrations are over the unit sphere.

Now we use (1) to express the vector of received signals as

$$\boldsymbol{v} = \boldsymbol{H}\boldsymbol{u} + \boldsymbol{n}, \tag{2}$$

where $\boldsymbol{v} = [v_1, v_2, \ldots, v_P]'$, $\boldsymbol{n} = [n_1, n_2, \ldots, n_P]$, with $[\cdot]'$ the vector transpose, and $\boldsymbol{H}$ is a $P \times Q$ channel matrix with (p, q) element given by

$$\{\boldsymbol{H}\}_{pq} = \int\int A(\widehat{\phi}, \widehat{\varphi})\, e^{ik\boldsymbol{x}_q\cdot\widehat{\phi}}\, e^{-ik\boldsymbol{z}_p\cdot\widehat{\varphi}}\, d\widehat{\phi}\, d\widehat{\varphi}. \tag{3}$$

Equation (3) is a physically sound way of modelling a spatial channel where the factors $e^{ik\boldsymbol{x}_q\cdot\widehat{\phi}}$ and $e^{-ik\boldsymbol{z}_p\cdot\widehat{\varphi}}$ represents the free-space wave propagation inside the scattering free transmit and receiver regions respectively, and $A(\widehat{\phi}, \widehat{\varphi})$ captures complex gains due to scatterers. However, the integral representation (3) is not directly usable in applications. In the next section, we use modal analysis to convert (3) into a form which we can exploit and reveal the underlying structure of the channel matrix $\boldsymbol{H}$.

3. MODAL DECOMPOSITION OF CHANNEL MATRIX

Our analysis in this paper considers the 2-dimensional space[1] using spatial basis functions on a circle. However, the results can be extended to 3-dimensional space by using spherical harmonics on a sphere and will be reported in a future publication.

Consider the Fourier series expansion of $e^{ik\boldsymbol{x}\cdot\widehat{\phi}}$ given by the Jacobi-Anger expansion [10, p. 67],

$$e^{ik\boldsymbol{x}\cdot\widehat{\phi}} = \sum_{m=-\infty}^{\infty} \left[J_m(kx) e^{-im(\phi_x - \pi/2)}\right] e^{im\phi} \tag{4}$$

where $J_m(\cdot)$ are the Bessel functions of integer order m, $\boldsymbol{x} = (x, \phi_x)$ and $\widehat{\phi} = (1, \phi)$ in polar coordinate system. Bessel functions[2] $J_m(\cdot)$ for $|m| \geq 1$ in (4) have a spatial high pass character ($J_0(\cdot)$ is spatially low pass). That is, $J_m(\cdot)$ starts small increasing monotonically to its maximum at arguments around $O(m)$ before decaying slowly. It was shown in [11] that $J_m(kr) \approx 0$ for $|m| > ker/2$. Thus, we can safely truncate the series (4) by $2M + 1$ terms where $M = \lceil ker/2 \rceil$ where $\lceil\cdot\rceil$ is the ceiling operator and $e \approx 2.7183$. Now, we write

$$e^{ik\boldsymbol{x}_q\cdot\widehat{\phi}} = \sum_{m=-M_{\mathrm{T}}}^{M_{\mathrm{T}}} \left[J_m(kx_q) e^{-im(\phi_q - \pi/2)}\right] e^{im\phi}, \tag{5}$$

for $x_q < r_{\mathrm{T}}$, $q = 1, \ldots, Q$ where $\boldsymbol{x}_q = (x_q, \phi_q)$ and $M_{\mathrm{T}} = \lceil ker_{\mathrm{T}}/2 \rceil$. Note that we have chosen M_{T} such that (5) hold for each antenna within a circular region of radius r_{T}. Similarly we can write

$$e^{-ik\boldsymbol{z}_p\cdot\widehat{\varphi}} = \sum_{m=-M_{\mathrm{R}}}^{M_{\mathrm{R}}} \left[J_m(kz_p) e^{im(\varphi_p - \pi/2)}\right] e^{-im\varphi}, \tag{6}$$

[1] This models the situation in 3D where the multipath propagation is restricted to the horizontal plane, having no component arriving at significant elevations. Thus the signal field is assumed to be height invariant.

[2] Note that $J_{-m}(\cdot) = (-1)^m J_m(\cdot)$.

for $z_p < r_R$, $p = 1, \dots, P$ where $z_p = (z_p, \varphi_p)$ and $M_R = \lceil ker_R/2 \rceil$.

By substituting (5) and (6) into (3) we express the channel matrix as

$$H = J_R H_s J_T', \tag{7}$$

where

$$J_R = \tag{8}$$
$$\begin{pmatrix} J_{-M_R}(kz_1)e^{iM_R(\varphi_1-\frac{\pi}{2})} & \dots & J_{M_R}(kz_1)e^{-iM_R(\varphi_1-\frac{\pi}{2})} \\ \vdots & \ddots & \vdots \\ J_{-M_R}(kz_P)e^{iM_R(\varphi_P-\frac{\pi}{2})} & \dots & J_{M_R}(kz_P)e^{-iM_R(\varphi_P-\frac{\pi}{2})} \end{pmatrix}$$

$$J_T = \tag{9}$$
$$\begin{pmatrix} J_{-M_T}(kx_1)e^{-iM_T(\phi_1-\frac{\pi}{2})} & \dots & J_{M_T}(kx_1)e^{iM_T(\phi_1-\frac{\pi}{2})} \\ \vdots & \ddots & \vdots \\ J_{-M_T}(kx_Q)e^{-iM_T(\phi_Q-\frac{\pi}{2})} & \dots & J_{M_T}(kx_Q)e^{iM_T(\phi_Q-\frac{\pi}{2})} \end{pmatrix}$$

and H_s is a $(2M_R + 1) \times (2M_T + 1)$ matrix with (ℓ, m) element given by

$$\{H_s\}_{\ell m} = \int_0^\pi \int_0^\pi A(\phi, \varphi)\, e^{-i(\ell - M_R - 1)\varphi} e^{i(m - M_T - 1)\phi}\, d\phi d\varphi \tag{10}$$

for $\ell = 1, \dots, 2M_R+1$, $m = 1, \dots, 2M_T+1$. We name J_R as the receiver configuration matrix, J_T as the transmitter configuration matrix and H_s as the scattering matrix.

Note that the scattering gain function $A(\phi, \varphi)$ is periodic in both ϕ and φ, hence it is natural to expand it as a 2-dimensional Fourier series

$$A(\phi, \varphi) = \sum_{\ell=0}^{\infty} \sum_{m=0}^{\infty} \beta_{\ell m} e^{i\ell\varphi} e^{-im\phi}. \tag{11}$$

It follows that

$$\{H_s\}_{\ell m} = \beta_{\ell - M_R - 1, m - M_T - 1} \tag{12}$$

for $\ell = 1, \dots, 2M_R + 1$, $m = 1, \dots, 2M_T + 1$.

We have following comments regarding the channel matrix given by (7) and the subsequent developments:

i. Equation (7) decomposes the conventional block channel matrix H into a product of three matrices J_R, J_T, and H_s according to the three different regions of propagation in a MIMO wireless communication system as shown in Fig.1.

ii. The receiver configuration matrix J_R is known and fixed for a given receiver antenna array structure. It includes antenna positions and orientations relative to the receiver origin and characterizes the effect of finite separation of antennas on the overall channel matrix H.

iii. Similarly, the transmitter configuration matrix J_T is known for a given transmit antenna structure and describes its effects on the overall channel.

iv. The scattering matrix H_s is generally unknown and parameterized by $\{\beta_{\ell m}\}$, $\ell \in [-M_R : M_R]$, $m \in [-M_T : M_T]$. For a random scattering environment, β_{lm} are random variables and for a rich scattering environment, β_{lm} are independent of each other.

v. In the wireless literature on MIMO systems, the elements of H are modelled as random variables. However in our model, deterministic portions due to transmit and receiver antenna configurations are factored out leaving only the parameters of the scattering environment to be modelled as random variables. This is a clear advantage of the current model over existing models since one can explicitly see the role of antenna configuration within the model.

4. DEGREES OF FREEDOM IN SPATIAL CHANNELS

In this section, we wish to quantify the Spatial Degrees of Freedom (SPDoF) in a given MIMO system. In other words, what is the number of free parameters available when we have Q transmit antennas and P receiver antennas inside balls of radii r_T and r_T respectively. Intuitively, it is not the number of antennas but the area or volume of the spatial region which contains the antennas, controls the number of SPDoF in a given MIMO system. In this section, we outline the number of SPDoF offered by three critical regions in our MIMO channel model.

The rank of the channel matrix H determines the effective number of independent parallel channels between the transmit array to the receiver array. In some of the literature, it is considered to have rank of $\min\{Q, P\}$ by assuming independent channel elements. However, when the antenna configuration take into account the elements of H are not independent of each other. Equation (7) factors out effects of antenna configuration to separate matrices, thus

$$\text{rank}\{H\} = \min\left\{\text{rank}\{J_R\}, \text{rank}\{H_s\}, \text{rank}\{J_T\}\right\}. \tag{13}$$

Recall that the number of columns M_R and M_T in receive and transmit configuration matrices are determined by the radii of the receive and transmit balls (circular regions for 2D case) r_R and r_T, respectively. Thus, the rank of these matrices cannot be exceed M_R and M_T by increasing the number of antennas. Therefore, if $P > M_R$ and $Q > M_T$ antennas are located inside the circular regions of radius r_R and r_T then the maximum rank of the antenna configuration matrices will be $2M_R + 1$ and $2M_T + 1$, respectively.

A rich scattering environment is capable of providing the maximum number of independent links between transmitter modes m to receiver modes[3] ℓ, thus it has sufficient independent scattering coefficients $\beta_{\ell m}$. Therefore, the scattering matrix $\boldsymbol{H}_s$ of a rich environment has the greatest rank of $\min\{2M_{\rm R}+1, 2M_{\rm T}+1\}$. However if the scattering media is not rich, i.e., not enough reflectors/scatteres such as trees, buildings etc., then the scattering coefficients could depend on each other and the rank of $\boldsymbol{H}_s$ could be less than $\min\{2M_{\rm R}+1, 2M_{\rm T}+1\}$. To conclude this section, we make the following proposition:

Proposition 1 *Spatial degrees of freedom of a MIMO wireless system in a rich scattering environment is given by*

$$\min\{2\lceil\frac{ker_R}{2}\rceil+1, P, 2\lceil\frac{ker_T}{2}\rceil+1, Q\} \qquad (14)$$

where $k = 2\pi/\lambda$ *is the wavenumber corresponding to wavelength* λ *of the carrier, P and Q are the number of receiver and transmitter antenna elements located inside the balls of radii* r_R *and* r_T*, respectively.*

5. APPLICATIONS

We believe that the development in this paper has wide range of applications. In this section we briefly discuss some of the possible applications.

There has been larger number of papers showing the capacity gain of MIMO systems (e.g., [2]). Recent work has shown the limitation of MIMO systems due to channel correlation [6, 7]. Using channel decomposition developed in this paper, we can further analyze the MIMO capacity in terms of physical antenna configuration.

Basic spatial diversity systems assumed uncorrelated signals at each diversity branch. This assumption is invalid in most practical systems where finite antenna separation and non isotropic scattering cause correlated signals. With the current channel model, we can redevelop spatial diversity systems taking into account the effects of antenna separation.

The model can also be used to design new spatial multiplexing systems or improve existing systems taking into consideration of deterministic portions of the channel matrix. Recall that the BLAST [2] and most of the space-time coding systems assumed uncorrelated received signals.

6. CONCLUSIONS

The channel matrix of a MIMO system can be factored into deterministic and random matrices where the deterministic portion depends on receiver and transmitter antenna configurations.

[3]Note the distinction between the mode to mode channels provided by $\boldsymbol{H}_s$ and the antenna to antenna channels provided by $\boldsymbol{H}$.

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