Coherent Broadband Source Localization by Modal Space Processing

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Abstract

A novel method for coherent broadband direction of arrival (DOA) estimation is introduced based on physics of signal propagation. This technique does not require any preliminary knowledge of DOA angles nor the number of sources to be estimated. As an illustration, two simulation examples covering single and multi-group scenarios are presented.

I. Introduction

The problem of estimation of direction of arrival (DOA) broadband sources has renewed applications in wireless communication systems due to the use of multiple antenna receivers. In a complex multipath environment, received signal from different directions may be correlated, which prevents the application of narrowband DOA estimation techniques to estimate DOA.

Wang and Kaveh [1] introduced the use of focusing matrices for the purpose of coherent signal-subspace processing for DOA estimation of farfield wideband sources. These focusing matrices are used for the alignment of the signal subspaces of narrowband components within the bandwidth of the signals, followed by the averaging of narrowband array data covariance matrices into a single covariance matrix, thus achieving a substantial reduction in data. Now, any signal subspace direction finding procedure, (such as MUSIC [2] or its variants), maximum likelihood (ML), or minimum variance (MV), can be applied to this averaged covariance matrix to obtain the desired parameter estimates. The problem with the above theory being that it requires knowledge of the direction of arrivals which are unknown and is applicable to only a pair of sources. In later years, the technique was developed and refined [3], [4] to account for multiple sources but the problem of prior information about the DOA still remained. In this paper, we have used modal analysis technique to propose novel focusing matrices that do not require preliminary DOA estimates and are completely independent of the signal environment.

The spatial resampling method is one technique that does not require preliminary knowledge of DOA in order to localize wideband sources. It was first introduced by Krolik and Swinger [5] and is motivated by treating the output of a discrete array as being the result of spatially sampling a continuous linear array. The same concept is also known as an interpolated array approach used in [6]. Krolik and Swingler [5] used digital interpolation methods to resample the array data. An alternative technique is suggested in this paper. Under this technique a set of resampling matrices has been proposed which is same for the full field of view of the array data, unlike in the case of [6].

The number of computations can be reduced by combining the focusing matrices and spatial resampling matrices to form modal covariance matrices. Simulation results show that the method works well for single group as well as multi-group sources at lower and upper frequency band ratios without preliminary DOA estimates.

II. Problem Formulation

Let us consider a double sided linear array of $2Q+1$ sensors, located at distances x_q, $q = -Q, \cdots, 0, 1, \cdots, Q$ from the array origin, which receives signals from V wideband sources in space. Let $\Theta = [\theta_1, \theta_2, \ldots, \theta_v]$, be a vector containing bearings of each source with reference to the array axis where θ_v is the direction of the vth source. We assume that the source signal and the noise are confined in a bandwidth of $k \in [k_l, k_u]$, where k_l and k_u are lower and upper band edges respectively. We use wavenumber $k = 2\pi f/c$ where f is the frequency in Hz and c is the speed of wave propagation, to represent frequency in this paper. The signal received at each sensor is Discrete Fourier Transformed into M distinct frequency bins within the design bandwidth. The array output in the mth frequency bin can be represented as:

$$z(k_m) = \sum_{v=1}^{V} a(\theta_v; k_m) s_v(k_m) + n(k_m), \qquad (1)$$

where, $s_v(\cdot)$ is the signal received from the vth source at the origin, $n(\cdot)$ is the uncorrelated noise data and

$$a(\theta; k) = [e^{-ikx_{-Q}cos\theta}, \ldots, e^{-ikx_Q cos\theta}]' \qquad (2)$$

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where $[\cdot]'$ denotes the transpose operator and $i = \sqrt{-1}$. We write (1) in matrix form as

$$z(k_m) = A(\Theta; k_m)s(k_m) + n(k_m), \quad (3)$$

for $m = 1, \ldots, M$ where

$$A(\Theta; k) = [a(\theta_1; k), \ldots, a(\theta_v; k)], \quad (4)$$

and

$$s(k) = [s_1(k), \ldots, s_v(k)]'. \quad (5)$$

We wish to determine the direction of arrival (DOA) Θ from the observed data $z(k_m)$.

The correlation matrix of the observed data in the mth frequency bin is defined as

$$R_z(k_m) = E\{z(k_m)z(k_m)^H\}, \quad (6)$$

where $[\cdot]^H$ denotes conjugate transpose operation and E is the expectation operator. Substituting (3) in (6), we get

$$\begin{aligned} R_z(k_m) =& A(\Theta; k_m)R_s(k_m)A^H(\Theta; k_m) \quad (7) \\ &+ E\{n(k_m)n(k_m)^H\} \end{aligned}$$

where

$$R_s(k_m) = E\{s(k_m)s(k_m)^H\}, \quad (8)$$

is the source correlation matrix. Here, we assume that the source signals and noise are uncorrelated.

III. Focusing Matrices for Coherent Wideband Processing

In this section, we briefly outline the focusing method. The first step following the frequency decomposition of the array data vector is to align or focus the signal space at all frequency bins into a common one at a reference frequency by focusing matrices $T(k_m)$ that satisfy

$$T(k_m)A(\Theta; k_m) = A(\Theta; k_0), \quad m = 1, \ldots, M, \quad (9)$$

where $k_0 \in [k_l, k_u]$ is some reference frequency and $A(\Theta; k)$ is the direction matrix defined by (4). Applying the M focusing matrices to the respective array data vectors (3) gives the following focused array data vector,

$$\begin{aligned} T(k_m)z(k_m) =& A(\Theta; k_0)s(k_m) + T(k_m)n(k_m) \\ & m = 1, \ldots, M \end{aligned}$$

Then the focused and frequency averaged data covariance matrix is given by

$$R = \sum_{m=1}^{M} T(k_m)\, E\{z(k_m)z^H(k_m)\}\, T^H(k_m). \quad (10)$$

We use (6), (7) and (9) to get

$$R = A(\Theta; k_0)\overline{R}_s A^H(\Theta; k_0) + R_{\text{noise}} \quad (11)$$

where

$$\overline{R}_s = \sum_{m=1}^{M} R_s(k_m), \quad (12)$$

and

$$R_{\text{noise}} = \sum_{m=1}^{M} T(k_m)\, E\{n(k_m)n^H(k_m)\}\, T^H(k_m). \quad (13)$$

The focused data covariance matrix (11) is now in a form in which almost any narrowband direction finding procedure may be applied. Here, we apply the minimum-variance (MV) method of spatial spectral estimation [7] to the frequency averaged data covariance matrix R.

Several methods of forming focusing matrices have been suggested in the literature. The focusing methods of [1,3,4, 8] require preliminary DOA estimates in order to construct the focusing matrices. This constitutes a severe disadvantage in practical applications since it leads to biased DOA estimates.

We now use modal analysis techniques to propose novel focusing matrices which do not require preliminary DOA estimates and are completely independent of the signal environment. Here we only consider a linear (possibly nonuniform) array but it may be generalized to arbitrary array configurations.

Using Jacobi-Anger expansion [9] we may write

$$e^{-ikx\cos\theta} = \sum_{n=0}^{\infty} i^n(2n+1)j_n(kx)\, P_n(\cos\theta), \quad (14)$$

where $j_n(\cdot)$ is the spherical Bessel function and $P_n(\cdot)$ is the Legendre function. The series expansion (14) gives an insight into the spatial wavefield along a linear array. Observe that in each term of the series, the arrival angle θ dependency is separated out from the sensor location x_q and the frequency k. Therefore we may use the above expansion to write the array DOA matrix $A(\Theta; k)$ as a product of two matrices, one depending on DOA angles and the other depending on frequency and sensor locations. Before reaching this step, there are certain hurdles to overcome; for example, expansion (14) has an infinite number of terms, thus we can not use it to represent finite-dimensional matrices.

For a finite aperture array with finite bandwidth signal environment, the series (14) can be safely truncated by finite number of terms (say N) without generating significant modelling errors. We show this somewhat informally below.

Figure 1 shows plots of a few spherical Bessel functions $j_n(\cdot)$ against its argument. We can observe from Figure 1 that for a given kx, the function $j_n(kx) \rightarrow 0$ as n becomes large. This observation is supported by the following asymptotic form [10]

$$j_n(kx) \approx \frac{(kx)^n}{1 \cdot 3 \cdot 5 \ldots (2n+1)} \quad \text{for } kx \ll n. \quad (15)$$

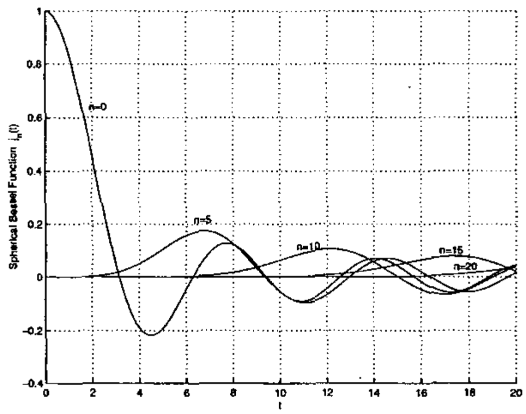

Fig. 1: spherical Bessel functions of order $n = 0, 5, 10, 15, 20$.

Therefore, we can notice that the factor $(2n+1)j_n(kx_q)$ in (14) decays as n grows larger beyond $n = kx_q$. Suppose that the minimum frequency of the signal band is k_l. Then we can truncate (14) to N terms if $N > k_l x_Q$, where x_Q is the distance to the Qth sensor (the maximum array dimension). It is difficult to derive an analytical expression for N, but a convenient rule of thumb [11] is $N \sim 2k_l x_Q$. More recent work [12] shows that the series (14) can be truncated by $N = kex/2$ terms with negligible error.

Now we substitute the first $N+1$ terms of (14) into (2) and thus write the array steering vector for farfield sources as

$$a(\theta;k) = J(k)\begin{bmatrix} P_0(\cos\theta) \\ \vdots \\ P_N(\cos\theta) \end{bmatrix}, \tag{16}$$

where

$$J(k) = \tag{17}$$
$$\begin{bmatrix} i^0(2\cdot 0+1)j_0kx_{-Q} & \dots & i^N(2N+1)j_Nkx_{-Q} \\ \vdots & \ddots & \vdots \\ i^0(2\cdot 0+1)j_0kx_Q & \dots & i^N(2N+1)j_Nkx_Q \end{bmatrix}.$$

We use (16) in (4) to write the array DOA matrix for farfield signal environment as

$$A(\Theta;k) = J(k)P(\Theta), \tag{18}$$

where the $(N+1)\times V$ matrix

$$P(\Theta) = \begin{bmatrix} P_0(\cos\theta_1) & \dots & P_0(\cos\theta_V) \\ \vdots & \ddots & \vdots \\ P_N(\cos\theta_1) & \dots & P_N(\cos\theta_V) \end{bmatrix}. \tag{19}$$

The $(2Q+1)\times(N+1)$ matrix $J(k)$ depends on the frequency k and the sensor locations and is independent of the DOA of the signals. Suppose $(2Q+1) > (N+1)$ and $J(k)$ has full rank $N+1$ if the sensor locations are chosen appropriately. With this assumption and using (18), we can propose a set of focusing matrices $T(k_m)$ given by

$$T(k_m) = J(k_0)\left[J^H(k_m)J(k_m)\right]^{-1}J^H(k_m)$$
$$m = 1,\dots,M \tag{20}$$

which satisfies the focusing requirement (9); recall that k_0 is the reference frequency.

The major advantage of the focusing matrices (20) over the existing methods is that these matrices do not need preliminary DOA estimates and accurately focus signal arrivals from all directions. Also note that these matrices can be calculated beforehand for a given array geometry and frequency band of interest.

IV. Spatial Resampling Methods

Spatial resampling is another method [5] used to focus the wideband array data to a single frequency so that existing narrowband techniques may be used to estimate the DOA. The basic idea of spatial sampling is outlined below.

Suppose we have a separate uniform array with half wavelength spacing for each frequency bin with the same effective array aperture in terms of wavelength. Thus for M frequencies, there are M arrays and the sensor separation of the mth array is $\lambda_m/2$ where $\lambda_m = 2\pi/k_m$. If each array has $2Q+1$ sensors, then the aperture length is the same for all frequencies in terms of corresponding wavelength. Then the mth array steering vector for farfield sources is given by

$$a(\theta;k_m) =$$
$$[e^{i\pi Q\cos\theta},\cdots,e^{i\pi\cos\theta},1,e^{-i\pi\cos\theta},\dots,e^{-i\pi Q\cos\theta}]$$
$$= a(\theta),\quad m = 1,\dots,M.$$

That is the steering vectors of all arrays are equal and hence from (4) the DOA matrices of all arrays are the same:

$$A^{(m)}(\Theta;k_m) = A(\Theta),\quad m = 1,\dots,M, \tag{21}$$

where $A^{(m)}(\Theta;k)$ is the DOA matrix of the mth subarray. Hence if we have M arrays for each frequency bin with the same aperture, then their covariance matrices can be averaged over frequency without losing DOA information. The average covariance matrix can then be used with existing narrowband DOA techniques to estimate DOA angles.

Of course it is not actually practical to have a separate array for each frequency. This problem can be overcome by having a single array and using the received array data to form the array data for M (virtual) arrays by interpolation/extrapolation of the received array data. This is tantamount to constructing a continuous sensor using the received array data and resampling it. There are several methods reported in the literature. In [6] the field of view of the array is divided into several sectors, and a different interpolation matrix is calculated for each sector using a least squares fit.

We will now show how to use the modal techniques to find a transformation matrix to calculate array data for M virtual arrays given the output of a single array. Sensor locations for the real array can be arbitrary on a line, i.e., there is no requirement for it to be a uniformly spaced array. From (18) the real array DOA matrix in the mth frequency bin is given by

$$A(\Theta; k_m) = J(k_m)P(\Theta), \qquad (22)$$

and the DOA matrix of the mth virtual array at frequency k_m would be

$$A^{(m)}(\Theta; k_m) = J(k_m)P(\Theta), \qquad (23)$$

where from (17) with $k_m x_q = q\pi$,

$$J(k_m) = \begin{bmatrix} i^0(2\cdot 0+1)j_0(-\pi Q) & \dots & i^N(2N+1)j_N(-\pi Q) \\ \vdots & \ddots & \vdots \\ i^0(2\cdot 0+1)j_0(\pi Q) & \dots & i^N(2N+1)j_N(\pi Q) \end{bmatrix} = a\overline{J}, \quad m = 1, \dots, M$$

which is a constant matrix, independent of m and k_m. Therefore we can write

$$\begin{aligned} A^{(m)}(\Theta; k_m) &= \overline{J}P(\Theta), \\ &= A(\Theta) \quad m = 1, \dots, M, \end{aligned} \qquad (24)$$

which is same for all frequency bins. By manipulating, (22) and (24), and using the pseudo inverse of $J(k_m)$ we obtain the least-square solution

$$A^{(m)}(\Theta; k_m) = T(k_m)A(\Theta; k_m), \quad m = 1, \dots, M, \qquad (25)$$

where

$$T(k_m) = \overline{J}[J^H(k_m)J(k_m)]^{-1}J^H(k_m), \quad m = 1, \dots, M, \qquad (26)$$

are the spatial resampling matrices. Now these spatial resampling matrices (they act as focusing matrices) can be used to align the array data in different frequency bins, so that narrowband DOA techniques can be applied. Similar to the focusing matrices (20), these spatial resampling matrices (26), do not require preliminary DOA estimation and depend only on the array geometry and the frequency.

V. Modal Space Processing

Observe that the proposed focusing matrices (20) and the spatial re-sampling matrices (26) have a common (generalized inverse) matrix factor

$$G(k_m) \equiv [J^H(k_m)J(k_m)]^{-1}J^H(k_m), \quad m = 1, \dots, M, \qquad (27)$$

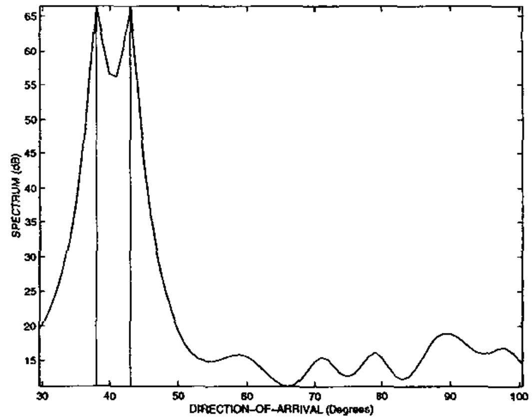

Fig. 2: The estimated spatial spectrum of the correlated sources using the algorithm of [1].

and only differ by the frequency independent factors $J_0(k_0)$ and $\overline{J}$. Also note that from (18),

$$G(k_m)A(\Theta; k) = P(\Theta), \quad m = 1, \dots, M, \qquad (28)$$

i.e., $G(k_m)$ transforms the array DOA matrix into a frequency invariant DOA matrix. Therefore we can use $G(k_m)$ instead of $T(k_m)$ to align the broadband array data to form a frequency averaged covariance matrix. Intuitively, one can say that the matrices $G(k_m)$ transform the $2Q+1$ array data vector $z(k_m)$ into a $N+1$ *modal data* vector. Now we can estimate the frequency averaged *modal covariance matrix* as

$$\hat{R} = \sum_{m=1}^{M} G(k_m)\, z(k_m) z^H(k_m)\, G^H(k_m) \qquad (29)$$

and the MV spectral estimate

$$\hat{Z}(\theta) = \frac{1}{\begin{bmatrix} P_0(\cos\theta) \\ \vdots \\ P_N(\cos\theta) \end{bmatrix} \hat{R}^{-1} \left[P_0(\cos\theta), \dots P_N(\cos\theta)\right]}. \qquad (30)$$

Comments:

1. This method (one can refer it as the *Modal Space Processing (MSP) method*) involves less computation compared to the other two methods since the modal space has less dimensions ($N+1$) than the signal subspace ($2Q+1$).
2. As for the other two methods, the modal space method does not require preliminary DOA estimates.
3. One can consider the modal space method as a superset of focusing matrices and spatial resampling methods.

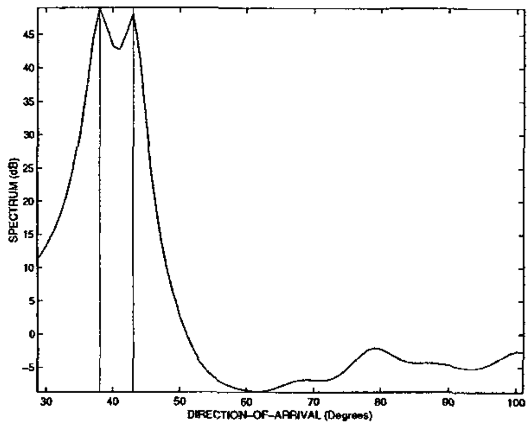

Fig. 3: The estimated spatial spectrum of the correlated sources using Modal Space Processing.

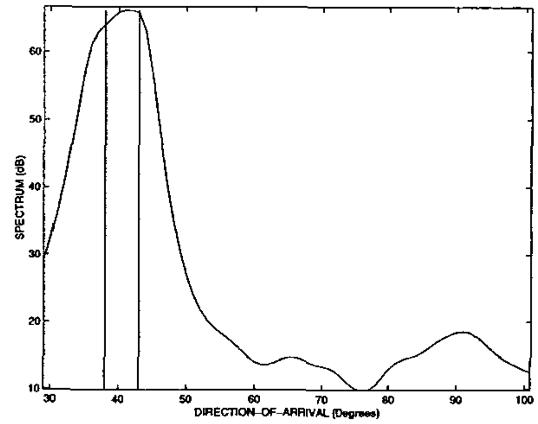

Fig. 4: The estimated spatial spectrum of the correlated sources using prior angle estimation of 53°.

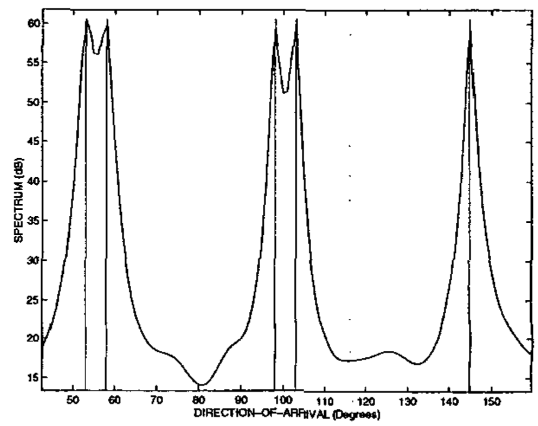

Fig. 5: The estimated spatial spectrum of the Multigroup sources using algorithm of [3]

VI. Simulation

In this section, the simulation results have been presented in order to demonstrate the effectiveness of MSP method. A linear array of 19 nonuniformly spaced sensors has been used for MSP technique. The use of nonuniformly spaced sensor array for broadband application has been discussed in [13]. The sensor spacing is kept uniform while performing the simulation of examples that follow the algorithms suggested in past literature [1,3]. These simulations are presented in this section for comparison of results. The source signal and the noise are stationary zero-mean white Gaussian processes. Noise at each sensor is independent of the other. Signal received at each sensor is Discrete Fourier Transformed to get 33 uniformly spaced narrow-band frequency bins within the desired bandwidth. For each trial, 64 independent snapshots are generated for every frequency bins. The frequency averaged modal covariance matrix is calculated using the relation (29). The sources are then localized by using Minimum Variance (MV) direction finding procedure (30) as implemented for narrow-band source localization.

VI.1. A group of two sources

The signal environment consists of two completely correlated sources at angles $\Theta = [38° \ 43°]$. Let $s_1(t)$ be the source at 38°, and the source at 43° is delayed version of $s_1(t)$ and is given by $s_2(t) = s_1(t - t_o)$ with $t_o = 0.125s$ or equivalently in frequency domain $s_2(f) = s_1(f)e^{-jft_o}$. Here, $s_1(f)$ is Fourier Transformed signal $s_1(t)$. The signal-to-noise ratio is 10dB.

The signals used lie within a bandwidth of 40Hz with midband frequency at 100Hz. This gives a lower band edge (f_l = 80Hz) to upper band edge (f_u = 120Hz) ratio of 2 : 3. All the signal parameters are kept identical to those described in [1]. The signals are captured by a linear array of 19 sources. Fig. 3 shows the spectral estimate obtained using MSP. The vertical lines indicate the correct direction of arrival of the sources. For comparison, the results obtained using the method described in [1] has shown in Fig. 2. A preliminary angle estimate of 40.4° has been necessary to correctly estimate the direction of arrivals using the later technique whereas no prior knowledge of angles is required for MSP technique. The graphs reveal that both processes localize the sources with fine accuracy. However, a focusing angle of 53° in the case of [1] will result in Fig. 4 which cannot resolve the true direction of arrivals.

VI.2. Three groups of five sources

The number of sources are now increased to five with bearings $\Theta = [53° \ 58° \ 98° \ 103° \ 145°]$. Complete correlation exists between first and second source. A frequency band of

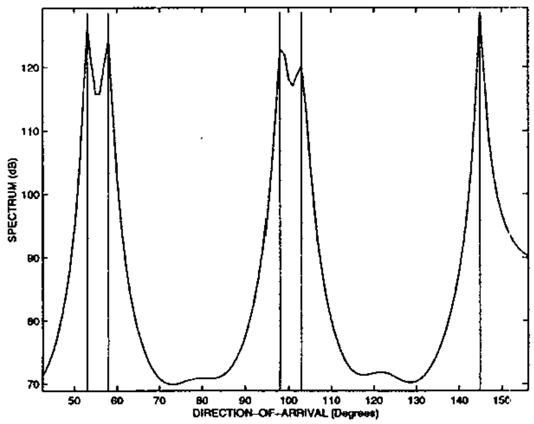

Fig. 6: The estimated spatial spectrum of the Multigroup sources using Modal Space Processing

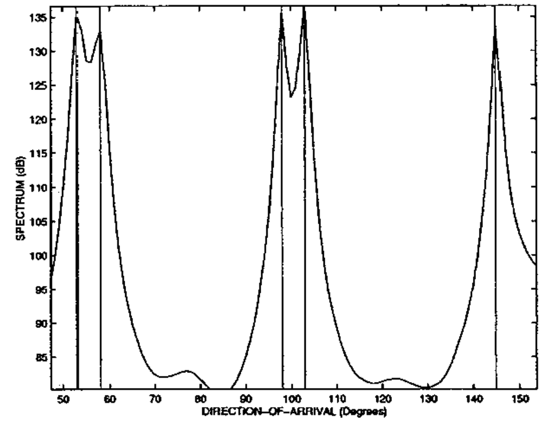

Fig. 7: The estimated spatial spectrum of the Multigroup sources using Modal Space Processing (Wider frequency band [300 : 3000])

$f = [80 : 120]$Hz is used to compare the results with those obtained using the focusing matrix proposed in [3] . Fifteen independent trials were carried out that showed similar results.

Fig. 6 shows one realization obtained by using MSP with MV spectral estimate. Here the number of modes used is N=15. Reducing the value of N degrades the performance of the procedure while increasing its value produces no appreciable improvement. The number of sensors is 19 and are nonuniformly spaced. All the sources are clearly detected without any prior knowledge of source environment. These results can be compared with Fig. 5 that shows the spatial spectrum of multigroup sources using the technique described in [3]. However, prior knowledge of source directions is required by this technique and preliminary angle estimates used for this example is $\beta = [53° \; 55° \; 59° \; 96.7° \; 100.5° \; 104.3° \; 144°]$.

The above simulation is performed for wider band of frequency, bandwidth [300 : 3000]Hz, and the results show that MSP produces better results (Fig 7) as compared to the technique proposed in [3] (Fig 8). A total number of 45 sensors and 55 frequency bins are used in the simulation.

VII. Conclusion

A novel method (MSP) for coherent broadband direction of arrival estimation is introduced. The method does not require any preliminary knowledge of DOA angles nor the number of sources.

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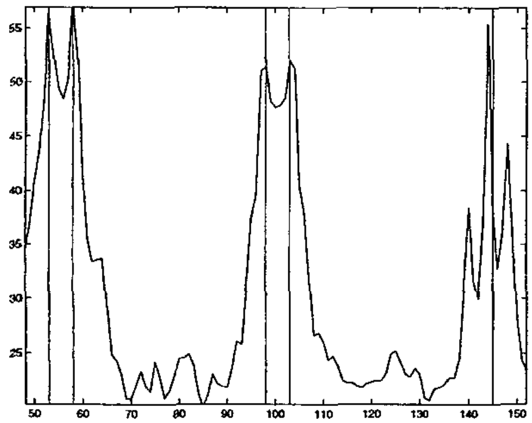

Fig. 8: The estimated spatial spectrum of the Multigroup sources using algorithm of [3] (Wider frequency band [300 : 3000])

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