Analytical Description of Signal Characteristics and Interference For Time Hopped UWB System

Jian Zhang, Rodney A. Kennedy, Thushara D. Abhayapala[1]
Department of Telecommunications Engineering,
Research School of Information Sciences and Engineering,
The Australian National University,
Canberra ACT 0200, Australia
e-mail: {jian.zhang,rodney.kennedy,thushara.abhayapala}@anu.edu.au

Abstract — **In this paper, signal and interference characterisation of Time Hopped Ultra Wideband (TH-UWB) systems is considered. Some special problems often neglected in theoretical studies are discussed, such as the bound between nearfield and farfield and the distortions of wave shape. The transmitted TH-UWB signal is shown to be a cyclostationary process, as is the interference caused by TH-UWB signal on another (victim's) receiver. Further, the statistical properties of the system output are investigated when the input is cyclostationary process and when the system is either linear time-invariant or a stationary process independent of the input. To evaluate the influence of interference, the standard method of Power Spectral Density (PSD) is used. The result shows that the PSD of UWB interference signal is made up of many discrete spectral lines. Since the PSD measure, originating in harmonic analysis, has been argued to be an inappropriate tool to analyze transient signals like UWB, a new method, focusing on the aggregate of time jitter, is developed and tested.**

I. Introduction

With the definition of relative bandwidth,

$$\eta = 2(f_H - f_L)/(f_H + f_L),$$

where f_H and f_L are the high and low frequencies measured at the -10 dB emission points, radio signals can be classified into three types: ultra wideband (UWB), broadband and narrowband, corresponding to $\eta > 25\%$, $1\% < \eta < 25\%$ and $\eta < 1\%$, respectively. For traditional sinusoidal systems, the carrier will need to be very high to maintain a low η if a broad signal bandwidth is required. However, attenuation increases rapidly with frequency increasing, and absorption caused by weather prevents the use of frequency bands above 10GHz. Therefore, high η systems seem to be the only choice when ultra broad signal bandwidth is required.

Since the UWB energy will be spread over the frequency bands allocated to many other wireless systems, it is a major problem to evaluate the mutual interference and solve the satisfactory coexistence of UWB with other systems. It is expected that UWB receivers could collect enough power and protect them against the interference from other systems according to the use of fast spread spectrum techniques. To reduce the interference on other systems caused by UWB, the critical design parameters are: hopping Sequence, pulse shaping, chip and frame period. Further, a low upper bound is imposed on the radiated power of UWB system by regulatory bodies such as FCC.

In this paper, we first highlight some important differences on the signal properties between UWB and conventional sinusoidal systems, including the bound of near-far field, the distortions of wave shape during propagation and transformations. After verifying that TH-UWB signals and the interference against victim's receivers caused by TH-UWB signals are both a cyclostationary process, we investigate the statistical properties of the output when the input is cyclostationary process and the system is either Linear Time Invariant (LTI) or a stationary process independent of the input. Finally, interference problem is determined based on the PSD method and compared with the aggregate of time-jitter mothod.

II. System Model

UWB signals need not have their pulse shapes restricted to Gaussian. However, due to its effective simultaneous high time and frequency resolution, and wide usage in the previous research, it is used in this paper too. A Gaussian monocycle wave can be represented as below:

$$p(t) = A \exp\big(-0.5(t/\sigma)^2\big) \tag{1}$$

and its first derivative

$$p'(t) = 2At/\sigma^2 \exp\big(-0.5(t/\sigma)^2\big) \tag{2}$$

For Time Hopped UWB systems, a transmitter's signal

[1]T.D. Abhayapala also has a joint appointment with the Department of Engineering, Faculty of Engineering and Information Technology, ANU.

is

$$s(t) = \sum_{n=-\infty}^{\infty} p(t - nT_f - c_nT_c - d_nT_d) \quad (3)$$

$$= \sum_{n=-\infty}^{\infty} \delta(t - nT_f - a_n) \otimes p(t) \quad (4)$$

where $c_n \in [1, N_c]$ are the time hopping sequence, $d_n \in [1, -1]$ are the bits to be transmitted. T_c is chip length, T_f is frame length and T_d is a time offset of the pulse position modulation (PPM). A discrete stationary random process a_n is defined as $a_n = c_nT_c + d_nT_d$ and "$\otimes$" represents operation of convolution. So the signal input to the transmitter's antenna can be regarded as the output of a sequence of time-shifted Delta functions input to a shaper $p(t)$.

When I transmitters are used, the aggregated signals (due to multipath) in a receiver become

$$r(t) = \sum_{i=1}^{I} r_i(t) = \sum_{i=1}^{I}\sum_{k=1}^{K} g_{ik}(t)s(t - \tau_{ik}) \quad (5)$$

where $g_{ik}(t)$ are the channel fading coefficients and τ_{ik} are time delays.

III. Propagation of UWB Signals

Carrier Systems are mainly based on sinusoidal signals that have the unique property of keeping their shape unchanged during propagation and conversions (such as addition, differentiation, integration). This property simplifies the process of analysis and design. On the contrary, without this property, non-sinusoidal signals used in UWB add many extra problems while processing these transformations. In some of the existing literature on UWB, system models similar to traditional wireless systems are adopted to simplify analysis. However, those results could lead to false conclusions because the models are untested.

Next we examine how UWB differs from conventional systems.

III.A. Antenna and Near-Far Field

It is well known that when the distance r between transmitter and receiver is much larger than the wavelength of the carrier signal, the receiver can be assumed to be in the far field, and the electric field $\vec{E}$ at that point will be the derivative of the current exciting the antenna. For sinusoidal signals, the shape of waveform does not change after derivation. In contrast, for the Gaussian wave, the shape will change. What we argue and highlight here is the location of the bound between far and near field where the derivation or integration of the exciting current will be involved, respectively. For the Hertizian dipole, the following distance condition is derived to distinguish far field from near field in [3]:

$$r^2 \gg c^2 \left| \int s(t)\,dt / s'(t) \right|, \qquad \text{for } \vec{E}(\vec{r}, t) \quad (6)$$

where c is the speed of light in free space. This equation follows from the fact that the electric field, $\vec{E}$, in the near field should be much larger than that in the far field. When the current $s(t)$ is sinusoidal, $r \gg \lambda/(2\pi)$ can be obtained. While for a Gaussian wave defined in (1), (6) becomes

$$r \gg r_0 = \sqrt{\frac{1.25c^2\sigma^3\ \mathrm{erf}\big((t-m)/(1.4\sigma)\big)}{(t-m)\ \exp(-0.5(t-m)^2/\sigma^2)}} \quad (7)$$

where m represents the possible time jitter, and $texterf(\cdot)$ is the error function. Fig. 1 is the sketch map of (7), where $\sigma = 2$ns, $m = 30$ns, and the pulse is supposed to be transmitted periodically with $T_f = 200$ns. Only finite values in a period are shown (only when the values of denominator are not zero). The figure highlights that far and near field is not well-defined since not only the far/near fields vary with time, but also such a r for far field could not exist for many indoor applications. The basic questions are: should we accept the near zone where E should be proportional to the integration of current when we deal with UWB in those indoor communications? Or we have to reconsider the definition for determining the near field and far field?

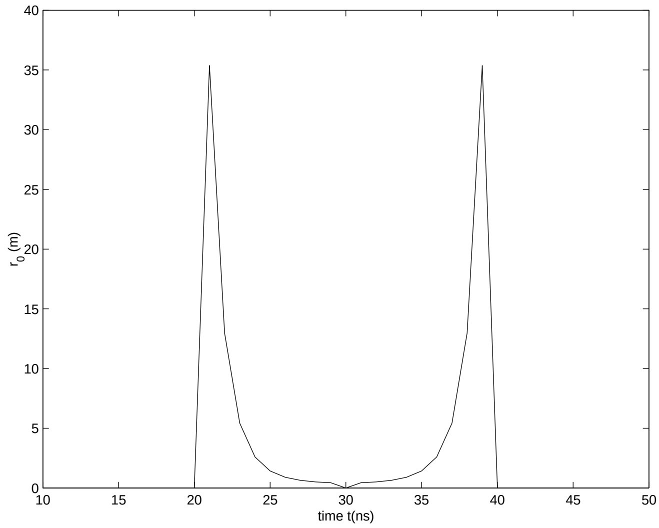

Fig. 1: Plot of (7), the bound of near/far field

III.B. Distortions of Wave Shape

Many distortions of shape could happen to UWB signals during transmission, propagation and detection, as is addressed in [4]. These changes include nonlinear transformation, production of new pulses, and attenuation. Together with multipath, these changes make it very hard to describe such signals analytically and set up an adequate channel model. Conventional optimal receiver techniques using matched filters or correlators may be unsuitable for these signals because of the changed waveform. Also, the TH-UWB system with PPM will experience more difficulties in the process of pseudo-noise code Acquisition, Synchronization and detection than other spread spectrum techniques with less stringent time requirements.

In this situation, the Cluster Model can be expected to take an active role. Based on it, the channel characteristic has been studied reasonably [1]. Similar to the idea of the Cluster Model, the Tapped Delay Line (TDL) model could be applied to the channel description, and maximizing the signal/noise ratio should be an effective metric when designing detection algorithm.

III.C. Channel, ISI and Frequency Selectivity

There is not yet an adequate indoor channel model for UWB signal. But some properties on channel can be confirmed which is helpful to the design of systems.

A periodic signal's *amplitude spectrum* can be represented by the coefficients of its Fourier series. Fig. 2 shows the discrete spectrum of un-modulated Gaussian periodic signal and its derivative. The bandwidth of both is as wide as several GHz. The coherence bandwidth of channel is approximately the reciprocal of maximum multipath delay, which is about $200ns$ given by some experiments [1]. So the coherence frequency is much less than the signal frequency and the channel is frequency selective. But the relation between signal period T_f and coherence bandwidth is still unclear, thus the assumption of slow fading needs to be verified further.

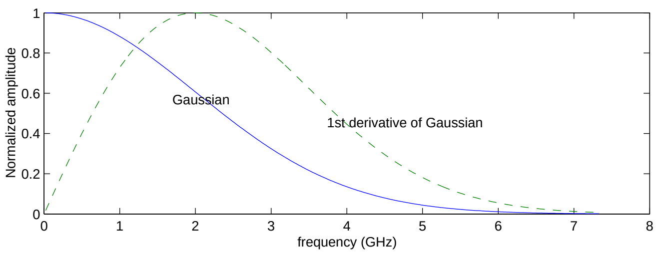

Fig. 2: Amplitude spectrum of periodic UWB signals

Since the maximum multipath delay is considerably less than the signal duration, the channel introduces a negligible amount of intersymbol inteference (ISI). So UWB is a ISI-free system. This conclusion together with frequency selectivity of the channel shows a distinct property of UWB, because ISI and frequency selectivity are generally bound together in a conventional wireless system.

IV. Cyclostationary Process

Let $i(t) = \sum_{n=-\infty}^{\infty} \delta(n - T_f - a_n)$. Taking advantage of the Fourier transformation to separate random variables, we obtain the mean and autocorrelation functions:

$$E[i(t)] = \frac{1}{T_f} \sum_{k=-\infty}^{\infty} E[e^{-j\omega_0 k a_n}] \cdot e^{j\omega_0 k t} \tag{8}$$

$$\phi_{ii}(t, t-\tau) = \frac{1}{{T_f}^2} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} e^{j\omega_0[(k-l)t + l\tau]} \times E[e^{j\omega_0(ka_n - la_m)}] \tag{9}$$

where $\omega_0 = 2\pi/T_f$.

It is obvious that both functions of $i(t)$ are periodic with T_f, and such a stochastic process is called *cyclostationary*. With the definition of *time-average autocorrelation*, we have

$$\bar{\phi}_{ii}(\tau) = \frac{1}{{T_f}^2} \sum_{l=-\infty}^{\infty} e^{j\omega_0 l\tau} \times E[e^{j\omega_0 l(a_n - a_m)}] \tag{10}$$

and power spectral density

$$\phi_{ii}(f) = \frac{1}{{T_f}^2} \sum_{l=-\infty}^{\infty} E[e^{j\omega_0 l(a_n - a_m)}] \times \delta(f - \frac{l}{T_f}) \tag{11}$$

When a cyclostationary signal $x(t)$ is input to a LTI System $h(t)$, it can be proved that the output $y(t)$ is also a cyclostationary process with

$$E[y(t)] = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} H(n\omega_0) \tag{12}$$

$$\bar{\phi}_{yy}(\tau) = \int_\beta \int_\alpha \bar{\phi}_{xx}(t, t-\tau+\alpha-\beta) h(\alpha) h^*(\beta)\, d\alpha\, d\beta \tag{13}$$

$$\phi_{yy}(f) = \phi_{xx}(f) \cdot |H(f)|^2 \tag{14}$$

If $h(t)$ is a Wide Sense Stationary (WSS) random process and independent of $x(t)$, $y(t)$ will be a cyclostationary process too, and

$$E[y(t)] = \sum_{n=-\infty}^{\infty} \left(F_n\, e^{jn\omega_0 t}\, E[h(t)]\, \delta(n\omega_0)\right) \tag{15}$$

$$\phi_{yy}(f) = 2\phi_{xx}(f) \cdot \phi_{hh}(f) \tag{16}$$

where

$$F_n = \frac{1}{T_f} \int_{-T_f/2}^{T_f/2} E[x(t)] e^{jn\omega_0 t}\, dt \tag{17}$$

are the coefficients when $E[x(t)]$ is expressed as a Fourier series.

V. Evaluation of Interference of UWB Signal

V.A. PSD Method

A UWB signal $s(t)$ can be regarded as the output of a LTI system where the input signal is $i(t)$, and system impulse response is $p(t)$. The Fourier coefficients for $E[i(t)]$ are $F_k = 1/T_f\, E[e^{-j\omega_0 k a_n}]$. According to the results in (12)-(14), we obtain

$$E[s(t)] = \frac{1}{T_f} \sum_{k=-\infty}^{\infty} \left(E[e^{-jk\omega_0 a_n}]\, e^{jk\omega_0 t}\, S(k\omega_0)\right) \tag{18}$$

$$\phi_{ss}(f) = \frac{1}{T_f^2} \sum_{l=-\infty}^{\infty} \Big(E[e^{-j\omega_0 l(a_n - a_m)}] \big|S(1/T_f)\big|^2 \times \delta(f - \frac{l}{T_f})\Big) \tag{19}$$

This implies that the PSD of UWB interference signal is made of many discrete spectral lines isolated by $1/T_f$.

This is similar to the amplitude spectrum shown in Fig. 2 where spectrum lines are separated by $2\pi/T_f$. The random time jitter caused by TH sequence and PPM will influence the intensity, but will not eliminate all spectral lines. However, it is possible to eliminate or reduce part of spectral lines in some special part of the spectrum to avoid interference by the careful construction of the TH sequence.

Supposing $T_c = 4T_d$, a_n will be uniformly distributed on $[3T_d, (4N_c+1)T_d]$, and time aliasing can be eliminated completely. Fig. 3(a) shows PSD of two UWB signals. The shapers are Gaussian wave and its first derivative. Parameters are T_d = 2ns, N_c = 10, T_f = 300ns, and σ = 0.5ns. Fig. 3(b) highlights the individual spectral lines over a limited range ($0-0.28$GHz) where the shaper is the derivative of Gaussian. Under the assumption above, $E[e^{-j\omega_0 l(a_n - a_m)}]$ is periodic and Fig. 3(c) shows part of it. Then the interference problem can be evaluated through the in-band interference power calculated from the power spectrum samples over the victim receiver's IF bandwidth. It has reported that the interference is not serious in the view of PSD [2].

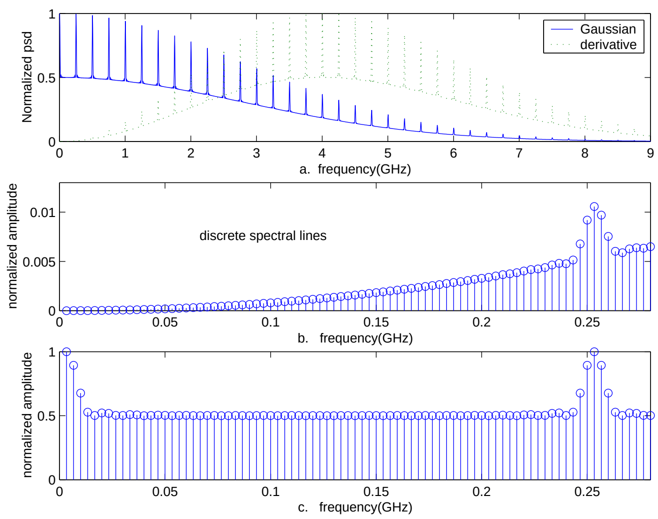

Fig. 3: PSD of UWB signals

With the results in Section IV, the statistical characteristics of both signals after propagating in a stationary-random-process-modeled channel and the interference power in the output of optimal detectors can be obtained. However, it is argued that the PSD measure, originating in harmonic analysis and relating to autocorrelation function, is an entirely inappropriate measure of transient signals like UWB [5]. At the same time, the phase information corresponding to the time jitter position, which takes a key role in describing the UWB signal, disappears in PSD method. Actually, it is known that the performance of a Direct Sequence Spread Spectrum (DSSS) system in the presence of *pulsed interference* is extremely poor where the *pulsed interference* is defined as the jam caused by pulses of spectrally flat noise that covers the entire signal bandwidth [7]. A single UWB signal just looks like that interference. What is worse, its duty cycle is so small that the peak power could affect the victim's signal more seriously. Fortunately, as we will see below, the influence by the aggregated signals of some UWB transmitters seems to be less serious in regard to *pulsed interference*.

V.B. Effect of Random Time Jitter

The interference caused by different transmitters could be assumed independently and identically distributed (i.i.d). According to the Central Limit theorem [6], if the number of UWB transmitters, I, is larger enough, the aggregate of interference will resemble the AWGN from the standpoint of most present victim receivers. Unfortunately, I is always small in UWB's typical multiple access application. Besides, since periodic UWB signals can only be expressed as the form of summation rather than compact sinusoidal functions where the period is hidden in the form of $e^{j\omega t+\varphi_0}$, the precise distribution of (5) is even unable to be derived because of the complicated forms of the high-order moments and the characteristic function. So instead of calculating the Probability Density Function (PDF) of amplitude, we will discuss the aggregated "phase" distribution and its influence.

In one period, time jitter for i^{th} transmitter's signal is $j_i = a_i + \tau_i$, where τ_i is multipath delay. I transmitters' signals can be assumed i.i.d. Unlike the phase in sinusoidal, time jitter for I UWB transmitters can not be summed up directly. A new continuous random variable θ_i, representing the start of time-of-arrival of i^{th} transmitter's signal, needs to be added to j_i. Then, in any point concerned, the aggregated time jitter of all I transmitters is

$$\varphi = \sum_{i=1}^{I} \varphi_i = \sum_{i=1}^{I} (a_i + \tau_i + \theta_i) \tag{20}$$

The PDF of φ is $f(\varphi) = \otimes_{i=1}^{I} f(\varphi_i)$, which means convolution from $f(\varphi_1)$ to $f(\varphi_I)$. Multipath time delay τ_i should be a discrete random variable if the TDL model is adopted or the Poisson model is used. For convenience, it is supposed to be a continuous random variable uniformly distributed on $[0, T_m]$ and θ_i uniformly distributed on $[0, T_f]$. It makes sense if many new pulses are produced during propagation. The distribution of a_i and other variables are the same with those in Section V.A. Fig. 4(a) shows $f(a_i)$ and $f(a_i + \tau_i)$. Fig. 4(b) is the convolution of 4 transmitters' time jitter distribution. It spreads over some periods. So the aggregated time jitter distributions for one period will be the sum of samples over $f(\varphi)$ in the duration of T_f , as shown in Fig. 4(c). We find that when the number of transmitters is larger than 4, the unbalanced time jitter in a period caused by a_i has been smoothed and φ is approximately uniformly distributed in a period. It implies that interference caused by aggregated UWB signal appears evenly in the whole time domain. Thus the aggregated interference will have a short period, that is, high duty cycle, and the position of pulses could be considered to be regular and known. So it is much different from *pulsed interference* and will be a continuous wave in ideal situation. It has to be stated that this conclusion is drawn under some ideal assumptions. In reality, the situation could be a little different since τ_i and θ_i could not be continuous and uniformly distributed when I is small.

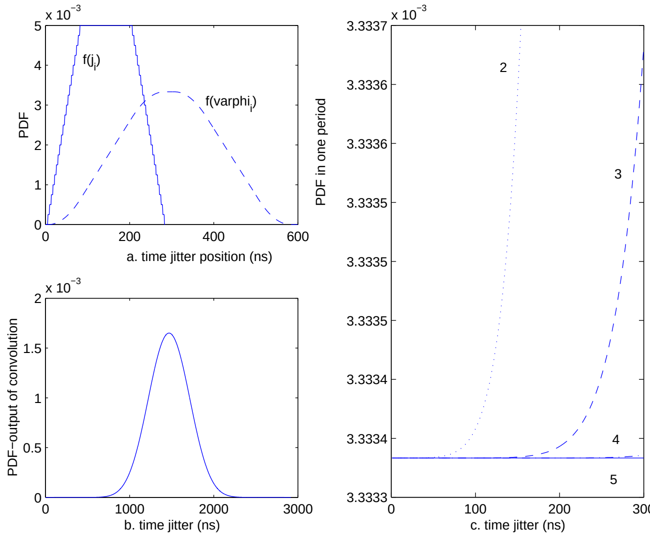

Fig. 4: PDF of aggregated UWB signals' time jitter

VI. Conclusions

UWB signals exhibit quite different characteristics from traditional sinusoidal transmission systems. Some characteristics may influence the effectiveness of conventional analysis and design methods particularly in regard to the near-far zone problem and the optimal receiver. Studying the signal characteristic is a direct way to find out how to transplant existing analysis and design techniques from high-level perspective.

The UWB interference problem was evaluated using the methods of PSD and time jitter in this paper. Analysis of PSD is a general but inaccurate and incomplete method for UWB as pointed in Section V.A. But it still provides important information and can direct the design of UWB system. The result of time jitter analysis shows the interference of aggregated UWB signals can be regarded as uniformly distributed and will not influence wireless systems so seriously as *pulsed interference*.

During the research, we also find the Gaussian wave maybe not the most suitable pulse in the view of analysis. A complete wave series with periodic information implicit in compact expression is preferable.

References

[1] J. M. Cramer, R. A. Scholtz and M. Z. Win, "Spatio-Temporal Diversity in Ultra-Wideband Radio", *IEEE WCNC, New Orleans, LA*, pp888-892, Sep, 1999.

[2] M. Hamalainen and J. Iinatti, "In-Band Interference of Three Kind of UWB signals in GPS L1 Band and GSM900 Uplink Band", *PIMRC2001, San Diego, CA, USA*, pp D-76 - D-80, Sep, 2001.

[3] H. F. Harmuth, "Transmission of Information by Orthogonal Functions", *Springer-Verlag*, 1970

[4] J. D. Taylor, "Ultra-Wideband Radar Technology", *CRC Press*, 2000

[5] T. W. Barrett, "History of Ultra WideBand Radar Communications: Pioneers and Innovators", em PIERS2000, Cambridge, MA, July, 2000

[6] P. Beckmann, "Probability in Communication Engineering", *Harcourt, Brace World Inc.*, 1967

[7] J. G. Proakis, "Digital Communications", *McGraw-Hill*, Third Edition, 1995.