Generalized Clarke Model for Mobile Radio Reception

Rauf Iqbal, Thushara D. Abhayapala and Tharaka A. Lamahewa

Abstract

Clarke's classical model of mobile radio reception assumes isotropic rich scattering around the mobile receiver antenna. The assumption of isotropic scattering is valid only in limited circumstances. In this contribution we develop a generalized Clarke model, which is applicable to mobile radio reception in general scattering environments. We give expressions for the autocorrelation and power spectral density (PSD) of the channel fading process and demonstrate the generality of the model by applying it to different non-isotropic scattering scenarios. Using the generalized model, we analyze the effect of mobile direction of travel and the non-isotropicity on the statistics of the channel fading process. We also show that if the mobile direction of travel is equiprobable in all directions, a non-isotropic scattering environment on average is as good as an isotropic scattering environment.

Index Terms

Non-Isotropic scattering, Rayleigh fading, Autocorrelation, Power Spectral Density, Power Azimuth Spectrum, mobile fading channel

I. INTRODUCTION

In real world communication scenarios, the transmitter and/or the receiver may be in motion. In a mobile-radio situation in which the transmitter is fixed in position while the receiver is moving, the direct line between the transmitter and receiver may be obstructed by buildings. At ultra high frequencies and above, therefore, the mode of propagation of the electromagnetic energy from transmitter to receiver is largely by way of scattering [1]. The amplitude fluctuations of the received signal have been shown to follow Rayleigh distribution in this communication scenario. By assuming a 2-dimensional (2D) uniform power azimuth spectrum (PAS) which corresponds to uniform distribution of angle-of-arrival (AOA) around the receive antenna, the autocorrelation of the received signal was shown to be strictly real valued and the resulting power spectral density (PSD) was U-shaped symmetric [1].

It has been argued and experimentally demonstrated (see [2] and references therein) that the scattering encountered in many suburban and rural environments is non-isotropic i.e., the distribution of AOA of waves is not uniform as assumed in [1]. The use of a directional antenna with non-uniform gain pattern at the receiver also results in non-isotropic PAS as seen by the antenna.

The authors are with the Applied Signal Processing group, Department of Information Engineering, Research School of Information Sciences and Engineering, the Australian National University, Canberra ACT 0200, Australia, email:{rauf.iqbal, thushara.abhayapala, tharaka.lamahewa}@anu.edu.au.

The assumption of uniform PAS resulted in closed form expressions for the statistics of the channel in [1]. For non-uniform PAS, the resulting expressions for the statistics of the channel are not in closed form [3]. A quadratic form for the probability distribution function (pdf) of the AOA proposed in [4] resulted in a closed form expression for the correlation of the complex envelope of the received signal in a non-isotropic scattering environment. In [5], von Mises distribution was used to model the pdf of AOA and effects of non-isotropic scattering on the correlation properties and velocity estimation in a Rician fading channel were discussed. In practice, PAS under non-isotropic conditions has also been shown to be well modelled by truncated Laplacian, truncated Cosine, and truncated Gaussian distribution [6]. Some of the other work related to mobile fading channels are [7]–[9].

The assumption of uniform PAS introduces small errors on the first order statistics of the received signal but a significant error on the second order statistics [10], like correlation function or, equivalently, PSD, and level crossing rates or, equivalently, the fading rate. There are certain communication system parameters like the estimation of vehicle velocity [5] for handoff decisions and the achievable information rates [11] without channel state information (CSI) that depend on the correlational properties of the received signal. It is, therefore, of some interest to develop a model that accurately models the statistics of the mobile Rayleigh fading channel in isotropic and non-isotropic scattering environments. In this sequel we use the term *generalized Rayleigh fading* to denote Rayleigh fading in general scattering environments.

The main contribution of this paper is the generalization of the Clarke model of mobile radio reception to generalized Rayleigh fading. This model can be used for the accurate performance prediction and evaluation of a communication system in generalized Rayleigh fading. Using the proposed generalized model, the impact on channel statistics of different parameters like the mobile direction of travel, mean angle of arrival (AOA) and the angular spread of the scattering environment can be accurately determined.

The rest of the paper is organized as follows. In Section II, we develop a generalized discrete time Rayleigh fading model and derive expressions for the autocorrelation and the PSD. In Section III, we show that the Clarke's isotropic scattering model is a special case of this generalized model. In Section III, we also analyze the effects of non-isotropicity and mobile velocity on the channel statistics. Some of the possible applications of the model are described in Section IV. Finally, the conclusions are drawn in Section V.

Throughout the paper, the following notation will be used: Bold lower (upper) letters denote vectors (matrices). The superscript * denotes the conjugate transpose. $\Re(\cdot)$ represents the real part operator. The notation $\mathrm{E}\{\cdot\}$ denotes the mathematical expectation and the symbol $\oint$ represents integration over a circle.

II. CHANNEL MODEL

We consider a downlink transmission system where the transmitter is stationary while the receiver is moving with some speed $|\boldsymbol{v}|$ at an angle of ϕ_v with respect to the x-axis, where $\boldsymbol{v} \equiv (|\boldsymbol{v}|, \phi_v)$ is the velocity of the mobile as shown in Fig. 1. We assume that the scatterers are distributed in the far-field from the transmitter and receiver antennas. We also assume that the channel between the transmitter and the mobile receiver is a strictly bandlimited, flat-fading (frequency non-selective), wide-sense stationary, zero mean circularly symmetric Gaussian fading process.

Continuous-Time Model

The baseband-equivalent received signal $y(t)$ at time t in continuous-time form is given as

$$y(t) = \sqrt{\rho}\, h_{\mathrm{c}}(t)\, x(t) + n(t), \quad -\infty < t < \infty \tag{1}$$

where $x(t) \in \mathbb{C}$ and $y(t) \in \mathbb{C}$ denote the channel input and the corresponding output at time instant t, respectively. The additive noise $n(t)$ is modeled as a zero-mean circularly symmetric[1] complex Gaussian white noise process with $\mathrm{E}\left\{n(s)n^{\dagger}(t)\right\} = \delta(s-t)$ where $\delta(\cdot)$ is Dirac's delta function. The fading process $h_{\mathrm{c}}(t)$ is modeled as a wide-sense stationary zero-mean circularly symmetric complex Gaussian process with unit variance $\mathrm{E}\left\{h_{\mathrm{c}}(t)h_{\mathrm{c}}^{\dagger}(t)\right\} = 1$. The input has variance $\mathrm{E}\left\{x(t)x^{\dagger}(t)\right\} = 1$. Equation (1) is normalized in such a way that ρ represents signal-to-noise ratio (SNR). We have the following remarks:

1) Channel variations in a mobile fading channel are caused by the relative movement between the mobile and scatterers. Since this relative movement is always limited due to physical limitations, the channel process can be assumed to be bandlimited.
2) The assumption of zero-mean complex Gaussian channel fading process implicitly assumes that the scattering is rich enough with roughly equally strong paths with independent phases rectangularly distributed on $[0, 2\pi)$, so that the central limit theorem applies.
3) The assumption of circular symmetry of the channel fading process implies that the *pseudo-autocorrelation* [14] is identically zero, and , therefore, the fading process is completely characterized by its complex autocorrelation.
4) Our approach is similar to that employed in [15] for space-time MIMO channel model with some important differences. First, unlike [15] which considers continuous time fading process, we employ a discrete time channel model. Secondly, we assume that the channel fading process is wide-sense stationary which is a much milder requirement than the channel fading process to be ergodic as implicitly assumed in [15].

Discrete-Time Model

If the output $y(t)$ is processed through a matched filter[2], we get the following discrete-time model

$$y[j] = \sqrt{\rho}\, h_{\mathrm{d}}[j]\, x[j] + n[j], \quad -\infty < j < \infty \tag{2}$$

where $\{n[j]\}$ is a sequence of samples of an i.i.d. zero mean[3] proper complex additive Gaussian noise process with unit variance, $\{h_{\mathrm{d}}[j]\}$ is a sequence of zero-mean circular complex Gaussian channel process with unit variance and $\{x[j]\}$ is input process of unit variance. In the sequel, we will use $\{h[j]\}$ instead of $\{h_{\mathrm{d}}[j]\}$ to denote sampled channel process for notational convenience. We assume that symbol duration is T_s. The

[1]The circular symmetry of a complex random process implies that the real and imaginary parts of the random process are uncorrelated and zero-mean. In case of a circularly symmetric *Gaussian* random process, the circular symmetry also implies that the real and imaginary parts of the process have equal variances, are jointly Gaussian and, hence, are independent of each other [12], [13].

[2]The matched filter is optimal for a white Gaussian channel but is suboptimal, in general, for fading channels. We still have considered matched filter because it is commonly employed in many practical communication systems. We assume that the filter is matched to the symbol rate and the output of the filter is sampled at the end of the symbol period. Moreover, it is customary to normalize the filter response to have unit-energy over the duration of the symbol.

[3]If a circularly symmetric continuous-time random process is sampled, the sampled process is also circularly symmetric [12], [16] where circular symmetry implies that the process is zero-mean (see footnote 1).

continuous-time and discrete-time models ((1) and (2) respectively) are related through the following:

$$x[j] = \frac{1}{\sqrt{T_s}} \int_{jT_s}^{(j+1)T_s} x(t)\, dt, \tag{3}$$

$$y[j] = \frac{1}{\sqrt{T_s}} \int_{jT_s}^{(j+1)T_s} y(t)\, dt, \tag{4}$$

$$n[j] = \frac{1}{\sqrt{T_s}} \int_{jT_s}^{(j+1)T_s} n(t)\, dt, \tag{5}$$

$$h_{\mathrm{d}}[j] = \frac{1}{\sqrt{\int_0^{T_s}\int_0^{T_s} \Phi(s-t)\, dt\, ds}} \int_{jT_s}^{(j+1)T_s} h_{\mathrm{c}}(t)\, dt, \tag{6}$$

where $\Phi(\cdot)$ in (6) is the covariance of the continuous channel process, $h_{\mathrm{c}}(t)$. Equations (3)−(6) are based on the facts that matched filter is an integrator, the output of the filter is sampled at the end of the each symbol interval T_s and discrete-time input, output, noise and channel processes must be normalized to have the same (co)variances as their continuous-time counterparts. It is to be noted that while most of the material on channel modeling deals with continuous-time model, we have chosen to work in the discrete-time domain, firstly, because the two domains are equivalent as long as the matched filter output is sampled at least at the Nyquist rate corresponding to rate of channel variation (for an idea of equivalence between continuous-time and discrete-time models please see [17]). Secondly, whenever it comes to practical implementation of the model, discrete-time models are computationally more efficient. There are some instances in literature of discrete-time modeling e.g., see [18] and references therein. Equation (2) can be obtained as a special case of the discrete-time triply (i.e., time-frequency-space) selective model of [18].

With no loss of generality, let the mobile be at some arbitrary point 'O' at the signaling instant j'. Thus, at the signaling interval $j = j' + k$, the mobile will be at the point $(|\boldsymbol{v}|\eta k T_s, \phi_v)$ with respect to 'O', where T_s is the symbol duration, $\eta = 2\pi/\lambda$ is the free space phase constant, λ is the wavelength, and k is an integer, which represents the time lag. Assume that the scattered signals are impinging on the mobile receiver from all directions on the 2D (horizontal) plane. Let $f(\beta)$ be the scattering gain at the origin due to the signals arriving from the direction β with respect to the x-axis (see Fig. 1). Then we write the channel gain as

$$h[j] = \oint f(\beta) \exp\left(i\eta j T_s\, \boldsymbol{v} \cdot \hat{\boldsymbol{\beta}}\right) \mathrm{d}\beta, \tag{7}$$

where $\hat{\boldsymbol{\beta}} \equiv (1, \beta)$ represents a unit vector along the direction β, '$\cdot$' is the scalar product between two vectors and $i = \sqrt{-1}$. Note that the factor $\exp(i\eta j T_s \boldsymbol{v} \cdot \hat{\boldsymbol{\beta}})$ in ((7)) reflects the phase delay of the incoming signal from the direction β at the mobile receiver with respect to the origin.

It is crucial to highlight an important conceptual difference between our approach and that used in [1] for modelling mobile radio reception. In [1], firstly, a probability of arrival of waves is associated with each direction in the azimuth. Secondly, the complex scattering gain, $f(\beta)$, from a certain direction of arrival is also random. In this contribution, we essentially assume that there is no probability distribution associated with AOA i.e., waves are assumed to be impinging on the mobile receive antenna from all the directions in the azimuth. Only the complex random scattering gain, $f(\beta)$, is assumed to be random and has, associated with it, a probability distribution. An azimuth direction with zero associated probability of arrival of wave is *equivalent* to a direction with waves assumed to be impinging from that particular direction but zero complex scattering gain. In other

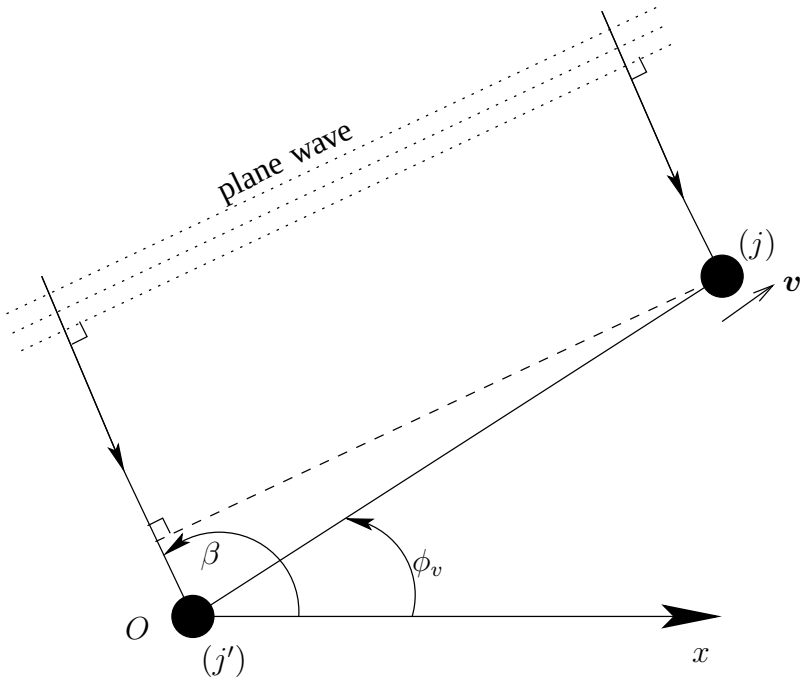

Fig. 1. Illustration of the key parameters: direction of mobile travel ϕ_v, mobile velocity $\boldsymbol{v}$, direction of signal (wave) arrival β, time instances j' and j, and the origin O of the co-ordinate system.

words, our approach is a simplified yet equivalent form of that employed in [1] but proves more convenient, as we would see, for arriving at a generalized Rayleigh fading model for mobile radio reception.

A. Autocorrelation of the Channel Fading Process

Using (7) we write the correlation between the channel gain at the signaling intervals j' and j as

$$\begin{aligned}\Phi(j,j') &= \mathrm{E}\left\{h_j h_{j'}^*\right\} \\ &= \oint\oint \mathrm{E}\left\{f(\beta)f^*(\beta')\right\} \exp\left(i\eta T_s(j\,\boldsymbol{v}\cdot\hat{\boldsymbol{\beta}} - j'\,\boldsymbol{v}\cdot\hat{\boldsymbol{\beta}}'\right) \mathrm{d}\beta\mathrm{d}\beta', \end{aligned} \tag{8}$$

where $\mathrm{E}\{\cdot\}$ stands for mathematical expectation. Assuming that the scattering gain from two distinct directions are uncorrelated and are zero mean [3], i.e.,

$$\mathrm{E}\left\{f(\beta)f^*(\beta')\right\} = \mathrm{E}\left\{|f(\beta)|^2\right\}\delta(\beta-\beta'). \tag{9}$$

where δ is the Dirac delta function. Making use of (9) in (8) , we can write

$$\Phi(j,j') = \Phi(j-j') = \Phi(k) = \oint \Psi(\beta)\,\exp\left(i\eta T_s k\,\boldsymbol{v}\cdot\hat{\boldsymbol{\beta}}\right)\mathrm{d}\beta \tag{10}$$

where $k = j - j'$ and

$$\Psi(\beta) = \mathrm{E}\left\{|f(\beta)|^2\right\}, \tag{11}$$

normally termed as the *angular power distribution* of the received signal. Thus $\Psi(\beta)$ is the average power received from the direction β.

Notice that $\Psi(\beta)$ and $\exp\left(i\eta T_s k\, \boldsymbol{v}\cdot\hat{\boldsymbol{\beta}}\right)$ are both periodic in β with period 2π. We can, therefore, express $\Psi(\beta)$ using Fourier series form as follows:

$$\Psi(\beta) = \sum_{m=-\infty}^{\infty} \gamma_m \, \exp\left(im\beta\right), \tag{12}$$

where γ_m are the Fourier series coefficients. Also we can express the factor $\exp\left(i\eta T_s k\, \boldsymbol{v}\cdot\hat{\boldsymbol{\beta}}\right)$ in Fourier series [19, page 67] form (also known as Jacobi-Anger expansion) as

$$\exp\left(i\eta T_s k\, \boldsymbol{v}\cdot\hat{\boldsymbol{\beta}}\right) = \sum_{m=-\infty}^{\infty} \underbrace{i^m \, J_m(\eta k T_s|\boldsymbol{v}|)\exp\left(-im\phi_v\right)}_{\xi_m} \exp\left(im\beta\right), \tag{13}$$

where ξ_m are the Fourier series coefficients and $J_m(\cdot)$ is the Bessel function of integer order m. Using (12), (13) and the convolution property[4] of the Fourier series, (8) can be written as

$$\begin{aligned}
\Phi(k) &= \oint \sum_{n=-\infty}^{\infty}\left(\sum_{m=-\infty}^{\infty} \gamma_m\, \xi_{n-m}\right)\exp\left(in\beta\right)\mathrm{d}\beta, \\
&= \sum_{n=-\infty}^{\infty}\left(\sum_{m=-\infty}^{\infty} \gamma_m\, \xi_{n-m}\right)\oint \exp\left(in\beta\right)\mathrm{d}\beta, \\
&= 2\pi \sum_{m=-\infty}^{\infty} \gamma_m \xi_{-m} \\
&= 2\pi \sum_{m=-\infty}^{\infty} i^m\, \gamma_m \; J_m(\eta k T_s|\boldsymbol{v}|)\; \exp\left(im\phi_v\right),
\end{aligned} \tag{14}$$

where the inner summation in the first equality represents the convolution of the Fourier series coefficients γ_m and δ_m, the third equality is the result of the following fact:

$$\oint \exp\left(in\beta\right)\mathrm{d}\beta = \begin{cases} 2\pi, & n = 0; \\ 0, & \text{otherwise.} \end{cases}$$

and the relation $J_{-m}(\cdot) = (-1)^m J_m(\cdot)$. If we let $\omega_d = \eta|\boldsymbol{v}|$ be the maximum angular Doppler spread (maximum doppler frequency, $f_d = v/\lambda$), (14) can be rewritten as

$$\boxed{\Phi(k) = \sum_{m=-\infty}^{\infty} i^m\, \gamma_m\, J_m(\omega_d k T_s)\; \exp\left(im\phi_v\right).} \tag{15}$$

Equation (15) is the complex autocorrelation function of the channel fading process as a function of time lag k. It is obvious from (15) that the autocorrelation of the fading process, in general, not only depends on the distribution of power (and, hence, mean scattering angle) but also on the direction of mobile travel, ϕ_v, maximum Doppler spread ω_d, and the symbol duration, T_s.

B. Spectral Density of the Channel Fading Process:

Using the well-known Wiener-Khintchine theorem [21], we obtain the PSD of the fading process by taking discrete-time Fourier transform (DTFT) of (15) as follows:

$$\Phi(\omega) = \sum_{m=-\infty}^{\infty} \gamma_m\, e^{im\phi_v}\left\{\sum_{p=-\infty}^{\infty} J_m(\omega_d p T_s)\exp\left(-i\omega p T_s\right)\right\}, \tag{16}$$

[4]The Fourier series coefficients of the product of two periodic functions are equal to the convolution of the coefficients of the Fourier series expansions of the individual functions [20].

where $\omega \in [-\pi, \pi]$ is the continuous radian frequency variable. We simplify (16) in Appendix A to obtain

$$\boxed{\Phi(\omega) = \frac{1}{\omega_D} \sum_{m=-\infty}^{\infty} \gamma_m \, e^{im\phi_v} \, F_m\left(\frac{\omega}{\omega_D}\right),} \tag{17}$$

where $\omega_D = \omega_d T_s$ is the maximum Doppler spread normalized by the symbol rate, and $f_D = \omega_D/2\pi$ is the normalized maximum Doppler frequency which is also called the normalized fading rate. It is easy to see that (17) is equivalent to the following

$$\Phi(\omega) = \frac{1}{\omega_D}\left\{\frac{1}{\sqrt{1-(\omega/\omega_D)^2}} + 2\sum_{m=0}^{\infty} \Re\left(\gamma_m \, e^{im\phi_v}\right) \, F_m\left(\frac{\omega}{\omega_D}\right)\right\}, \tag{18}$$

where $\Re(\cdot)$ is the real part of the argument. Equation (18) clearly shows that the PSD is real-valued as expected.

C. Truncation of series expansions

Equations (15) and (18) involve summation over infinite number of terms. However it is almost always possible in cases of practical interest to safely truncate the respective series up to a finite number of terms in the light of the following two facts:

i) For a fixed order m, $J_m(x)$ starts small and reaches its maximum at argument $x \approx O(m)$ before it starts decaying slowly. It was shown in [22] that $J_m(x) \approx 0$ for $|m| > 2\lceil x/2 \rceil + 1$ with $e = 2.7183\ldots$. Since the argument of $J_m(k\omega_{\mathrm{D}})$ depends on the lag variable k, the approximation would also depend on k for a fixed normalized fading rate.

ii) The Fourier coefficients γ_m must decay with m for Fourier series to be convergent (see e.g., [23]). The rate of decay depends on the smoothness of the function which is, in turn, related to the number of continuous derivatives of the function. In fact, the Fourier coefficients of an analytic (infinitely differentiable) function decay exponentially with m. Fourier coefficients of a Gaussian distribution, for example, decay exponentially with m, and decay polynomially for Laplace distribution [24]. The Fourier coefficients for a uniform distribution decay as $1/m$. In all these cases, the Fourier coefficients tend to zero[5] with m. The rapid the decay of the Fourier series coefficients, the less the number of Fourier modes (γ_m) with significant contribution and vice versa.

For small k, the fact (i) above is useful in approximating (15) due to the presence of $J_m(k\omega_{\mathrm{D}})$. When $k = 0$, for example, $J_m(k\omega_{\mathrm{D}}) = 1$ for $m = 0$ and zero otherwise. In other words, none of the Fourier modes γ_m except $m = 0$ contribute to the autocorrelation of the channel process. For $k = 1$, $J_m(k\omega_{\mathrm{D}}) \approx 0$ after [6] $|m| > (2\lceil k\omega_{\mathrm{D}}/2 \rceil + 1)$. As k increases, the number of Fourier coefficients that are "allowed" to contribute also increases. In the limit $k \to \infty$, the number of Fourier modes that contribute to (15) also approaches infinity. This is where the fact (ii) comes into effect. Due to the decay (e.g., exponential for a Gaussian distribution) of γ_m with m, the values of γ_m become increasingly small so that there must exist some finite m_0 such that $\gamma_m \approx 0$ for all $|m| > m_0$. We can , therefore, truncate the infinite summation in (15) to a summation over $|m| = m_0$ terms.

[5]The Delta distribution is an exception to this behavior. The Fourier coefficients of a Delta distribution do not tend to zero at all, indicative of the fact that it is not an ordinary function and its Fourier series does not converge in the standard sense.

[6]For a normalized fading rate $f_{\mathrm{D}} = 0.05$ and maximum time lag of $k = 10$ symbols, this approximation requires only $|m| = 3$ terms.

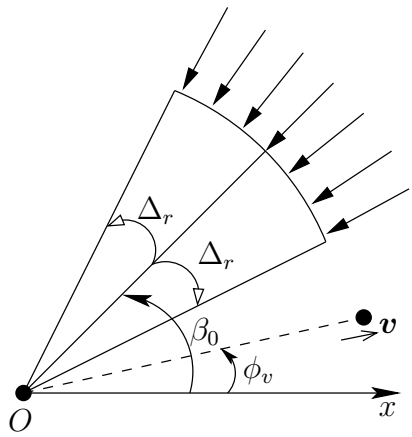

Fig. 2. Illustraion of an uniform-limited scattering scenario where the scattered power is uniformly distributed with magnitude $1/2\Delta_r$ over a part of the azimuth with a mean angle β_0 and a maximum deviation of Δ_r on each side of the mean. The direction of the mobile travel ϕ_v is also shown.

The approximation of the PSD of the fading process in (18) seems difficult due to the presence of the factor[7] $F_m(\omega/\omega_D)$ ((27) with $x = \omega/\omega_D$) which, for all m, sharply increases as ω approaches ω_D becoming infinite at $\omega_{\rm D}$. The approximation, therefore, would depend on the the decay of the Fourier coefficients of a particular scattering distribution. As long as the decay of γ_m is sufficiently fast, we can use the fact (ii) to approximate (18) to some finite $|m| = m_0'$ with the approximation error that approaches zero as m_0 approaches infinity. The accuracy of this approximation, moreover, depends on the angular spread[8] of the scattering distribution and the mobile direction of travel.

III. Effect of Non-Isotropicity and Mobile Velocity on Channel Statistics

Equations (15) and (18) give the second order channel statistics, namely autocorrelation and power spectral density (PSD) for any 2D scattering environment around the receiver. For illustration purposes, throughout this section, we consider the so-called uniform limited scattering scenario, i.e., scattered waves are arriving uniformly from an angular sector as shon in Fig. 2. For this case, we have

$$F(\beta) = \begin{cases} 1/(2\triangle_r), & \text{if } |\beta - \beta_0| \leq \triangle_r \\ 0, & \text{otherwise} \end{cases} \tag{19}$$

where β_0 is the mean angle of arrival, β is any other angle of arrival, and $2\triangle_r$ is the angular spread of the uniform limited scattering arrival signals as illustrated in Fig. 2. The values of γ_m for this distribution were derived in closed form in [**?**] and are given as

$$\gamma_m = \exp\left(-im\beta_0\right) \operatorname{sinc}(m\Delta_r). \tag{20}$$

[7]Notice that the Fourier transformation of $J_m(k\omega_{\rm D})$ in (16), that gave rise to the factor $F_m(\omega/\omega_D)$, involves summation over $k = \pm\infty$ which implies that infinite number of modes are allowed to contribute to the PSD of the fading process.

[8]The angular spread, Λ, is defined as the standard deviation of scattering distribution (PAS).

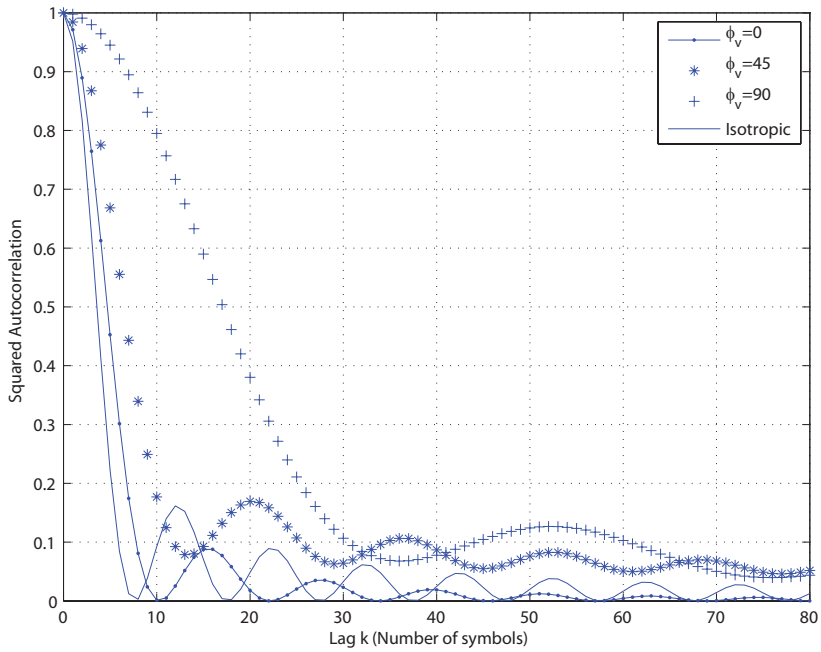

Fig. 3. Squared auto correlation $|\Phi(k)|^2$ as a function of time lag k for different direction of mobile receiver $\phi_v = 0°, 45°$, and $90°$, when $\beta_0 = 90°$, $\Delta_r = 60°$ and $f_{\mathrm{D}} = 0.05$. Auto-correlation for the isotropic case is also shown, which is independent of ϕ_v.

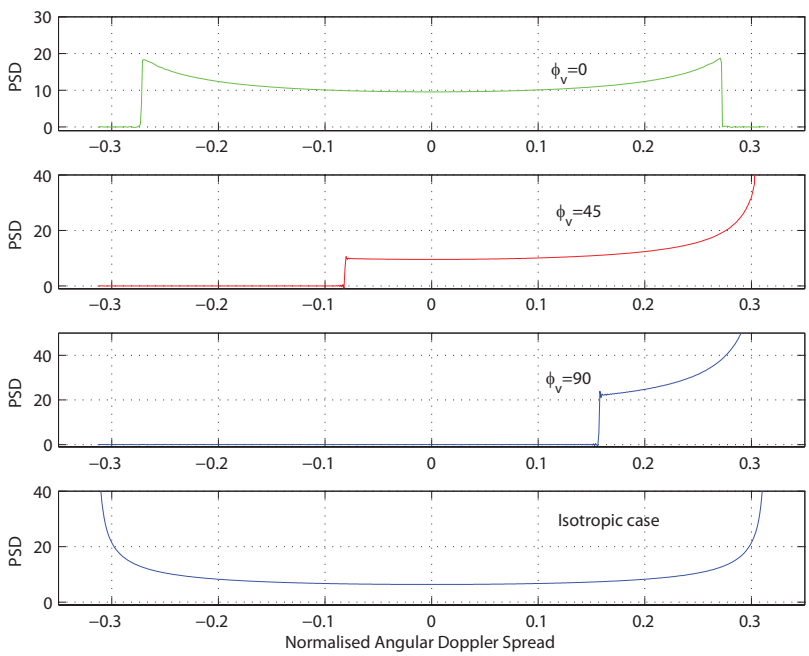

Fig. 4. Effect of the mobile direction of travel $\phi_v = 0°, 45°$, and $90°$ on the Power Spectral Density (PSD) when $\beta_0 = 90°$, $\Delta_r = 60°$ and $f_{\mathrm{D}} = 0.05$. Also shown is the symmetric U-shaped PSD in the case of isotropic scattering, which does not change with the direction of travel.

Using (20) in (15) and (18), we can write the autocorrelation and the PSD respectively, as

$$\Phi(k) = \sum_{m=-\infty}^{\infty} i^m \operatorname{sinc}(m\Delta_r)\, J_m(\omega_D k)\, \exp\left(-im\left(\beta_0 - \phi_v\right)\right), \tag{21a}$$

$$\Phi(\omega) = \frac{1}{\omega_D} \sum_{m=-\infty}^{\infty} \operatorname{sinc}(m\Delta_r)\, F_m\left(\frac{\omega}{\omega_D}\right) \exp\left(-im\left(\beta_0 - \phi_v\right)\right). \tag{21b}$$

In the rest of this section, we first show that the classical Clarke model is a special case of the generalized Clarke model introduced in this contribution. We then explore the effect of the non-isotropicity and mobile velocity on channel autocorrelation and PSD.

A. Clarke's model as a special case

When $\Delta_r = \pi$, i.e., when the scattered power is uniformly distributed over the full azimuth plane around the mobile receiver, it can easily be verified that (21a) and (21b) collapse to the following equations for autocorrelation and PSD, respectively,

$$\Phi(k) = J_0(\omega_{\mathrm{D}} k), \tag{22}$$

$$\Phi(\omega) = \frac{2}{\omega_{\mathrm{D}}\left(1 - (\omega/\omega_{\mathrm{D}})^2\right)}, \tag{23}$$

which are the well-known Clarke's model [1] for 2D isotropic scattering around the receive antenna. Thus Clarke's model is a special case of the generalized model developed in this contribution. Notice that, in general, the autocorrelation in (21a) is complex valued unlike that given in (22) for isotropic scattering environment which is strictly real valued.

B. Effect of mobile velocity

For a uniform-limited scattering scenario, the effect of changing the mobile direction of travel on the autocorrelation and PSD of the received signal has been shown in Figures 3 and 4 respectively. The autocorrelation and PSD for isotropic case have also been plotted for comparison. A marked deviation from the isotropic case can be observed. The skewness of the PSD is easily observed: If the mobile is moving into the non-isotropic scattering environment, the Doppler spectrum becomes (emphasized and) concentrated towards positive Doppler frequency axis. On the other hand, the Doppler spectrum is skewed towards negative Doppler frequency axis if the mobile moves away from the scatterers. The spectrum is symmetric about the mean only when the mobile moves at right angles to the mean scattering angle.

The above discussion of the autocorrelation and PSD *implicitly* assumes that the direction of mobile travel is perfectly known which is usually not the case in practice. It may be of some interest to find out the autocorrelation and PSD if the mobile direction of travel is unknown at the receiver. Suppose the mobile direction of travel is equiprobable in all directions i.e., $p\left(\phi_v\right) = 1/2\pi$, then, from (21a), the average autocorrelation, $\Phi_{\mathrm{avg}}(k)$ is given by

$$\Phi_{\mathrm{avg}}(k) = \sum_{m=-\infty}^{\infty} i^m \operatorname{sinc}(m\Delta_r)\, J_m(\omega_{\mathrm{D}} k) \exp\left(-im\beta_0\right) \int_0^{2\pi} \exp\left(im\phi_v\right) \mathrm{d}\phi_v, \tag{24}$$

and using (21b), the average PSD, $\Phi_{\mathrm{avg}}(\omega)$, is given by

$$\Phi_{\mathrm{avg}}(\omega) = \frac{1}{\omega_{\mathrm{D}}} \sum_{m=-\infty}^{\infty} \operatorname{sinc}(m\Delta_r) F_m\left(\frac{\omega}{\omega_{\mathrm{D}}}\right) \exp\left(-im\beta_0\right) \int_0^{2\pi} \exp\left(im\phi_v\right) \mathrm{d}\phi_v, \tag{25}$$

It is not hard to see that, irrespective of Δ_r and the β_0, the integrals in (24) and (25) are zero for all $m \neq 0$, and these two equations converge, respectively, to (22) and (23) i.e., the Clarke's isotropic case. In other words, if the mobile direction of travel is equiprobable in all directions, a non-isotropic scattering environment on average is as good as an isotropic scattering environment.

C. Effect of non-isotropicity

For a fixed direction of mobile travel, the effect of changing the degree of non-isotropicity on the autocorrelation and PSD of the fading process has been plotted in Figures 5 and 6 respectively. It can be seen from Fig. 5 that the channel fading process can have significantly higher correlation over time in non-isotropic scattering environments as compared to the isotropic scattering environment. With increasing Δ_r (or, in other words, decreasing the non-isotropicity) the correlation curves tend towards those of isotropic case.

Figure 6 shows the effect of changing the degree of non-isotropicity on the PSD of the channel process. The normalized Doppler spread seems to be directly proportional to Δ_r. For a fixed carrier frequency, the normalized fading rate f_{D} depends directly on the mobile speed. In our case none of the parameters, except the scattering environment (Δ_r), is being changed. It can, therefore, be concluded that changing the degree of non-isotropicity is actually equivalent to changing the normalized fading rate to some effective value, $f_{\mathrm{D}\mathit{eff}}$. Since the second order statistics are directly related to the normalized fading rate, this verifies the point of view of [10] that the assumption of isotropic distribution of scattered power in a non-isotropic distribution of power introduces significant errors in the second order statistics.

IV. Applications of the Generalized Clarke model

The generalized Clarke model presented in this contribution can be used for accurate performance evaluation and prediction of a communication system in Rayleigh fading under any scattering environment. Among many possible applications, following are some important applications of the generalized model:

- The modeling approach adopted herein to arrive at the generalized model lends itself easily to be extended to a generalized 3D Rayleigh fading environment.
- Most velocity and Doppler estimators are based on second order channel statistics. The performance of different velocity estimators in a non-isotropic Rician fading environment where the angle-of arrival (AOA) distribution is modelled by Von-Mises distribution has been evaluated in [5]. The generalized Rayleigh fading model developed herein can be extended to include the Rician component and, then, it can be used to quantify the difference in performance of various velocity estimators (designed to operate in a particular environment) in general scattering environments.
- Blind and semi-blind channel estimation and equalization techniques based on second order channel statistics have been proposed in the literature [25]. The performance of these estimators and equalizers can be predicted accurately in Rayleigh fading in any scattering environment with the help of the proposed model.
- Second order statistics based blind and semi-blind channel identification techniques have been proposed in the literature [26]. The effect of having a different scattering environment on the performance of these identification techniques can be analyzed using the generalized model.

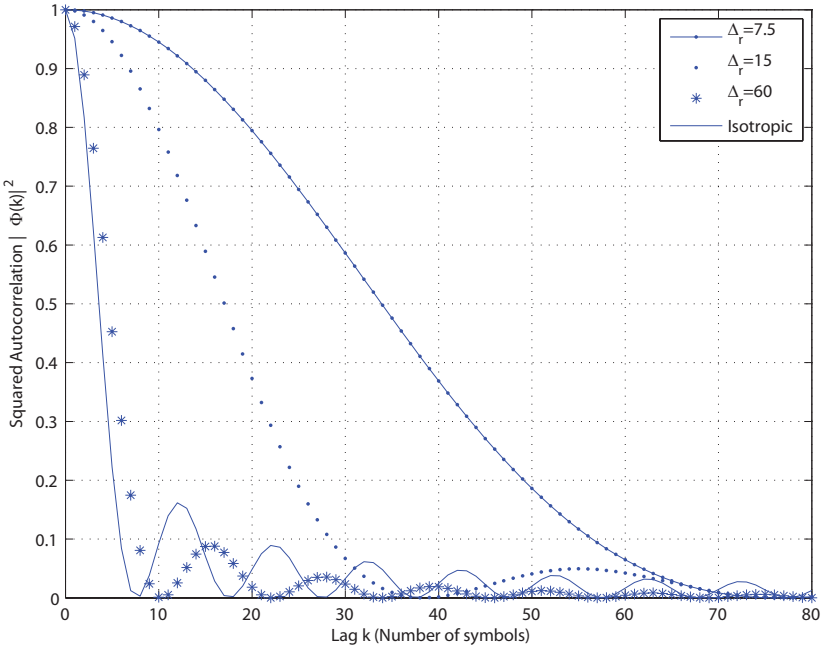

Fig. 5. Effect of the degree of non-isotropicity on the squared autocorrelation $|\Phi(k)|^2$ as a function of time lag k when $\beta_0 = 90°$, $\phi_v = 0°$ and $f_{\mathrm{D}} = 0.05$. Degree of ono-isotropicity is varied by setting $\Delta_r = 7.5°, 15°, 60°$ and $180°$ (isotropic case). The parameters corresponds to the scenario when the mobile is moving at a right angle to the mean scattering angle.

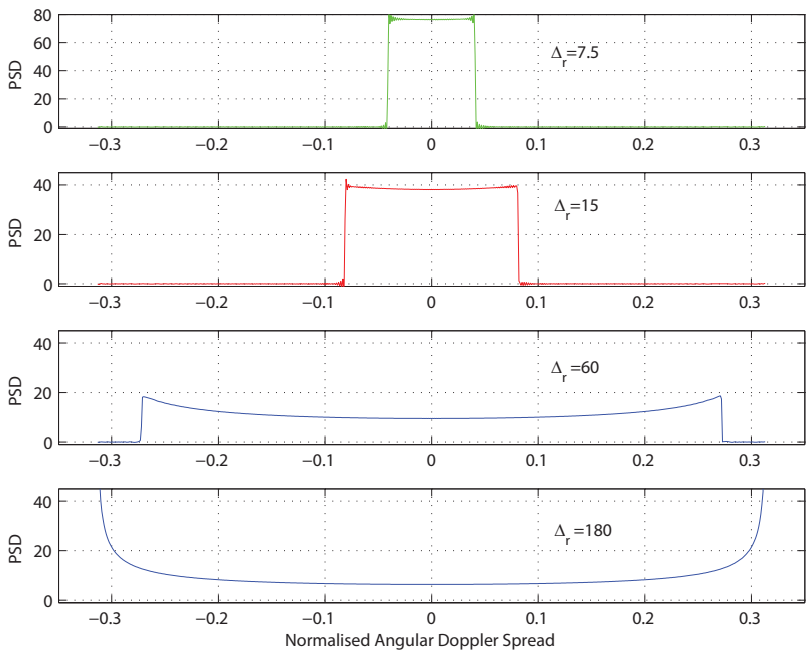

Fig. 6. Effect of the degree of non-isotropicity on the PSD as a function of normalised angular Doppler spread when $\beta_0 = 90°$, $\phi_v = 0°$ and $f_{\mathrm{D}} = 0.05$. Degree of ono-isotropicity is varied by setting $\Delta_r = 7.5°, 15°, 60°$ and $180°$ (isotropic case). The parameters corresponds to the scenario when the mobile is moving at a right angle to the mean scattering angle.

- The work of [11] on noncoherent communication over Rayleigh fading channel assumes isotropic scattering environment. Using the generalized model, we can extend this work to generalized Rayleigh fading with the results of [11] as a special case.

V. CONCLUSIONS

A time-selective generalized Rayleigh fading model was presented that extends the Clarke's classical isotropic scattering model of mobile radio reception to general scattering environments. It turned out that the statistics of a generalized Rayleigh fading channel depend on the direction of mobile travel and the mean angle of arrival, and can be significantly different from those in an isotropic environment. In general, the autocorrelation is complex-valued and the power spectral density is asymmetric. Moreover, if the mobile direction of travel is equiprobable in all directions and the mean angle of arrival is fixed, then a non-isotropic scattering environment on the average is identical to the isotropic scattering environment. The simulation results verified that the assumption of isotropic scattering in a non-isotropic environment introduces significant errors in the second order statistics.

APPENDIX A
PROOF OF (17)

The following identity exists for *continuous time Fourier transform* (CTFT) of the Bessel function of the first kind and integer order μ [3]

$$\int_{-\infty}^{\infty} J_\mu(\omega_d \Delta t)\, e^{-i\omega\Delta t} d\Delta t = \frac{F_\mu\left(\frac{\omega}{\omega_d}\right)}{i^\mu \omega_d}, \tag{26}$$

where

$$F_\mu(x) \triangleq 2\frac{\cos\left(\mu \cos^{-1}(x)\right)}{\sqrt{1-x^2}}. \tag{27}$$

We know from the basic Fourier theory that the DTFT of a sampled process is essentially a magnitude and frequency scaled version of CTFT of the continuous-time process with 2π periodicity [20]. Making use of (26) and (27), the DTFT of the sampled Bessel function of the first kind and integer order μ is given by

$$\begin{aligned}\sum_{p=-\infty}^{\infty} J_\mu(\omega_d p T_s)\, e^{-i\omega p T_s} &= \frac{1}{T_s}\sum_{\ell=-\infty}^{\infty} \frac{F_\mu\left(\frac{1}{\omega_d T_s}(\omega - 2\pi\ell)\right)}{i^\mu \omega_d}, \\ &= \sum_{\ell=-\infty}^{\infty} \frac{F_\mu\left(\frac{1}{\omega_D}(\omega - 2\pi\ell)\right)}{i^\mu \omega_D},\end{aligned} \tag{28}$$

where $\omega_D = \omega_d T_s$ is the maximum Doppler spread normalized by the symbol rate.

Since we are working at the baseband level, only the low pass term ($\ell = 0$) in (28) is of interest to us, i.e.,

$$\sum_{p=-\infty}^{\infty} J_\mu(\omega_d p T_s)\, e^{-i\omega p T_s} = \frac{F_\mu\left(\frac{\omega}{\omega_D}\right)}{i^\mu \omega_D}. \tag{29}$$

Using (27) and (29), we can rewrite (16) to get the desired expression (17) for PSD

$$\Phi(\omega) = \frac{1}{\omega_D}\sum_{m=-\infty}^{\infty} \gamma_m\, e^{im\phi_v}\, F_m\left(\frac{\omega}{\omega_D}\right). \tag{30}$$

REFERENCES

[1] R. H. Clarke, "A statistical theory of mobile-radio reception," in *Bell System Technical Journal*, vol. 47, 1968, pp. 957–1000.

[2] A. Abdi, J. A. Barger, and M. Kaveh, "A parametric model for the distribution of the angle of arrival and the associated correlation function and power spectrum at the mobile station," vol. 51, no. 3, pp. 425–434, May 2002.

[3] J. S. Sadowsky and V. Kafedziski, "On the correlation and scattering functions of the WSSUS channel for mobile communications," in *IEEE Transactions on Vehicular Technology*, vol. 47, Feb 1998, pp. 270–282.

[4] K. Anim-Appiah, "Complex envelope correlations for non-isotropic scattering," *IEE Electronics Letters*, vol. 34, no. 9, pp. 1039–1052, Apr 1998.

[5] C. Tepedelenlioglu and G. Giannakis, "On velocity estimation and correlation properties of narrow-band communication channels," *IEEE Trans. Vehic. Technol.*, vol. 50, no. 4, pp. 1039–1052, July 2001.

[6] P. D. Teal, T. D. Abhayapala, and R. A. Kennedy, "Spatial correlation for general distributions of scatterers," *IEEE Signal Processing Letters*, vol. 9, no. 10, pp. 305–308, Oct. 2002.

[7] T. Aulin, "A modified model for the fading signal at a mobile radio channel," *IEEE Trans. on Vehicular Technology*, vol. VT-28, no. 3.

[8] J. D. Parsons, *The mobile radio propagation channel*. London, UK: Pentech Press, 1992.

[9] J. D. Parsons and A. M. D. Turkmani, "Characterisation of mobile radio signals: model description," in *IEE Proceedings-I*, vol. 138, no. 6, 1991, pp. 549–556.

[10] W. C. Y. Lee, "Finding the aproximate angular probability density function of wave arrival by using a directional antenna," in *IEEE transactions on Antennas and Propagation*, vol. AP-21, 1973, pp. 328–334.

[11] X. Deng and A. M. Haimovich, "Information rates of time varying Rayleigh fading channels," in *Proc. IEEE International Conference on Communication , ICC'2004*, Paris, France, 2004.

[12] D. Torrieri, *Principles of* S*pread*-S*pectrum Communication Systems*. Springer, 2005.

[13] D. Tse and P. Viswanath, *Fundamentals of wireless communication*. New York, NY, USA: Cambridge University Press, 2005.

[14] F. D. Neeser and J. L. Massey, "Proper complex random processes with applications to information theory," vol. 39, pp. 1293–1302, Jul. 1993.

[15] T. A. Lamahewa, T. D. Abhayapala, R. A. Kennedy, and J. T. Y. Ho, "Space-time cross correlation and space-frequency cross spectrum in non-isotropic scattering environment," *International Conference on Acoustics, Speech, and Signal Processing*, vol. 4, May 2006.

[16] J. R. Barry, D. G. Messerschmitt, and E. A. Lee, *Digital Communication: Third Edition*. Norwell, MA, USA: Kluwer Academic Publishers, 2003.

[17] V. K. N. Lau and Y. R. Kwok, *Channel-Adaptive Technologies and Cross-Layer Designs for Wireless Systems with Multiple Antennas: Theory and Applications (Wiley Series in Telecommunications and Signal Processing)*. Wiley-Interscience, 2006.

[18] C. Xiao, J. Wu, S. Leong, Y. R. Zheng, and K. Letaief, "A discrete-time model for triply selective MIMO Rayleigh fading channels," *IEEE Trans. on Wireless Comm.*, vol. 3, no. 5, pp. 1678–1688, Sep. 2004.

[19] D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, 2nd ed. New York: Springer, 1998, vol. 93.

[20] A. V. Oppenheim and R. Shafer, Eds., *Discrete-Time Signal Processing*, 2nd ed. New Jersey: Prentice Hall, 1989.

[21] D. C. Champeney, *A Handbook of* F*ourier Transforms*. U.K.: Cambridge University Press, 1987.

[22] H. M. Jones, R. A. Kennedy, and T. D. Abhayapala, "On dimensionality of multipath fields: Spatial extent and richness," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing, ICASSP'02*, vol. 3, Orlando, Florida, May 2002, pp. 2837–2840.

[23] G. B. Folland, *Real Analysis*, 2nd ed. New York: Wiley-Interscience, 1972.

[24] F. Comte, Y. Rozenholc, and M. L. Taupin, "Penalized contrast estimator for density deconvolution," in *The Candian Journal of Statistics*, vol. 34, no. 3, 2006.

[25] A. A. Farid, Z. Q. Luo, and Z. Ding, "Blind channel equalization based on second order statistics," *International Conference on Acoustics, Speech, and Signal Processing*, vol. 3, pp. 557–560, 2005.

[26] L. Tong, G. Xu, B. Hassibi, and T. Kailath, "Blind channel identification based on second-order statistics: A frequency-domain approach," *IEEE Trans. on Inform. Theory*, pp. 329–334, 1995.