Chapter 1

Coherent Broadband DOA Estimation Using Modal Space Processing

1.1 Introduction

The problem of estimation of direction of arrival (DOA) of coherent broadband sources has applications in wireless communication systems, especially systems with smart antennas [1]. DOA estimation and adaptive beamforming are the main issues in smart antenna systems. However, in a complex multipath environment, received signal from different directions may be correlated, which prevents the application of narrowband DOA estimation techniques to estimate DOA of broadband sources. This chapter introduce a novel coherent broadband DOA estimation technique based on *modal decomposition* of wavefields.

Wang and Kaveh [2] introduced the use of focusing matrices for the purpose of Coherent Signal Subspace (CSS) processing for DOA estimation of farfield wideband sources. In this method, the wideband array data are first decomposed into several narrowband components. The focusing matrices are used for the alignment of the signal subspaces of narrowband components within the bandwidth of the signals, followed by the averaging of narrowband array data covariance matrices into a single covariance matrix. Then, any signal subspace direction finding procedure, such as MUSIC [3] or its variants, maximum likelihood (ML), or minimum variance (MV), can be applied to this averaged covariance matrix to obtain the desired parameter estimates. The design of focusing matrices in CSS method requires preliminary estimates of the direction of arrivals. Further, this method is applicable to only a pair of sources. In later years, the CSS technique was further developed and refined [4], [5] to account for multiple sources but the problem of prior information about the DOA still remained. In this chapter, we use *modal decomposition* of wavefields to propose novel focusing matrices that do not require preliminary DOA estimates and are completely independent of the signal environment.

The spatial resampling method is a technique that does not require preliminary knowledge of DOA in order to localize wideband sources. It was first introduced by Krolik and Swinger [6] and is motivated by treating the output of a discrete array as being the result of spatially sampling a continuous linear array. The same concept is also known as an interpolated array approach used in [7]. Krolik and Swingler [6] used digital interpolation methods to resample the array data. The spatial resampling method requires more than one resampling matrix to be constructed for different field of view or sectors. An alternative technique is suggested in this chapter using modal decomposition of wavefields. Under this technique a set of resampling matrices has been proposed which is same for the full field of view of the array data, unlike in the case of [7].

The application of modal decomposition of wavefields enables to gain insights into the structure of focusing and spatial resampling matrices. It can be observed that the computational complexity of CSS and spatial resampling methods can be reduced by combining the focusing matrices and spatial resampling matrices to form modal covariance matrices. Use of modal decomposition of wavefields in any array signal processing application is termed as *Modal Space Processing* as it converts measured array data into *modal space*. The modal space can be viewed as a vector space with a predefined orthogonal basis set which are the natural basis set for wavefields.

1.2 System Model

Let us consider a double sided linear array of $2Q+1$ sensors, located at distances x_q, $q = -Q, \cdots, -1, 0, 1, \cdots, Q$ from the array origin, which receives signals from V sources in space. Let $\Theta = [\theta_1, \theta_2, \ldots, \theta_v, \ldots, \theta_V]$, be a vector containing bearings of each source with reference to the array axis where θ_v is the direction of the vth source. We assume that the source signal and the noise are confined in a bandwidth of $k \in [k_\mathrm{l}, k_\mathrm{u}]$, where k_l and k_u are lower and upper band edges respectively. We use wavenumber $k = 2\pi f/c$ where f is the frequency in Hz and c is the speed of wave propagation, to represent frequency in this chapter. The signal received at each sensor is Discrete Fourier Transformed (DFT) into M distinct frequency bins within the design bandwidth. The array output in the mth frequency bin can be represented as:

$$\boldsymbol{z}(k_m) = \sum_{v=1}^{V} \boldsymbol{a}(\theta_v; k_m) s_v(k_m) + \boldsymbol{n}(k_m), \tag{1.1}$$

where, $s_v(\cdot)$ is the signal received from the vth source at the origin, $\boldsymbol{n}(\cdot)$ is the uncorrelated noise data and

$$\boldsymbol{a}(\theta; k) = [e^{-ikx_{-Q}\cos\theta}, \ldots, e^{-ikx_Q\cos\theta}]' \tag{1.2}$$

where $[\cdot]'$ denotes the transpose operator and $i = \sqrt{-1}$. We write (1.1) in matrix form as

$$\boldsymbol{z}(k_m) = \boldsymbol{A}(\Theta; k_m)\boldsymbol{s}(k_m) + \boldsymbol{n}(k_m), \tag{1.3}$$

for $\qquad m = 1, \ldots, M$ where

$$\boldsymbol{A}(\Theta; k) = [\boldsymbol{a}(\theta_1; k), \ldots, \boldsymbol{a}(\theta_V; k)], \tag{1.4}$$

and

$$\boldsymbol{s}(k) = [s_1(k), \ldots, s_V(k)]'. \tag{1.5}$$

The objective of any DOA technique is to determine the direction of arrival (DOA) Θ from the observed data $\boldsymbol{z}(k_m)$, $m = 1, \ldots, M$.

Most DOA techniques use correlation matrix of the received data in their DOA algorithms. The correlation matrix of the observed data in the mth frequency bin is defined as

$$\boldsymbol{R}_z(k_m) \triangleq E\{z(k_m)z(k_m)^H\}, \tag{1.6}$$

where $[\cdot]^H$ denotes conjugate transpose operation and $E\{\cdot\}$ is the expectation operator. Substituting (1.3) in (1.6), we have

$$\begin{aligned}\boldsymbol{R}_z(k_m) =& \boldsymbol{A}(\Theta; k_m)\boldsymbol{R}_s(k_m)\boldsymbol{A}^H(\Theta; k_m) \\ &+ E\{\boldsymbol{n}(k_m)\boldsymbol{n}(k_m)^H\}\end{aligned} \tag{1.7}$$

where

$$\boldsymbol{R}_s(k_m) \triangleq E\{\boldsymbol{s}(k_m)\boldsymbol{s}(k_m)^H\}, \tag{1.8}$$

is the source correlation matrix. Here, we assume that the source signals and noise are uncorrelated.

1.3 Focusing Matrices for Coherent Wideband Processing

In this section, we briefly outline the focusing method. The first step following the frequency decomposition of the array data vector is to align or focus the signal space at all frequency bins into a common one at a reference frequency by focusing matrices $\boldsymbol{T}(k_m)$ that satisfy

$$\boldsymbol{T}(k_m)\boldsymbol{A}(\Theta; k_m) = \boldsymbol{A}(\Theta; k_0), \quad m = 1, \ldots, M, \tag{1.9}$$

where $k_0 \in [k_l, k_u]$ is some reference frequency and $\boldsymbol{A}(\Theta; k)$ is the direction matrix defined by (1.4). Applying the M focusing matrices to the respective array data vectors (1.3) gives the following focused array data vector,

$$\boldsymbol{T}(k_m)\boldsymbol{z}(k_m) = \boldsymbol{A}(\Theta; k_0)\boldsymbol{s}(k_m) + \boldsymbol{T}(k_m)\boldsymbol{n}(k_m),$$
$$m = 1, \ldots, M.$$

Then the focused and frequency averaged data covariance matrix can be defined by

$$\begin{aligned}\boldsymbol{R} &\triangleq \sum_{m=1}^{M} E\{\boldsymbol{T}(k_m)\boldsymbol{z}(k_m)(\boldsymbol{T}(k_m)\boldsymbol{z}(k_m))^H\} \\ &= \sum_{m=1}^{M} \boldsymbol{T}(k_m)\, E\{\boldsymbol{z}(k_m)\boldsymbol{z}^H(k_m)\}\, \boldsymbol{T}^H(k_m).\end{aligned} \tag{1.10}$$

We use (1.6), (1.7) and (1.9) to get

$$\boldsymbol{R} = \boldsymbol{A}(\Theta; k_0)\overline{\boldsymbol{R}}_s \boldsymbol{A}^H(\Theta; k_0) + \boldsymbol{R}_{\text{noise}} \tag{1.11}$$

where

$$\overline{\boldsymbol{R}}_s \triangleq \sum_{m=1}^{M} \boldsymbol{R}_s(k_m), \tag{1.12}$$

and the transformed noise covariance matrix

$$\boldsymbol{R}_{\text{noise}} \triangleq \sum_{m=1}^{M} \boldsymbol{T}(k_m)\, E\{\boldsymbol{n}(k_m)\boldsymbol{n}^H(k_m)\}\, \boldsymbol{T}^H(k_m). \tag{1.13}$$

The focused data covariance matrix (1.11) is now in a form in which almost any narrowband direction finding procedure may be applied. Here, we apply the minimum-variance (MV) method of spatial spectral estimation [8] to the frequency averaged data covariance matrix $\boldsymbol{R}$.

Several methods of forming focusing matrices have been suggested in the literature. The focusing methods of [2, 4, 5, 9] require preliminary DOA estimates in order to construct the focusing matrices. This constitutes a severe disadvantage in practical applications since it leads to biased DOA estimates. In Section 1.5, we show how to design focusing matrices without preliminary DOA estimates, using modal decomposition of wavefields.

1.4 Spatial Resampling Method

Spatial resampling is another method [6] used to focus the wideband array data to a single frequency so that existing narrowband techniques may be used to estimate the DOA. The basic idea of spatial sampling is outlined below.

Suppose we have a separate uniform array with half wavelength spacing for each frequency bin with the same effective array aperture in terms of wavelength. Thus for M frequencies, there are M arrays and the sensor separation of the mth array is $\lambda_m/2$ where $\lambda_m = 2\pi/k_m$. If each array has $2Q+1$ sensors, then the aperture length is the same for all frequencies in terms of corresponding wavelength. Then the mth array steering vector for farfield sources is given by

$$\begin{aligned}
&\boldsymbol{a}(\theta; k_m) = \\
&[e^{i\pi Q\cos\theta}, \cdots, e^{i\pi\cos\theta}, 1, e^{-i\pi\cos\theta}, \ldots, e^{-i\pi Q\cos\theta}] \\
&= \boldsymbol{a}(\theta), \quad m = 1, \ldots, M.
\end{aligned}$$

That is the steering vectors of all arrays are equal and hence from (1.4) the DOA matrices of all arrays are the same:

$$\boldsymbol{A}^{(m)}(\Theta; k_m) = \boldsymbol{A}(\Theta), \quad m = 1, \ldots, M, \tag{1.14}$$

where $\boldsymbol{A}^{(m)}(\Theta; k)$ is the DOA matrix of the mth subarray. Hence if we have M arrays for each frequency bin with the same aperture, then their covariance matrices can be averaged over frequency without losing DOA information. The average covariance matrix can then be used with existing narrowband DOA techniques to estimate DOA angles.

However, it is not actually practical to have a separate array for each frequency. This problem can be overcome by having a single array and using the received array data to form the array data for M virtual arrays by interpolation/extrapolation of the received array data. This is tantamount to constructing a continuous sensor using the received array data and resampling it. There are several methods reported in the literature. In [7] the field of view of the array is divided into several sectors, and a different interpolation matrix is calculated for each sector using a least squares fit.

1.5 Modal Decomposition

At the physical level, sensor array signal processing is characterized by the classical wave equation. The general solution to the wave equation can be decomposed into *modes* which are orthogonal basis functions of the spatial coordinates. These modes exhibit interesting mathematical properties and form a useful basis set to analyze and synthesize an arbitrary wavefield, a response of an array of sensors, or a spatial aperture. Modes of a 2 and 3 dimensional wavefield are called cylindrical and spherical harmonics respectively. In this section, we show how to decompose any wavefield due to farfield sources.

In the antenna literature, these modes have been used to synthesize antenna shapes [10, 11], to represent electromagnetic fields radiated by circular antennas [12], and to compute antenna couplings [13]. Recently, they have been used for nearfield broadband design [14, 15], directional soundfield recording [16] and reproduction [17], spatial wireless channel characterization [18, 19] and capacity calculation of MIMO channels [20].

1.5.1 Linear Arrays

The term $e^{-ikx\cos\theta}$ represents the phase delay of a signal received at a point located distance x from the origin for a signal from the direction θ. This term is an integral part of any array processing algorithm. Here, we separate the distance dependency and angle dependency into two terms. This separation is very useful in any algorithm as it effectively separate the sensor geometry from the signal direction.

Using Jacobi-Anger expansion [21], we write

$$e^{-ikx\cos\theta} = \sum_{n=0}^{\infty} i^n (2n+1) j_n(kx)\, P_n(\cos\theta), \tag{1.15}$$

where n is a non-negative integer to index modes, $j_n(\cdot)$ is the spherical Bessel function and $P_n(\cdot)$ is the Legendre function. We have the following comments on this expansion:

1. The series expansion (1.15) gives an insight into the spatial wavefield along a linear array.

2. Equation (1.15) can be viewed as a Fourier series type expansion of a function where $P_n(\cos\theta)$, $n = 0, \ldots, \infty$ are the orthogonal basis set.

3. Observe that in each term of the series, the arrival angle θ dependency is separated out from the sensor location x and the frequency k. Therefore we may use the above expansion to write the array DOA matrix $\boldsymbol{A}(\Theta; k)$ as a product of two matrices, one depending on DOA angles and the other depending on frequency and sensor locations.

4. However, you may have noticed that the expansion (1.15) has an infinite number of terms. Thus, the usefulness of (1.15) depends on the number of significant terms need to be used in any numerical evaluation.

1.5.2 Modal Truncation

For a finite aperture array with finite bandwidth signal environment, the series (1.15) can be safely truncated by finite number of terms (say N) without generating significant modelling errors. We show this below. Figure 1.1 shows plots of a few spherical Bessel functions $j_n(\cdot)$ against its argument. We can observe from Figure 1.1 that for a given kx, the function $j_n(kx) \to 0$ as n becomes large. This observation is supported by the following asymptotic form [22]

$$j_n(kx) \approx \frac{(kx)^n}{1 \cdot 3 \cdot 5 \ldots (2n+1)} \quad \text{for } kx \ll n. \tag{1.16}$$

Therefore, we can notice that the factor $(2n+1)j_n(kx_q)$ in (1.15) decays as n grows larger beyond $n = kx_q$. Suppose that the minimum frequency of the signal band is k_l. Then we can truncate (1.15) to N terms if $N > k_l x_Q$, where x_Q is the distance to the Qth sensor (the maximum array dimension). It is difficult to derive an analytical expression for N, but a convenient rule of thumb [23] is $N \sim 2k_l x_Q$. More recent work [18] shows that the series (1.15) can be truncated by $N = \lceil kex/2 \rceil$ terms with negligible error.

1.6 Modal Space Processing

In this section, we use the modal decomposition developed in the previous section to (i) design focusing matrices, (ii) spatial resampling matrices, and (iii) introduce a novel modal space processing DOA technique.

1.6.1 Focusing Matrices

We use modal analysis techniques to propose novel focusing matrices which do not require preliminary DOA estimates and are completely independent of the signal environment. Here we only consider a linear (possibly nonuniform) array but it may be generalized to arbitrary array configurations.

We substitute the first $N+1$ terms of (1.15) into (1.2) and thus write the array steering vector for farfield sources as

$$\boldsymbol{a}(\theta; k) = \boldsymbol{J}(k) \begin{bmatrix} P_0(\cos\theta) \\ \vdots \\ P_N(\cos\theta) \end{bmatrix}, \tag{1.17}$$

where

$$\boldsymbol{J}(k) = \begin{bmatrix} i^0(2\cdot 0+1)j_0kx_{-Q} & \dots & i^N(2N+1)j_Nkx_{-Q} \\ \vdots & \ddots & \vdots \\ i^0(2\cdot 0+1)j_0kx_Q & \dots & i^N(2N+1)j_Nkx_Q \end{bmatrix}. \tag{1.18}$$

We use (1.17) in (1.4) to write the array DOA matrix for farfield signal environment as

$$\boldsymbol{A}(\Theta;k) = \boldsymbol{J}(k)\boldsymbol{P}(\Theta), \tag{1.19}$$

where the $(N+1)\times V$ matrix

$$\boldsymbol{P}(\Theta) = \begin{bmatrix} P_0(\cos\theta_1) & \dots & P_0(\cos\theta_V) \\ \vdots & \ddots & \vdots \\ P_N(\cos\theta_1) & \dots & P_N(\cos\theta_V) \end{bmatrix}. \tag{1.20}$$

The $(2Q+1)\times(N+1)$ matrix $\boldsymbol{J}(k)$ depends on the frequency k and the sensor locations and is independent of the DOA of the signals. Suppose $(2Q+1) > (N+1)$ and $J(k)$ has full rank $N+1$ if the sensor locations are chosen appropriately. With this assumption and using (1.19), we can propose a set of focusing matrices $\boldsymbol{T}(k_m)$ given by

$$\begin{aligned} \boldsymbol{T}(k_m) =& \boldsymbol{J}(k_0)\left[\boldsymbol{J}^H(k_m)\boldsymbol{J}(k_m)\right]^{-1}\boldsymbol{J}^H(k_m) \\ & m = 1,\dots,M \end{aligned} \tag{1.21}$$

which satisfies the focusing requirement (1.9); recall that k_0 is the reference frequency.

The major advantage of the focusing matrices (1.21) over the existing methods is that these matrices do not need preliminary DOA estimates and accurately focus signal arrivals from all directions. Also note that these matrices are fixed for a given array geometry and frequency band of interest. Thus they can be calculated in advance in time critical applications such as smart antennas to save computational time..

1.6.2 Spatial Resampling Matrices

In this section, we show how to use the modal decomposition to find a transformation matrix to calculate array data for M virtual arrays for the spatial resampling method described in Section 1.4. Sensor locations for the real array can be arbitrary on a line, i.e., there is no requirement for it to be a uniformly spaced array. From (1.19) the real array DOA matrix in the mth frequency bin is given by

$$\boldsymbol{A}(\Theta; k_m) = \boldsymbol{J}(k_m)\boldsymbol{P}(\Theta), \tag{1.22}$$

and the DOA matrix of the mth virtual array at frequency k_m would be

$$\boldsymbol{A}^{(m)}(\Theta; k_m) = \boldsymbol{J}^{(m)}(k_m)\boldsymbol{P}(\Theta), \tag{1.23}$$

where from (1.18) with $k_m x_q = q\pi$,

$$\begin{aligned}
&\boldsymbol{J}^{(m)}(k_m) = \\
&\begin{bmatrix} i^0(2\cdot 0+1)j_0(-\pi Q) & \dots & i^N(2N+1)j_N(-\pi Q) \\ \vdots & \ddots & \vdots \\ i^0(2\cdot 0+1)j_0(\pi Q) & \dots & i^N(2N+1)j_N(\pi Q) \end{bmatrix} \\
&= \overline{\boldsymbol{J}}, \;\; m = 1, \dots, M
\end{aligned}$$

which is a constant matrix, independent of m and k_m. Therefore we can write

$$\begin{aligned}
\boldsymbol{A}^{(m)}(\Theta; k_m) &= \overline{\boldsymbol{J}}\boldsymbol{P}(\Theta), \\
&= \boldsymbol{A}(\Theta) \;\; m = 1, \dots, M,
\end{aligned} \tag{1.24}$$

which is same for all frequency bins. We need to design the set of spatial resampling matrices $\boldsymbol{T}(k_m)$, $m = 1, \ldots, M$ such that

$$\boldsymbol{A}^{(m)}(\Theta; k_m) = \boldsymbol{T}(k_m)\boldsymbol{A}(\Theta; k_m), \quad m = 1, \ldots, M. \tag{1.25}$$

By substituting (1.22) and (1.24) into (1.25 and using the pseudo inverse of $\boldsymbol{J}(k_m)$ we obtain the least-square solution

$$\boldsymbol{T}(k_m) = \overline{\boldsymbol{J}}[\boldsymbol{J}^H(k_m)\boldsymbol{J}(k_m)]^{-1}\boldsymbol{J}^H(k_m), \quad m = 1, \ldots, M. \tag{1.26}$$

These spatial resampling matrices (they act as focusing matrices) can be used to align the array data in different frequency bins, so that narrowband DOA techniques can be applied. Similar to the focusing matrices (1.21), these spatial resampling matrices (1.26), do not require preliminary DOA estimation and depend only on the array geometry and the frequency. Also they are independent of the angle of arrival and fixed for full field of view.

1.7 Modal Space Algorithm

Observe that the proposed focusing matrices (1.21) and the spatial re-sampling matrices (1.26) have a common (generalized inverse) matrix factor

$$\boldsymbol{G}(k_m) \triangleq [\boldsymbol{J}^H(k_m)\boldsymbol{J}(k_m)]^{-1}\boldsymbol{J}^H(k_m), \quad m = 1, \ldots, M, \tag{1.27}$$

and only differ by the frequency independent factors $\boldsymbol{J}_0(k_0)$ and $\overline{\boldsymbol{J}}$. Also note that from (1.19),

$$\boldsymbol{G}(k_m)\boldsymbol{A}(\Theta; k) = \boldsymbol{P}(\Theta), \quad m = 1, \ldots, M, \tag{1.28}$$

i.e., $\boldsymbol{G}(k_m)$ transforms the array DOA matrix into a frequency invariant DOA matrix. Therefore we can use $\boldsymbol{G}(k_m)$ instead of $\boldsymbol{T}(k_m)$ to align the broadband array data to form a frequency averaged covariance matrix. Intuitively, one can say that the matrices $\boldsymbol{G}(k_m)$ trans-form the $2Q+1$ array data vector $\boldsymbol{z}(k_m)$ into a $N+1$ *modal data* vector in *modal space*. Now we can estimate the frequency averaged *modal covariance matrix* as

$$\hat{\boldsymbol{R}} = \sum_{m=1}^{M} \boldsymbol{G}(k_m)\, \boldsymbol{z}(k_m) \boldsymbol{z}^H(k_m)\, \boldsymbol{G}^H(k_m) \tag{1.29}$$

and the MV spectral estimate

$$\hat{Z}(\theta) = \frac{1}{\begin{bmatrix} P_0(\cos\theta) \\ \vdots \\ P_N(\cos\theta) \end{bmatrix} \hat{\boldsymbol{R}}^{-1} \begin{bmatrix} P_0(\cos\theta), \ldots P_N(\cos\theta) \end{bmatrix}}. \tag{1.30}$$

Comments:

1. This method (one can refer it as the *Modal Space Processing (MSP) method*) involves less computation compared to the other two methods since the modal space has less dimensions ($N+1$) than the signal subspace ($2Q+1$).

2. As for the other two methods, the modal space method does not require preliminary DOA estimates.

3. One can consider the modal space method as a superset of focusing matrices and spatial resampling methods.

4. Given the frequency averaged modal covariance matrix (1.29), any other narrowband DOA technique such as MUSIC or its variants, maximum likelihood (ML) can be used.

5. This method can be extend to find the range and angle of nearfield sources by using the modal expansion of a spherical wavefront [21]. Readers are referred to [15] for detail.

1.8 Simulation

In this section, the simulation results have been presented in order to demonstrate the effectiveness of modal space processing (MSP) method. A linear array of 19 nonuniformly spaced sensors has been used for MSP technique. The use of nonuniformly spaced sensor array for broadband application has been discussed in [14]. The sensor spacing is kept uniform while performing the simulation of examples that follow the algorithms suggested in past literature [2, 4]. These simulations are presented in this section for comparison of results. The source signal and the noise are stationary zero-mean white Gaussian processes. Noise at each sensor is independent of the other. Signal received at each sensor is Discrete Fourier Transformed to get 33 uniformly spaced narrow-band frequency bins within the desired bandwidth. For each trial, 64 independent snapshots are generated for every frequency bins. The frequency averaged modal covariance matrix is calculated using the relation (1.29). The sources are then localized by using Minimum Variance (MV) direction finding procedure (1.30) as implemented for narrow-band source localization.

1.8.1 A group of two sources

The signal environment consists of two completely correlated sources at angles $\Theta = [38^\circ \; 43^\circ]$. Let s_1(t) be the source at 38°, and the source at 43° is a delayed version of s_1(t) and is given by $s_2(t) = s_1(t - t_o)$ with $t_o = 0.125s$ or equivalently in frequency domain $s_2(f) = s_1(f)e^{-jft_o}$. Here, $s_1(f)$ is the Fourier Transformed signal of s_1(t). The signal-to-noise ratio is 10dB. The two signals $s_1(t)$ and $s_2(t)$ can be viewed as mutipath signals from a single source.

The signals used lie within a bandwidth of 40Hz with midband frequency at 100Hz. This gives a lower band edge ($f_l = 80$Hz) to upper band edge ($f_u = 120$Hz) ratio of $2 : 3$. All the signal parameters are kept identical to those described in [2]. The signals are captured by a linear array of 19 sources. Fig. 1.3 shows the spectral estimate obtained using MSP. The vertical lines indicate the correct direction of arrival of the sources. For comparison, the results obtained using the method described in [2] has shown in Fig. 1.2. A preliminary angle estimate of 40.4° has been necessary to correctly estimate the direction of arrivals using the later technique whereas no prior knowledge of angles is required for MSP technique. The graphs reveal that both processes localize the sources with fine accuracy. However, a focusing angle of $53°$ in the case of [2] will result in Fig. 1.4 which cannot resolve the true direction of arrivals.

1.8.2 Three groups of five sources

The number of sources are now increased to five with bearings $\Theta = [53° \ 58° \ 98° \ 103° \ 145°]$. Complete correlation exists between first and second source. A frequency band of $f = [80 : 120]$Hz is used to compare the results with those obtained using the focusing matrix proposed in [4] . Fifteen independent trials were carried out that showed similar results.

Fig. 1.6 shows one realization obtained by using MSP with MV spectral estimate. Here the number of modes used is N=15. Reducing the value of N degrades the performance of the procedure while increasing its value produces no appreciable improvement. The number of sensors is 19 and are nonuniformly spaced. All the sources are clearly detected without any prior knowledge of source environment. These results can be compared with Fig. 1.5 that shows the spatial spectrum of multigroup sources using the technique described in [4]. However, prior knowledge of source directions is required by this technique and preliminary angle estimates used for this example is $\beta = [53° \ 55° \ 59° \ 96.7° \ 100.5° \ 104.3° \ 144°]$.

The above simulation is performed for wider band of frequency of bandwidth $[300 : 3000]$Hz, and the results show that MSP produces better results (Fig 1.7) as compared to the technique proposed in [4] (Fig 1.8). A total number of 45 sensors and 55 frequency bins are used in the simulation.

1.9 Conclusion

In this chapter, Modal Signal Processing (MSP) is introduced as a tool to solve coherent broadband source localization problem. MSP direction of arrival estimation techniques do not require any preliminary knowledge of DOA angles nor the number of sources.

References

[1] T. Do-Hong and P. Russer, "Signal processing for wideband array applications," *IEEE Microwave Magazine*, pp. 54–67, Mar. 2004.

[2] H. Wang and M. Kaveh, "Coherent signal-subspace processing for the detection and estimation of angles of arrival of multiple wide-band sources," *IEEE Trans. Acoust. Speech Sig. Proc.*, vol. ASSP-33, no. 4, pp. 823–831, Aug. 1985.

[3] R.O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propagat.*, vol. AP-34, no. 3, pp. 276–280, Mar. 1986.

[4] H. Hung and M. Kaveh, "Focussing matrices for coherent signal-subspace processing," *IEEE Trans. Acoust. Speech Sig. Proc.*, vol. 36, no. 8, pp. 1272–1281, Aug. 1988.

[5] S. Sivanand, J. Yang, and M. Kaveh, "Focussing filters for wideband direction finding," *IEEE Trans. Sig. Proc.*, vol. 39, pp. 437–445, Feb. 1991.

[6] J. Krolik and D. Swingler, "Focused wide-band array processing by spatial resampling," *IEEE Trans. Acoust. Speech Sig. Proc.*, vol. 38, no. 2, pp. 356–360, Feb. 1990.

[7] B. Friedlander and A. Weiss, "Direction finding for wide-band signals using an interpolated array," *IEEE Trans. Sig. Proc.*, vol. 41, no. 4, pp. 1618–1634, Apr. 1993.

[8] N.L. Owsley, "Sonar array processing," in *Array Signal Processing*, S. Haykin, Ed. New Jersey: Prentice Hall, 1985.

[9] M.A. Doron and A.J. Weiss, "On focusing matrices for wideband array processing," *IEEE Trans. Sig. Proc.*, vol. 40, pp. 1295–1302, June 1992.

[10] R.J. Garbacz and D.M. Pozar, "Antenna shape synthesis using characteristic modes," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 340–350, May 1982.

[11] F.J. Harackiewicz and D.M. Pozar, "Optimum shape synthesis of maximum gain omnidirectional antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-34, pp. 254–258, Feb. 1986.

[12] L.W. Liand M.S. Leong, P.S. Kooi, and T.S. Yeo, "Exact solutions of electromagnetic fields in both near and far zones radiated by thin circular-loop antennas: A general representation," *IEEE Trans. Antennas Propagat.*, vol. 45, no. 12, pp. 1741–1748, Dec. 1997.

[13] A.D. Yaghjian, "Efficient computation of antenna coupling and fields within the nearfield region," *IEEE Trans. Antennas Propagat.*, vol. AP-30, no. 1, pp. 113–128, Jan. 1982.

[14] T.D. Abhayapala, R.A. Kennedy, and R.C. Williamson, "Nearfield broadband array design using a radially invariant modal expansion," *J. Acoust. Soc. Amer.*, vol. 107, pp. 392–403, Jan. 2000.

[15] D.B. Ward and T.D. Abhayapala, "Range and bearing estimation of wideband sources using an orthogonal beamspace processing structure," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing, ICASSP'2004*, vol. 2, May 2004, pp. 109–112.

[16] T.D. Abhayapala and D.B. Ward , "Theory and design of higher order sound field microphones using spherical microphone array," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing, ICASSP'2002*, vol. 2, May 2002, pp. 1949–1952.

[17] D.B. Ward and T.D. Abhayapala, "Reproduction of a plane-wave sound field using an array of loudspeakers," *IEEE Trans. Speech and Audio Proc.*, vol. 9, no. 6, pp. 697–707, Sept. 2001.

[18] H. M. Jones, R. A. Kennedy, and T. D. Abhayapala, "On Dimensionality of Multipath Fields: Spatial Extent and Richness," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing, ICASSP'2002*, vol. 3, Orlando, Florida, May 2002, pp. 2837–2840.

[19] T.D. Abhayapala, T.S. Pollock, and R.A. Kennedy, "Spatial decomposition of mimo wireless channels," in *Proc. IEEE Seventh International Symposium on Signal Processing and Its Applications*, vol. 1, July 2003, pp. 309–112.

[20] T.S. Pollock, T.D. Abhayapala, and R.A. Kennedy, "Introducing space into space-time mimo capacity calculation: A new closed form upper bound," in *Proc. Int. Conf. Telecommunicatons, ICT'2003*, Papeete, Tahiti, Feb. 2003, pp. 1536–41.

[21] D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, second edition ed. New York: Springer, 1998.

[22] E. Skudrzyk, *The Foundations of Acoustics*. New York: Springer-Verlag, 1971.

[23] T. Abhayapala, "Modal analysis and synthesis of broadband nearfield beamforming arrays," Ph.D. dissertation, Research School of Information Sciences and Engineering, Australian National University, Canberra ACT, Dec. 1999.

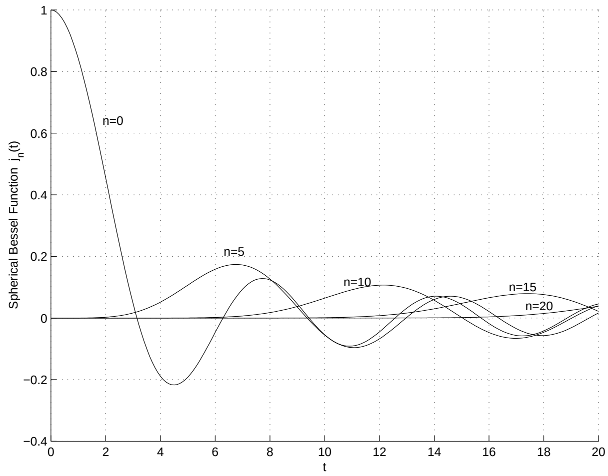

Figure 1.1: spherical Bessel functions of order $n = 0, 5, 10, 15$, and 20.

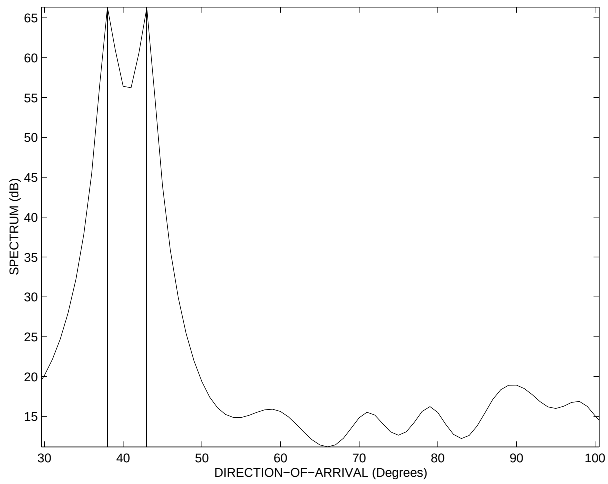

Figure 1.2: The estimated spatial spectrum of the correlated sources using the algorithm of [2].

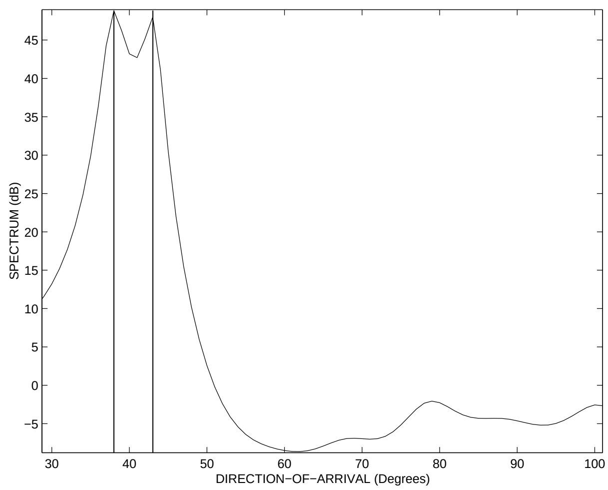

Figure 1.3: The estimated spatial spectrum of the correlated sources using Modal Space Processing (MSP) method.

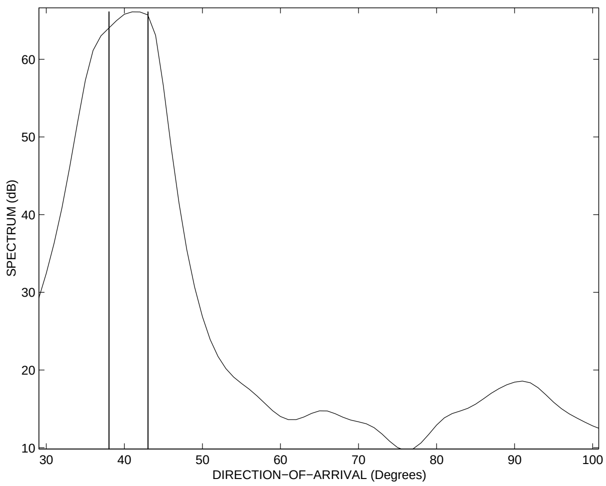

Figure 1.4: The estimated spatial spectrum of the correlated sources using prior angle estimation of $53°$.

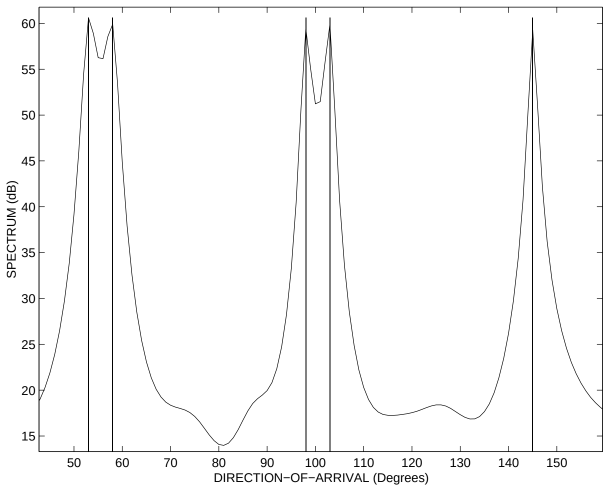

Figure 1.5: The estimated spatial spectrum of the Multigroup sources using algorithm of [4]

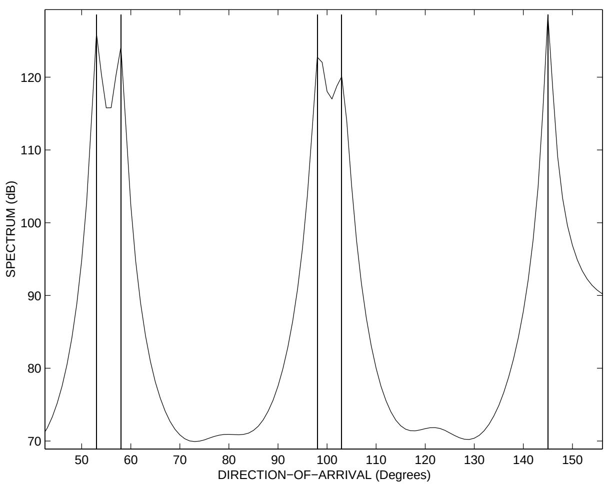

Figure 1.6: The estimated spatial spectrum of the Multigroup sources using Modal Space Processing (MSP) method.

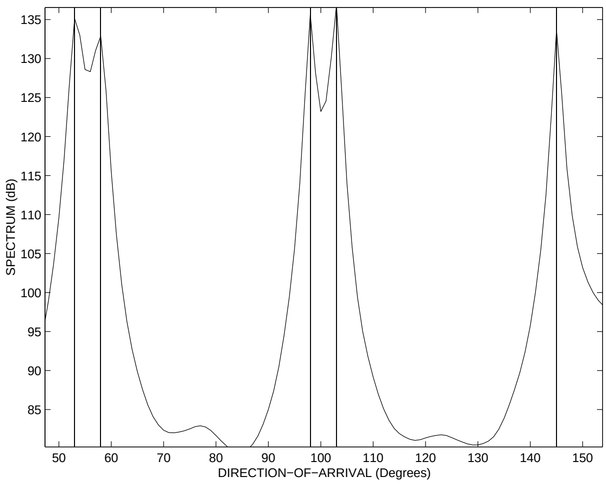

Figure 1.7: The estimated spatial spectrum of the Multigroup sources using Modal Space Processing for a wider frequency band $[300 : 3000]$Hz.)

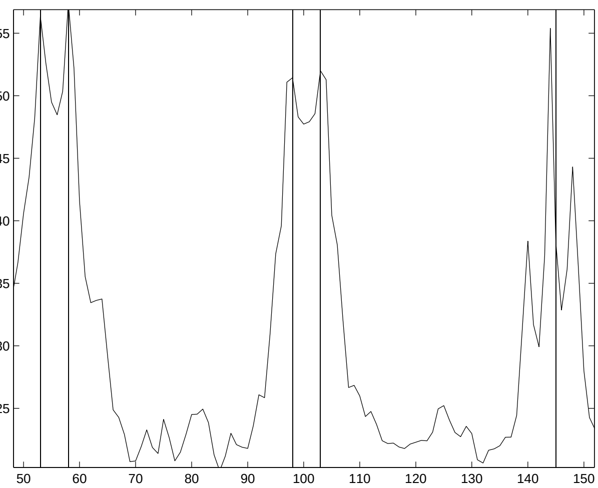

Figure 1.8: The estimated spatial spectrum of the Multigroup sources using algorithm of [4] for a wider frequency band $[300 : 3000]$Hz.