Simultaneous Learning of Structure and Value in Relational Reinforcement Learning

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Overview

1. **Relational RL: Advantages and challenges**
2. **Background and related work**
3. **An approach to structure and value RRL (SVRRL):**
   - Domain assumptions/restrictions
   - Efficiently learning value
   - Efficiently learning structure
4. **Experimental results**
5. **Conclusions and future work**

RRL: Advantages and Challenges

- **RRL is a natural representation/learning paradigm:**
  - Describe world as objects and relations between them
  - Compact descriptions: absence-as-negation, quantification
- **But, benefits are not without drawbacks:**
  - **Very large state spaces:** Combinatorial explosion of ground relations as domain size increases
  - **Need robust learning for sparse data:** Restrict hypothesis space initially, relax in presence of more data
  - **Must focus on good approximations:** Optimal/exact inference extremely difficult

RRL: Addressing these Challenges

- **General solution difficult, focus on restricted setting:**
  - Finite-horizon, undiscounted domains (assuming MDP setting)
  - Single terminal reward of success/failure
  - Applies to goal-oriented tasks (e.g. planning, games w/ stationary opp.)
- $\Rightarrow$ **Value function** $\equiv$ **probability of success**
- **Allows us to address previous RRL challenges:**
  - **Very large state spaces:** Repr. value function as naive Bayes net
  - **Robust learning:** Leverage Bayes net parameter & structure learning
  - **Good approximations:** Use max-likelihood (ML) and MDL principles

Background and Related Work

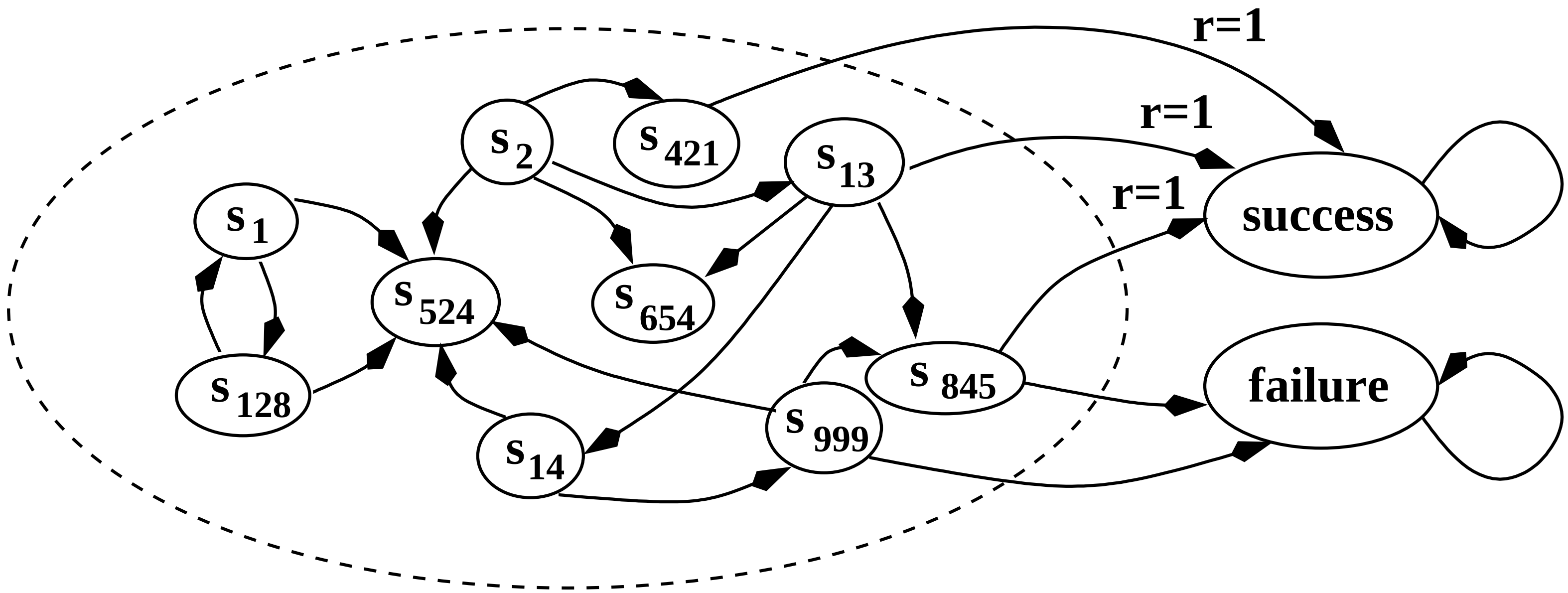
- **Model-free relational RL:**
  - **(Dzeroski et al, 1998)**: Logical regression trees for RRL (top-down)
  - **(Walker et al, 2004)**: Sample & weight relational features (bottom-up)
  - **(Croonenborghs et al, 2004)**: SVRRL can be viewed as instance of general QLARC framework (bottom-up)
- **Bayes net structure learning:**
  - **(Friedman and Goldschmidt, 1996)**: Tree-augmented naive Bayes (TAN) for classification; SVRRL leverages similar approach
  - **(Friedman et al, 1999)**: Probabilistic relational model (PRM) learning; full approach too computationally intensive for SVRRL

Notational Preliminaries

- $\{R_1, \ldots, R_i\}$: **Set of relations used to describe a state**
- $\{A_1, \ldots, A_j\}$: **Set of relation attribute types**
  - Example: $R_1(A_1, A_2)$, $A_1 = \{a, b\}$, $A_2 = \{1, 2\}$
  - 4 ground atoms: $\{R_1(a, 1), R_1(a, 2), R_1(b, 1), R_1(b, 2)\}$
  - 2^4 possible truth assignments = 16 states
- $F = \{F_1, \ldots, F_n\}$: **Ground rel. atoms (boolean features)**
- $f = \{f_1, \ldots, f_p, \bar{f}_{p+1}, \ldots, \bar{f}_n\}$: **Feature truth assignment**
  - Order true/positive features first, false/negative features last
  - Represent **state** f as $\{f_1, \ldots, f_p\}$, assume absence-as-negation
  - Space efficient because typically $p \ll n$

Theoretic Preliminaries

- Under a fixed policy π, MDP reduces to a Markov chain:



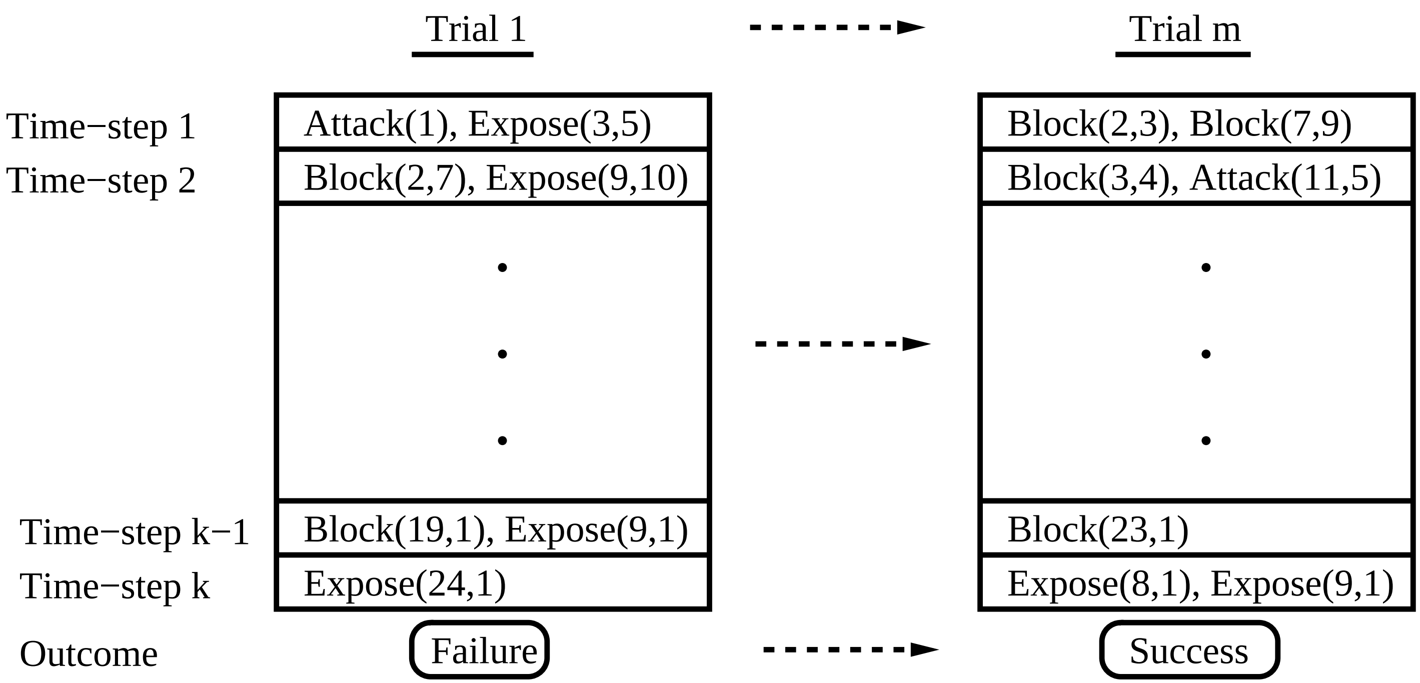
- Only non-zero reward r is initial transition to success

- Value function is prob. of reaching success in ∞ limit:

$$V_\pi(s) = E_\pi[\sum_{t=0}^{\infty} r^t | S^{t=0} = s] \; = \; P(S^{t=\infty} = success | S^{t=0} = s)$$

Overall Learning Framework

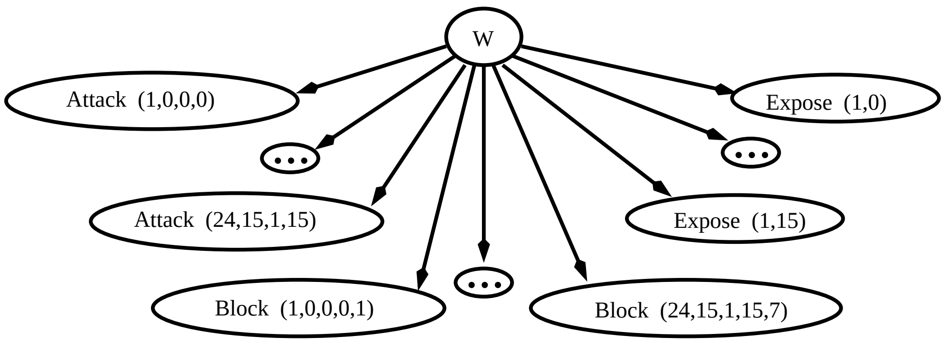
- **For each trial/time-step, record state & final outcome:**



- **Computational and representational issues aside:**
  - Let W be a boolean variable denoting eventual win/success
  - Optimal value function under a fixed policy is $P(W|F_1, \ldots, F_n)$
  - Learning = direct estimate of $P(W|F_1, \ldots, F_n)$ from trial data

Value Function Representation

- **Unfortunately, $P(W|F_1, \ldots, F_n)$ is intractably large... so approximate it with a naive Bayesian network, e.g.**



- **ML cond. prob. table (CPT) params just observed freq.**
- **Then value of a state can be easily calculated:**

$$\hat{P}(w|f) = \frac{\hat{P}(f|w)\hat{P}(w)}{\hat{P}(f)}$$

$$= \frac{\hat{P}(w)\prod_{i=1}^{p}\hat{P}(f_i|w)\prod_{i=p+1}^{n}\hat{P}(\bar{f}_i|w)}{\sum_{o\in\{w,\bar{w}\}}\hat{P}(o)\prod_{i=1}^{p}\hat{P}(f_i|o)\prod_{i=p+1}^{n}\hat{P}(\bar{f}_i|o)}$$

Efficient Policy Evaluation

- **Still many ground atoms, need to eval policy efficiently:**
  - Focus on policy evaluation via after-state analysis
  - Pol. execution is just choice of best state from possible set
  - Only need relative comp., use log winning odds $\log(\frac{P(w|f)}{P(\bar{w}|f)})$

$$\log\frac{P(w|f)}{P(\bar{w}|f)} = \log\frac{P(w)}{P(\bar{w})} + \sum_{i=1}^{p}\log\frac{P(f_i|w)}{P(f_i|\bar{w})} + \sum_{i=p+1}^{n}\log\frac{P(\bar{f}_i|w)}{P(\bar{f}_i|\bar{w})}$$

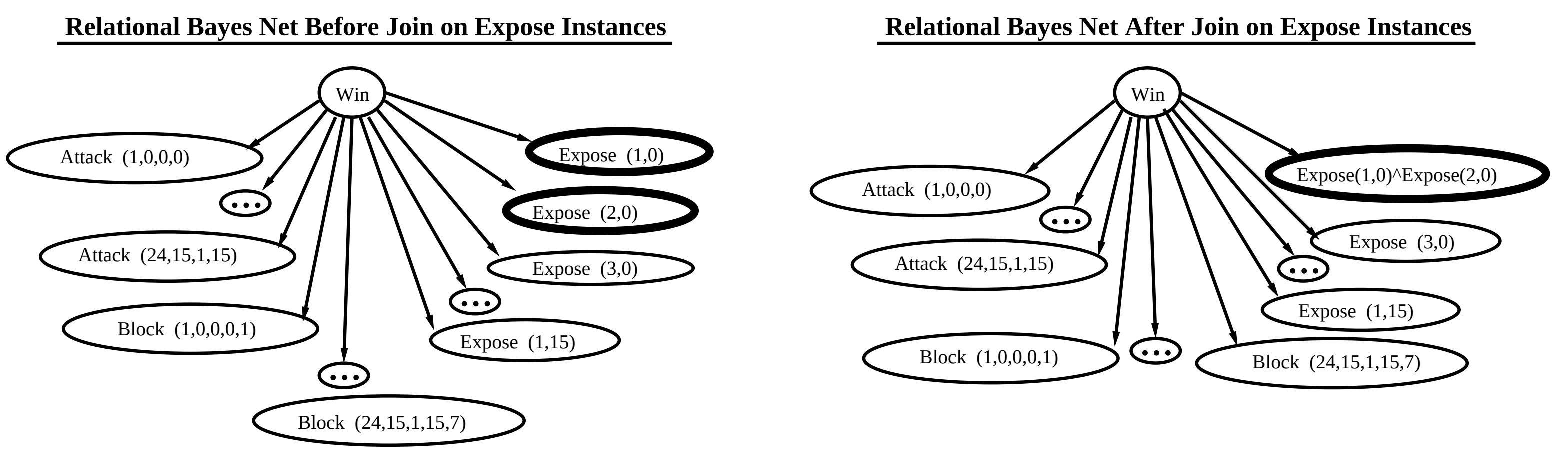
Let $C = \log\frac{P(w)}{P(\bar{w})} + \sum_{i=1}^{n}\log\frac{P(\bar{f}_i|w)}{P(\bar{f}_i|\bar{w})}$

$$\log\frac{P(w|f)}{P(\bar{w}|f)} = C + \sum_{i=1}^{p}\left(\log\frac{P(f_i|w)}{P(f_i|\bar{w})} - \log\frac{P(\bar{f}_i|w)}{P(\bar{f}_i|\bar{w})}\right)$$

- **Find best after-state by only looking at positive features!**

Structure Learning Overview

- **Feature attribute augmentation (FAA) learning:**
  - Each CPT is a conditional probability, e.g. $P(E(5,3,0)|W)$
  - Could approximate CPT probability using attribute estimates with don't cares ".": $P(E(5,.,.)|W) \cdot P(E(.,3,.)|W) \cdot P(E(.,.,0)|W)$
  - Need to determine which **joint attribute est.** are most informative (ML)
- **Feature conjunction (FC) learning:**



  - Can combine nodes to come up with joint probability estimates
  - Need to determine which **joint nodes** are most informative (ML)

Greedy Optimal Structure Learning

- **Given two independent features F_a and F_b:**
  - Want to determine increase in log-likelihood if features considered jointly:
    $\Delta l^*(\theta|D) = C + M \cdot I(F_a, F_b|W))$ (see paper for derivation)
  - In brief, change in log-likelihood due to join given by mutual conditional entropy $I(\cdot)$ times # of data samples M (C is a common constant)
  - Choose FAA or FC joins to maximize log-likelihood (greedy optimal)
- **Caveat: Statistical noise leads to structure overlearning**
  - Solution: Use MDL score: $MDL(B|D) = \frac{1}{2}log(M|B|) - l^*(\theta|D)$
  - Balances log-likelihood score vs. # parameters B in Bayes net
- **Why is this relational RL?**
  - FAA learning applies to **all ground relations** sharing learned attributes
  - Non-parametric CPT learning exploits rel. structure via similarity of attribute dimensions and sparseness of relation sampling (esp. for FC)

Empirical Results

- **Evaluated FAA-SVRRL on Backgammon (est. 10^{18} states)**
- **Learning efficiency data:**
  - Trains 5000 games of self-play in $<$ 10 min on 1 GhZ PIII, 128 Mb
  - Use non-parametric CPT learning: 240 instances, $<$ 10Kb RAM
  - FAA-SVRRL learns faster than static version starting with full structure
- **Asymptotic performance evaluation data:**

PLAYER	WINNING PCT	# TRAINING GAMES
TD-GAMMON 1-PLY, ESTIMATED	66.0 % $\pm$???	1,500,000
FAA-SVRRL	51.2 % $\pm$ 0.02	5,000
PUBEVAL (LINEAR REGRESSION)	50.0 % $\pm$ 0.00	UNKNOWN
HC-GAMMON (GENETIC PROG)	40.0 % $\pm$ 3.46	100,000

Conclusions and Future Work

- **Conclusions:**
  - FAA-SVRRL is efficient structure and value RRL algorithm
  - Achieves commendable performance in Backgammon
- **Future work:**
  - Full implementation and evaluation of FC-SVRRL
  - Experiment with domains other than Backgammon
  - Use other learning frameworks: Prob/ML vs. Winnow/COLT
  - Can we efficiently learn more complex tree-augmented naive Bayes (TAN) or PRM-style structure?