AlphabetSOUP: A Framework for Approximate Energy Minimization Errata

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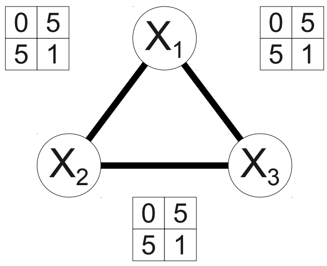
The paper Gould et al. [1] contains an error in Theorem 3.2 as demonstrated by the following counterexample over three binary variables:



Clearly $\boldsymbol{x}^\star = (0,0,0)$ and $E(\boldsymbol{x}^\star) = 0$. Now consider the sequence of γ-expansion moves:

$$A^\gamma \in \Big\{\{0\} \times \{0\} \times \emptyset, \{0\} \times \emptyset \times \{0\}, \emptyset \times \{0\} \times \{0\}, \{1\} \times \{1\} \times \{1\}\Big\}.$$

This set of moves is covering and satisfies the assumptions of [1, Theorem 3.2]. Moreover, $\boldsymbol{x}^\dagger = (1,1,1)$ is a local minimum with respect to these moves. However, $\frac{E(\boldsymbol{x}^\dagger)}{E(\boldsymbol{x}^\star)} = \frac{3}{0} = \infty$.

The theorem is correct for the case of disjoint γ-expansion moves and for the case of $\theta_c(\boldsymbol{x}_c) > 0$ for all c and $\boldsymbol{x}_c$. For the case of non-disjoint A_i^k and $\theta_c(\boldsymbol{x}_c) = 0$ for some c and $\boldsymbol{x}_c$ we need the following to hold: *There must exist some disjoint $\tilde{A}_i^k$ such that:*

- $A_i^k = \tilde{A}_i^k \bigcup_{k'} \tilde{A}_i^{k'}$ *is the union of* $\tilde{A}_i^k$ *with some of the remaining* $\tilde{A}_i^{k'}$*, i.e.,* $\tilde{A}_i^k \subseteq A_i^k$ *and for all* $k' \neq k$, $\tilde{A}_i^{k'} \subseteq A_i^k$ *or* $\tilde{A}_i^{k'} \cap A_i^k = \emptyset$*; and*
- *for all* $\boldsymbol{x}_c$ *such that* $\theta_c(\boldsymbol{x}_c) = 0$ *there exists a move* k *with* $\boldsymbol{x}_c \in \tilde{A}^k$.

This essentially requires the moves to be constructed from a set of disjoint moves satisfying $\boldsymbol{x}_c$ being considered in a move whenever $\theta_c(\boldsymbol{x}_c) = 0$. In particular for any c and c' such that there exist an $\boldsymbol{x}_c$ and $\boldsymbol{x}_{c'}$ with $\theta_c(\boldsymbol{x}_c) = 0$ and $\theta_c(\boldsymbol{x}_{c'}) = 0$ we must have that either $\boldsymbol{x}_c$ and $\boldsymbol{x}_{c'}$ are disjoint (i.e., do not share values) or are considered in the same move. This is a much stronger condition than originally stated in [1, Theorem 3.2].

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References

[1] Stephen Gould, Fernando Amat, and Daphne Koller. Alphabet SOUP: A framework for approximate energy minimization. In *CVPR*, 2009.