Tableau metatheory for propositional and syllogistic logics

Part III: Generalized relational semantics for propositional and syllogistic languages

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Program of lecture

Today we talk on:

- formalization of tableau methods
- generalized relational semantics as a pattern of interpretation for propositional and syllogistic languages
- how to reduce semantics for those languages to generalized relational semantics (in most cases).

Formalization of tableaux

- In the XXth century it was proposed a formal notion of axiomatic proof, that is still commonly accepted.
- It is accepted, since it is abstract and with a little modification is applicable to almost all deduction systems.

Formalization of proofs: axiomatic proofs

In the case of axiomatic proof systems we have the very general notions that under an assumption of some set of formulas For, enables almost straightforwardly to formulate an axiomatic system.

Having a set of formulas For of some language, we define *a rule of proving* as a set of pairs $\langle X, A \rangle$, where $X \subseteq \mathrm{For}$ and $A \in \mathrm{For}$. Of course, in case a rule is an axiom, X is empty set.

An axiomatic system is a pair $\langle \mathrm{For}, \mathbf{R} \rangle$, where $\mathbf{R}$ is some set of rules of proving.

Formalization of proofs: axiomatic proofs

For any axiomatic system $\langle \mathrm{For}, \mathbf{R} \rangle$, we have a general notion of proof. Let $X \subseteq \mathrm{For}$ and $A \in \mathrm{For}$.

Formula A is *provable from X* in $\langle \mathrm{For}, \mathbf{R} \rangle$ iff there exists such a finite sequence of formulas $B_1, \ldots, B_n$ that:

1. $B_n = A$
2. for all $1 \leq i \leq n$ at least one of the cases holds:
   2.1 $B_i \in X$
   2.2 there exist such a rule of proving $R \in \mathbf{R}$ and a pair $\langle Y, C \rangle \in R$ that
      - $C = B_i$
      - either Y is an empty set or for some $m > 0$ and some $0 < k_1, \ldots, k_m < i$, $Y = \{B_{k_1}, \ldots, B_{k_m}\}$.

Formalization of tableaux

The same we should expect from tableaux notions. A very general and abstract formalization of:

- what a tableau proof is
- what a tableau system is.

Formalization of tableaux: strategy

- Here, all notions are presented as set-theoretical ones (for example: branches are sequences of sets and tableaux are sets of those sequences).
- The rest of tableau notions are defined in a similar, formal way:
    1. tableau rules
    2. branches: open, closed, maximal (aka complete)
    3. tableaux: open, closed, complete
    4. also a new notion is presented — tableau consequence relation (as a very special set of branches).

Idea of generalized relational models

- A form of tableaux for a particular logical system depends on two things:
  1. syntax (language) of this logic
  2. semantic structures of this logic.
- Here, we deal with propositional and syllogistic logics.
- Consequently, we propose generalized relational models to have a uniformed semantic pattern for almost all logics of these kinds (maybe all — it's a hypothesis).

Program of tableau metatheory

The presented theory is the next step from more and more general approaches presented among others in:

Jarmużek Tomasz, “Construction of tableaux for classical logic: tableaux as combinations of branches, branches as chains of sets”, *Logic and Logical Philosophy*, 2007, 1(16), pp. 85-101.

Jarmużek Tomasz, “Tableau System for Logic of Categorial Propositions and Decidability”, *Bulletin of The Section of Logic*, 2008, 37 (3/4), pp. 223–231.

Jarmużek Tomasz, *Formalizacja metod tablicowych dla logik zdań i logik nazw* (*Formalization of tableau methods for propositional logics and for logics of names*), Wydawnictwo UMK, Toruń, 2013.

Jarmużek Tomasz, “Tableau Metatheorem for Modal Logics”, *Recent Trends in Philosphical Logic, Trends in Logic*, (Eds) Roberto Ciuni, Heinrich Wansing, Caroline Willkomennen, Springer Verlag, 2013, pp. 105–128.

Language of propositional and syllogistic logics

Symbols:

- $\mathrm{Var} = \{p_i : i \in \mathbb{N}\}$
- set of connectives $\mathrm{Con}_K^L = \{c_i^n : i \in K, n \in L\}$, where $K, L \subseteq \mathbb{N}$ (preferably non-empty)
- brackets:), (.

Formulas:

- *Set of formulas build over symbols* $\mathrm{Var} \cup \mathrm{Con}_K^L \cup \{),(\}$ is the least set of expressions X that:
  - (a) contains Var
  - (b) for all $n, i \in \mathbb{N}$ and $c_i^n \in \mathrm{Con}_K^L$, if $A_1, \ldots, A_n \in X$, then $c_i^n(A_1, \ldots, A_n) \in X$.

Syllogistic language as a special case

We will show that a syllogistic language is a special case of the presented approach.

Syllogistic language: diversity of connectives

A syllogistic language can contain:

Con_{IC} internal connectives, they:

- make terms of terms
- can be iterated.

Con_{EC} external connectives:

Con_{nItEC} some of them make sentences from terms – they can not be iterated

Con_{ItEC} some of them make sentences of sentences – they can be iterated.

Syllogistic language: internal connectives Con_{IC}

Internal connectives make terms of terms:

1. x is **a non-crocodile**
2. x is **a possible crocodile**
3. etc.
4. x is **a crocodile or a spider**
5. x is **a crocodile and a spider**
6. etc.

In case 4. and 5, the terms are fused in some way.

(0) Some **crocodile or spider** may be a philosopher.

(1) Less than n **non-crocodiles** are **a crocodile and spider**.

Notice, when we use connectives like Less than n ... are ... we always assume some natural number (including also 0) instead of variable n. So, we have in fact infinitely, but countably many external connectives of a similar kind.

Syllogistic language: fusion connectives

Fusion of terms can be:

1. of any set-theoretical nature, for example:
   - **a (crocodile or spider)** x that is a crocodile or a spider
   - **a (crocodile and spider and . . .)** x that is a crocodile and a spider and . . .
   - **a (crocodile that is not a spider)** x that is a crocodile and is not a spider
   - so, union, intersection, difference of terms etc.

2. of arbitrary arity, but reducible to binary fusions:
   - $T_1 \mathrm{f}^1 T_2 \mathrm{f}^2 \ldots T_n \mathrm{f}^m T_k$,

where any f^i is a fusion constant and any T_i is a simple term or a fusion of some simpler terms.

Syllogistic language: external binary, non-iterated connectives Con_{nItEC}

As we already know in syllogistic we have mainly binary connectives, like in the examples:

(0) All man are mortal.
(1) Some man may be a philosopher.
(2) Less than n people are logicians.

(0) All ... are ...
(1) Some ... may be ...
(2) Less than n ... are ...

The syllogistic binary connectives generally are not nested, since they make a sentence from two terms.

Syllogistic language: external, iterated connectives Con_{ItEC}

In a syllogistic we can have also external unary connectives, like in the examples:

(0) It is not the case that all man are mortal.

(1) It is possible that some man may be a philosopher.

(2) It is inevitable that less than n people are logicians.

(3) etc.

(0) It is not the case that ...

(1) It is possible that ...

(2) It is inevitable that ...

Syllogistic language: external, iterated connectives Con_{ItEC}

Clearly, external unary connectives can be iterated:

It is not the case that, it is possible that, it is inevitable that more than n people are logicians.

In fact they are propositional connectives – they make sentences of sentences.

If so, then we can add more — not only unary — propositional connectives to a syllogistic language. For example: $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ etc.

Surely, most of them can be also iterated.

Syllogistic language

Notice that, if we assume few things:

- Var are not propositional letters, but term letters
- $\mathrm{Con}_K^L = \{c_i^n : i \in K, n \in L\} = \mathrm{Con}_{IC} \cup \mathrm{Con}_{EC}$, where $\mathrm{Con}_{IC} \cap \mathrm{Con}_{EC} = \emptyset$
- $\mathrm{Con}_{EC} = \mathrm{Con}_{nItEC} \cup \mathrm{Con}_{ItEC}$, where:
  (a) $\mathrm{Con}_{nItEC} \cap \mathrm{Con}_{ItEC} = \emptyset$
  (b) and Con_{nItEC} contains only connectives of arity 2: c_m^2, where $m \in K$

than a certain subset of formulas may serve as a set of syllogistic formulas.

It is defined in the more sophisticated, succeeding way.

Syllogistic language: terms

First let us define a set of terms. The set of terms is the least set X that fulfills the conditions:

(a) $\mathrm{Var} \subseteq X$

(b) if $A_1, \ldots, A_n \in X$, then $c_i^n(A_1, \ldots, A_n) \in X$, for all $n, i \in \mathbb{N}$ and $c_i^n \in \mathrm{Con}_{IC}$.

The set of all terms is denoted by Term.

Here, we have iterations!

Syllogistic language: formulas

Second we define a set of formulas. The set of formulas is the least set X that fulfills the conditions:

(a) if $c_i^2 \in \mathrm{Con}_{nItEC}$ and $A, B \in \mathrm{Term}$, then $c_i^2(A, B) \in X$,

(b) if $c_i^n \in \mathrm{Con}_{ItEC}$ and $A_1, \ldots, A_n \in X$, then $c_i^n(A_1, \ldots, A_n) \in X$,

for all $n, i \in \mathbb{N}$.

Propositional vs. syllogistic language

Notice that, if we:

- interpret Var as propositional letters
- assume that $\mathsf{Con}_{IC} = \mathsf{Con}_{nItEC} = \emptyset$,

we have a propositional language.

On the other hand, if we:

- interpret Var as atomic terms (so, they are not included in For)
- assume that $\mathsf{Con}_{nItEC} \neq \emptyset$,

we have a syllogistic language.

Set of formulas

We assume some set of formulas (whether propositional or syllogistic) and denote it by For.

Generalized relational models

Definition (Models)

A generalized relational model is the following ordered triple: $\langle \{W_i\}_{i \in M}, \{R_j\}_{j \in N}, V \rangle$, where:

- (a) M, N are sets of indexes,
- (b) $\{W_i\}_{i \in M}$ is a non-empty family of indexed, non-empty sets (called *set of domains*),
- (c) $\{R_j\}_{j \in N}$ is a possibly empty family of such relations, that for any $j \in N$ there exist such $n \in \mathbb{N}$ and $i_1, \ldots, i_n \in M$, that $R_j \subseteq W_{i_1} \times \cdots \times W_{i_n}$,
- (d) V is a valuation of propositional letters in all domains, so $V\colon \bigcup_{i \in M} W_i \times \mathrm{Var} \longrightarrow \{0, 1\}$.

The set of all generalized models we denote by **GM**.

One pattern for semantic consequence

Definition (Set of formulas interpreted by subset of general models)

Let $\mathbf{M} \subseteq \mathbf{GM}$. For is *interpreted* by $\mathbf{M} \subseteq \mathbf{GM}$ iff for any formula $A \in$ For at any model $\mathfrak{M} = \langle \{W_i\}_{i \in M}, \{R_j\}_{j \in N}, V \rangle \in \mathbf{M}$, for any $i \in M$ at any w belonging to W_i:

- either $\mathfrak{M}, w \models A$,
- or $\mathfrak{M}, w \not\models A$,

by some truth–conditions.

In other words, it happens, if we can define (for example, inductively) relation $\mathfrak{M}, w \models A$ and — by negation — relation $\mathfrak{M}, w \not\models A$ — notions of *being satisfied* and of *being not satisfied* in model $\mathfrak{M}$ at any w.

Models with the ith domain

Definition (Model with the ith domain)

Let $\mathbf{M} \subseteq \mathbf{GM}$. Let $i \in \mathbb{N}$.

If for all models $\mathfrak{M} = \langle \{W_i\}_{i \in M}, \{R_j\}_{j \in N}, V \rangle \in \mathbf{M}$, $i \in M$, then $\mathbf{M}$ is called *set of models with the ith domain*.

We denote it by $\mathbf{M}^i$.

One pattern of semantic consequence

Definition (Semantic consequence relation)

Let $\mathbf{M}^i \subseteq \mathbf{GM}$, for some $i \in \mathbb{N}$. Let For be interpreted by $\mathbf{M}^i$. Let $X \subseteq \mathrm{For}$, $A \in \mathrm{For}$.

Formula A is *a semantic consequence of set of formulas* X *in respect of set of models* $\mathbf{M}^i$ (in short: (✠) $X \models_{\mathbf{M}^i} A$) iff

(a) for all $\mathfrak{M} = \langle \{W_i\}_{i \in M}, \{R_j\}_{j \in N}, V\rangle \in \mathbf{M}^i$,

(b) for all $w \in W_i$,

if for all $B \in X$: $\mathfrak{M}, w \models B$, then $\mathfrak{M}, w \models A$.

In this way we obtain semantically defined logic $\langle \models_{\mathbf{M}^i}, \mathrm{For}\rangle$ or just $\models_{\mathbf{M}^i}$, for each such set of models $\mathbf{M}^i \subseteq \mathbf{GM}$ that satisfies the initial assumptions.

For the further examination we assume logic $\langle \models_{\mathbf{M}^i}, \mathrm{For}\rangle$, for some $\mathbf{M}^i$. However we omit subscript $\mathbf{M}^i$, when possible.

Scope of general models

- Subsets of **GM** can serve as semantic structures for various propositional or syllogistic logics with one pattern of semantic consequence (✠).
- A domain W_i always serves in a model as an ultimate set of points of relativization.
- The rest of domains can serve as usual domains or as sets that code non–classical values or even formulas for semantics with a relation like in case of relating logics.
- Relations in a model can be: accessibility relations, functions (like ternary relations or Routley star $*$, heredity relation $\sqsubseteq$ etc.).

Scope of general models

- That is why in that semantic pattern can be determined:

  1. modal logics of various kind (intuitionist, conditional, relevant, paraconsistent etc.)
  2. multi–modal logics
  3. many–valued logics
  4. combinations of any of them
  5. various kinds of syllogistic, if we assume For to be a syllogistic language.

- So, if we can cover generalized relational models semantics by some universal tableau language Ex, we could also get abstract tableau notions for (✠) logics.

Two–valued logics, examples of applications: CPL

The most extreme and at the same time the most trivial case is CPL.

Take all models $\langle \{W_i\}_{i \in M}, \{R_j\}_{j \in N}, V \rangle$, where:

- $M = \{1, 2, 3, 4\}$,
- $W_1 = \{w\}$, $W_2 = \{w_0, w_1\} \supset W_3 = \{w_1\}$,
- $W_4 = W_1 \times W_2 = \{w\} \times \{w_0, w_1\}$,
- $\{R_j\}_{j \in N}$ is empty,
- $V'\colon W_1 \cup W_2 \times \mathrm{Var} \longrightarrow \{0\}$,
- $V''\colon W_4 \times \mathrm{Var} \longrightarrow \{0, 1\}$, limited by condition:

$$V''(\langle w, w_0 \rangle, p_i) = 1 \text{ iff } V''(\langle w, w_1 \rangle, p_i) = 0,$$

- $V = V' \cup V''$.

Two–valued logics, examples of applications: CPL

Now, assuming Boolean truth conditions for classical connectives and extending function V to $\overline{V}$, we have:

$$\mathfrak{M}, \langle w, w' \rangle \models A \text{ iff } \overline{V}(\langle w, w' \rangle, A) = 1,$$

for $w \in W_1$, all $w' \in W_2 = \{w_0, w_1\}$ and all formulas $A \in \mathsf{For}$.

Next, for all points of relativization in $W_1 = \{w\}$ we have:

$$\mathfrak{M}, w \models A \text{ iff for some } w' \in W_3,\ \mathfrak{M}, \langle w, w' \rangle \models A,$$

in fact if $w' = w_1$.

Then, by condition (✠) (where $i = 1$), we get Classical Propositional Logic: $\models_{\mathrm{CPL}}$.

Two–valued logics, examples of applications: CPL more easily

CPL can be obviously defined much more easily, since it is an extreme case: two–valued, with one point of relativization.

Taking all models $\langle \{W_i\}_{i \in M}, \{R_j\}_{j \in N}, V \rangle$, where:

- $M = \{1\}$, $W_1 = \{w\}$,
- $\{R_j\}_{j \in N}$ is empty,
- $V \colon W_1 \times \mathrm{Var} \longrightarrow \{0, 1\}$,

we have a usual valuation of sentential letters p_i:

$\mathfrak{M}, w \models p_i$ iff $V(w, p_i) = 1$.

Now, assuming Boolean truth conditions for classical connectives, we can extend it to all formulas, and this subset of generalized relational models works just as the set of classical valuations.

Then, by condition (✠), we get Classical Propositional Logic: $\models_{\mathrm{CPL}}$.

Two–valued logics, examples of applications: modal logic

It is obvious that models $\langle \{W_i\}_{i\in M}, \{R_j\}_{j\in N}, V\rangle$ are ready to be semantic structures for various:

- modal
- multi-modal
- normal as well as non-normal logics.

They are tailor-made.

We just take a proper numbers of domains and relations.

However, to be close to the general pattern we can define them in a more difficult way.

Two–valued logics, examples of applications: modal logic

We consider a two–valued logic with one unary modality (so, with one binary accessibility relation).

Take all models $\langle \{W_i\}_{i \in M}, \{R_j\}_{j \in N}, V \rangle$, where:

- $M = \{1, 2, 3, 4\}$,
- $W_1 \neq \emptyset$, $W_2 = \{w_0, w_1\} \supset W_3 = \{w_1\}$,
- $W_4 = W_1 \times W_2 = W_1 \times \{w_0, w_1\}$,
- $N = \{1\}$, so $\{R_j\}_{j \in N} = \{R_1\}$, where $R_1 \subseteq W_1 \times W_1$,
- $V' \colon W_1 \cup W_2 \times \mathrm{Var} \longrightarrow \{0\}$,
- $V'' \colon W_4 \times \mathrm{Var} \longrightarrow \{0, 1\}$, limited by condition:

  $$V''(\langle w, w_0 \rangle, p_i) = 1 \text{ iff } V''(\langle w, w_1 \rangle, p_i) = 0$$

  for all $w \in W_1$,
- $V = V' \cup V''$.

Two–valued logics, examples of applications: modal logic

Now, assuming Boolean truth conditions for classical connectives and modal conditions for modal ones in any world $w \in W_1$, and extending function V to $\overline{V}$, we have:

$$\mathfrak{M}, \langle w, u \rangle \models A \text{ iff } \overline{V}(\langle w, u \rangle, A) = 1,$$

for all $\langle w, u \rangle \in W_4$, where $w \in W_1$, $u \in W_2 = \{w_0, w_1\}$, and all formulas $A \in \text{For}$.

Next, for all points of relativization w in W_1 we have:

$$\mathfrak{M}, w \models A \text{ iff for some } u \in W_3,\ \mathfrak{M}, \langle w, u \rangle \models A,$$

so in fact if $u = w_1$.

Then, by condition (✠) (where $i = 1$), we get the modal logic we have considered.

Two–valued logics, examples of applications: modal logic more easily

The two–valued logic with one unary modality (one binary accessibility relation) we consider can be defined much more easily (not to say traditionally!).

Taking all models $\langle \{W_i\}_{i \in M}, \{R_j\}_{j \in N}, V \rangle$, where:

- $M = \{1\}$, $W_1 \neq \emptyset$,
- $N = \{1\}$, so $\{R_j\}_{j \in N} = \{R_1\}$, where $R_1 \subseteq W_1 \times W_1$,
- $V \colon W_1 \times \mathrm{Var} \longrightarrow \{0, 1\}$,

we have a usual valuation of sentential letters p_i:

$\mathfrak{M}, w \models p_i$ iff $V(w, p_i) = 1$.

Now, assuming Boolean truth conditions for classical connectives and modal conditions for modal ones, we can extend it for all formulas, and some subset of generalized relational models works as possible world models.

Then, by condition (✠), we get: $\models_{\mathrm{K}}$.

Examples of applications: many–valued logic

It is another extreme case (comparing to CPL).

Let us take logic with $m > 2$ logical values, where n values are designated.

In set **M** we choose the subset of models $\langle \{W_i\}_{i \in M}, \{R_j\}_{j \in N}, V \rangle$, where:

- $M = \{1, 2, 3, 4\}$,
- $W_1 = \{w\}$,
- $W_2 = \{w_1, \ldots, w_m\} \supset W_3 = \{w_{k_1}, \ldots, w_{k_n}\}$,
- $W_4 = W_1 \times W_2$
- $\{R_j\}_{j \in N}$ is empty,
- and valuation $V \colon \bigcup_{i \in M} W_i \times \mathsf{Var} \longrightarrow \{0, 1\}$, that satisfies conditions:

  (+) for all $p_i \in \mathsf{Var}$ there exists exactly one such $w' \in W_2$, that $V(\langle w, w' \rangle, p_i) = 1$.

Examples of applications: many–valued logic

Now, assuming some truth conditions for connectives and extending function V to $\overline{V}$, we have:

$$\mathfrak{M}, \langle w, w' \rangle \models A \text{ iff } \overline{V}(\langle w, w' \rangle, A) = 1,$$

for $w \in W_1$, all $w' \in W_2 = \{w_1, \ldots, w_m\}$ and all formulas $A \in \mathrm{For}$.

Next, for all points of relativization in $W_1 = \{w\}$ we have:

$$\mathfrak{M}, w \models A \text{ iff for some } w' \in W_3, \, \mathfrak{M}, \langle w, w' \rangle \models A,$$

so $w' \in \{w_{k_1}, \ldots, w_{k_n}\}$.

Then, by condition (✠) (where $i = 1$), we obtain some many–valued logic $\models_{m-val}$.

Clearly, we can not simplify this like two–valued logics, without extending V to an m–ary co-domain.

Examples of applications: combining many–valuedness with modalities

Let us take a modal logic with $m > 2$ logical values, where n values are designated, with one unary modality (surely, there can be more modalities with bigger arities!).

In set $\mathbf{M}$ we choose the subset of models $\langle \{W_i\}_{i \in M}, \{R_j\}_{j \in N}, V \rangle$, where:

- $M = \{1, 2, 3, 4\}$, $W_1 \neq \emptyset$,
  $W_2 = \{w_1, \ldots, w_m\} \supset W_3 = \{w_{k_1}, \ldots, w_{k_n}\}$,
- $W_4 = W_1 \times W_2$,
- $N = \{1\}$, so $\{R_j\}_{j \in N} = \{R_1\}$, where $R_1 \subseteq W_1 \times W_1$,
- valuation $V \colon \bigcup_{i \in M} W_i \times \mathrm{Var} \longrightarrow \{0, 1\}$, that satisfies condition:

(++) for all $p_i \in \mathrm{Var}$ and all $w_1 \in W_1$ there exists exactly one such $w \in W_2$, that $V(\langle w_1, w \rangle, p_i) = 1$.

Examples of applications: combining many–valuedness with modalities

Now, assuming Boolean truth conditions for classical connectives and modal conditions for modal ones in any world $w \in W_1$, and extending function V to $\overline{V}$ we have:

$$\mathfrak{M}, \langle w, u\rangle \models A \text{ iff } \overline{V}(\langle w, u\rangle, A) = 1,$$

for all $\langle w, u\rangle \in W_4$, where $w \in W_1$, $u \in W_2 = \{w_1, \ldots, w_m\}$, and all formulas $A \in \mathrm{For}$.

Next, for all points of relativization w in W_1 we have:

$$\mathfrak{M}, w \models A \text{ iff for some } u \in W_3,\ \mathfrak{M}, \langle w, u\rangle \models A,$$

so in fact if $u \in \{w_{k_1}, \ldots, w_{k_n}\}$.

Then, by condition (✠) (where $i = 1$), we get the modal logic we have considered.

Similarly, we can treat infinitely many–valued, modal logics.

Examples of applications: simple syllogistic CS

Let us remind, CS is a logic of categorial propositions:

- All P are Q
- No P are Q
- Some P is/are Q
- Some P is/are not Q.

Symbols of CS are:

- term letters: Term $= \{P_i : i \in \mathbb{N}\}$ (in practice we write: P, Q, R etc.)
- logical constants: Con $= \{\mathbf{a}, \mathbf{i}, \mathbf{o}, \mathbf{e}\}$.

Set of formulas $\mathsf{For}_{\mathrm{CS}}$ consists of:

1. $\Phi\mathbf{a}\Psi$
2. $\Phi\mathbf{i}\Psi$
3. $\Phi\mathbf{o}\Psi$
4. $\Phi\mathbf{e}\Psi$

where $\Phi, \Psi \in$ Term.

Examples of applications: simple syllogistic CS

In set **M** we take the subset of models $\langle \{W_i\}_{i \in M}, \{R_j\}_{j \in N}, V \rangle$, where:

- $M = \{1, 2, 3, 4, 5\}$,
- $W_1 = \{w\}$,
- W_2 is an arbitrary set (it is the set of objects that can be denoted by terms),
- $W_3 = \{w_1,\ w_0\} \supset W_4 = \{w_1\}$,
- $W_5 = W_1 \times W_2 \times W_3$,
- $\{R_j\}_{j \in N}$ is empty,
- valuation $V\colon \bigcup_{i \in M} W_i \times \mathrm{Var} \longrightarrow \{0, 1\}$, that satisfies condition:

(+++) for all $\Phi \in$ Term, all $w \in W_1$ and all $z \in W_2$ there exists exactly one such $u \in W_3$, that $V(\langle w, z, u \rangle, \Phi) = 1$.

Examples of applications: simple syllogistic CS

Now, let $\langle w, z, u \rangle \in W_5$. We define:

$$\mathfrak{M}, \langle w, z, u \rangle \models \Phi \text{ iff } V(\langle w, z, u \rangle, \Phi) = 1,$$

and finally for $z \in W_2$:

$$\mathfrak{M}, w, z \models \Phi \text{ iff for some } u \in W_4,\ \mathfrak{M}, \langle w, z, u \rangle \models \Phi,$$

but in this case $u \in W_4 = \{w_1\}$.

At this moment, we could give directly the truth–conditions for all categorial sentences or reconstruct traditional syllogistic models, and then define the truth–conditions for CS.

It will result in the same logic.

Examples of applications: simple syllogistic CS

Let us take the simplest way. We define truth-conditions directly:

(1) $\mathfrak{M}, w \models \Phi\mathbf{a}\Psi$ iff $\forall_{z\in W_2}(\mathfrak{M}, w, z \models \Phi \Rightarrow \mathfrak{M}, w, z \models \Psi)$

(2) $\mathfrak{M}, w \models \Phi\mathbf{i}\Psi$ iff $\exists_{z\in W_2}(\mathfrak{M}, w, z \models \Phi \,\&\, \mathfrak{M}, w, z \models \Psi)$

(3) $\mathfrak{M}, w \models \Phi\mathbf{e}\Psi$ iff $\forall_{z\in W_2}(\mathfrak{M}, w, z \models \Phi \;\Rightarrow\; \mathfrak{M}, w, z \not\models \Psi)$

(4) $\mathfrak{M}, w \models \Phi\mathbf{o}\Psi$ iff $\exists_{z\in W_2}(\mathfrak{M}, w, z \models \Phi \,\&\, \mathfrak{M}, w, z \not\models \Psi)$

for all $\Phi, \Psi \in \mathrm{Term}$.

Now, applying (as usual) condition (✠) (where $i = 1$), we get the syllogistic logic we have considered.

Examples of applications, simple syllogistic: possible extensions

In set **M** we can choose the subset of models $\langle \{W_i\}_{i \in M}, \{R_j\}_{j \in N}, V \rangle$, where:

- $M = \{1, 3, 4, 5\} \cup \{W_i\}_{i \in K}$,
- W_1 is a non-empty set of points of relativization,
- $\{W_i\}_{i \in K}$ is a family of sets and $K = W_1$,
- $W_3 = \{w_1, \ldots, w_m\} \supset W_4 = \{w_{k_1}, \ldots, w_{k_n}\}$, for $m, n \in \mathbb{N}$,
- $W_5 = \{\langle w, z, u \rangle \colon w \in W_1, z \in W_w \in \{W_i\}_{i \in K}, u \in W_3\}$,
- $\{R_j\}_{j \in N}$ is empty or non-empty if we have modal connectives,
- valuation $V \colon \bigcup_{i \in M} W_i \times \mathrm{Var} \longrightarrow \{0, 1\}$, that satisfies condition:

(++++) for all $\Phi \in \mathrm{Term}$, all $w \in W_1$ and all $z \in W_w$ there exists exactly one such $u \in W_3$, that $V(\langle w, z, u \rangle, \Phi) = 1$.

Now we can define a modal, many–valued syllogistic in the already known way.

Hypothesis on the range of generalized relational models

Hypothesis

Let $\vdash$ be a logic defined on $\mathsf{P}(\mathsf{For}) \times \mathsf{For}$, for some set of formulas For. There exists such a set of generalized models $\mathbf{M} \subseteq \mathbf{GM}$ that:

$$X \vdash A \text{ iff } X \models_{\mathbf{M}} A,$$

for all $X \cup \{A\}$.

Obviously, by *logic* we mean a consequence relation that can satisfy some important and desired conditions, like for example uniform substitution, reflexivity, monotony, idempotence, finitary etc.

If the hypothesis is true, then we are able to cover a lot of logics with the tableau methodology we propose in the subsequent parts.

A similar approach (but different in details) to general semantics is presented in:

Luis Estrada-González, *The (Non-)classicality of (Non-)classical Mathematics*, *Journal of Indian Council of Philosophical Research*, (2017) 34, pp. 365–377.

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