Throughput and Ergodic Capacity of Wireless Energy Harvesting Based DF Relaying Network


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Abstract—In this paper, we consider a decode-and-forward (DF) relaying network based on wireless energy harvesting. The energy constrained relay node first harvests energy through radio-frequency (RF) signals from the source node. Next, the relay node uses the harvested energy to forward the decoded source information to the destination node. The source node transfers energy and information to the relay node through two mechanisms, i) time switching-based relaying (TSR) and ii) power splitting-based relaying (PSR). Considering wireless energy harvesting constraint at the relay node, we derive the exact analytical expressions of the achievable throughput and ergodic capacity of a DF relaying network for both TSR and PSR schemes. Through numerical analysis, we study the throughput performance of the overall system for different system parameters, such as energy harvesting time, power splitting ratio, and signal-to-noise-ratio (SNR). In particular, the throughput performance of the PSR scheme outperforms the throughput performance of the TSR scheme for a wide range of SNRs.

I. INTRODUCTION

Recently, energy harvesting through wireless radio frequency (RF) signals has received significant attention [1], [2]. In wireless energy harvesting, ambient RF radiation is captured by the receiver antennas and converted into a direct current (DC) voltage through appropriate circuits [1]. The majority of the recent research in wireless energy harvesting and information processing has considered point-to-point communication systems and studied rate-energy trade-off assuming single-input-single-output (SISO) [1], [3], single-input-multiple-output (SIMO) [4], and multiple-input-multiple-output (MIMO) [5], [6] setups. The application of wireless energy harvesting to orthogonal frequency division multiplexing (OFDM) [7] and cognitive radio [2] based systems has also been considered. Energy beamforming through wireless energy harvesting has been studied for the multi-antenna wireless broadcasting system in [8]. Moreover, secure transmission in the presence of eavesdropper under wireless energy harvesting constraint has been studied in MISO beamforming systems [9].

An important application of wireless energy harvesting is in cooperative relaying networks, where an intermediate relay node assists in the transmission of the source information to the destination. The relay node may have limited battery reserves and thus relies on some external charging mechanism in order to remain active in the network [10]. Therefore, energy harvesting in such networks is particularly important as it can enable information relaying. Some studies have recently considered energy harvesting through RF signals in wireless relaying networks [11]–[16]. The different rate-energy trade-offs to achieve the optimal source and relay precoding in a MIMO relay system is studied in [11]. The outage performance of a typical cooperative communication system is studied in [12]. However, the authors in [11], [12] assume that the relay has its own internal energy source and does not need external charging. Multi-user and multi-hop systems for simultaneous information and power transfer are investigated in [13]. However, the optimization strategy in [13] assumes that the relay node is able to decode information and extract power simultaneously, which, as explained in [1], may not hold in practice. The outage performance of an amplify-and-forward (AF) relaying network under energy harvesting constraints is studied in [14], [15]. For a decode-and-forward (DF) relaying network, the power allocation strategies and outage performance under energy harvesting constraints is studied in [16]. It must be noted that the throughput analysis in DF relaying networks is fundamentally different from the analysis in AF relaying networks under wireless energy harvesting constraint, due to the different nature of the relaying protocols.

In this paper, we consider a DF relaying network and analyze the system throughput in the presence of energy harvesting constraints. We assume that the relay is an energy constrained node and thus harvests energy from the RF signal broadcasted by the source node. Unlike [13], we adopt time switching (TS) and power splitting (PS) receiver architectures proposed in [1], for separate information processing and energy harvesting at the relay node. In time switching-based relaying (TSR), the relay switches in time between energy harvesting and information processing, however, in power splitting-based relaying (PSR), the relay splits the received power for energy harvesting and information processing. Unlike [16], where the outage probability is analyzed in delay-limited transmission mode, we consider the delay-tolerant transmission mode. This implies that the destination node can buffer the received information blocks and can tolerate the delay in decoding the received signal. Thus, the code length can be kept very large compared to the transmission block time. Consequently, we analyze the ergodic capacity of a DF relaying network in the presence of wireless energy harvesting constraint. The main contributions of this work are as summarized below:

- We derive the closed-form analytical expressions for the ergodic capacity and achievable throughput of a DF relaying network in the presence of wireless energy harvesting constraints for both the TSR and the PSR schemes. The derived expressions provide practical design insights into the effect of various parameters on the system performance.
- Using our derived results, we compare the throughput performance of the TSR and PSR schemes for different system parameters, such as, energy harvesting time, power splitting ratio, or signal-to-noise-ratio (SNR). In particular, we show that the throughput performance of the PSR scheme outperforms that of the TSR scheme for a wide range of SNRs.

The rest of the paper is organized as follows. Section II presents the overall system model and assumptions. Sections III

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and IV detail the analytical derivation of the ergodic capacity and achievable throughput for the TSR and the PSR schemes, respectively. Section V presents the numerical results from which various design insights are obtained. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

We consider a DF relaying cooperative network, where the information is transferred from the source node, $S$, to the destination node, $D$, through an energy constrained intermediate relay node, $R$. All nodes are equipped with a single antenna. We assume no direct link between the source and the destination node. Thus, an intermediate DF relay assists the transmission of the source messages to the destination. First, the DF relay harvests energy from the source signal. Then, it uses the harvested energy as a source of transmit power to forward the source information to the destination. We assume that the processing power required by the information decoding circuitry at the relay is negligible as compared to the power used for signal transmission from the relay to the destination. This is justifiable when the transmission distances are large such that the energy transmitted is the dominant source of energy consumption [1], [10]. Throughout this paper, inter-node distances between $S \rightarrow R$ and $R \rightarrow D$ are denoted by $d_1$ and $d_2$, respectively.

The $S \rightarrow R$ and $R \rightarrow D$ channel gains, denoted by $h$ and $g$, respectively, are modeled as block-fading and frequency non-selective parameters. The channel is constant over the block time $T$ and independent and identically distributed from one block to the next. The fading is assumed to be frequency non-selective Rayleigh block fading. The use of such channels is motivated by prior research in this field [1], [3], [5], [10]–[12], [14], [16]. In this paper, we assume no availability of channel state information (CSI) at the source node. The CSI is only available at the relay and destination receivers, which is inline with the previous work in this research field [11], [12], [14], [16].

For the joint task of energy harvesting and information processing at the relay node, we adopt time switching and power splitting at the relay node with the power $P_{TS}$, which is achieved by the rectifier circuitry [1]. Using (1), the harvested energy, $E_{h}^{TS}$ during energy harvesting time $\alpha T$ is given by [1]

$$E_{h}^{TS} = \eta \frac{P_s |h|^2}{d_1^\alpha} \alpha T,$$

where $0 < \eta < 1$ is the energy conversion efficiency which depends on the rectification process and the energy harvesting circuitry [1].

B. $R \rightarrow D$ Information Transmission

After the source to relay information transmission, the DF relay decodes the source signal and forward it to the destination node with the power $P_{TS}$, which is available from the energy harvested during energy harvesting time. The received signal at the destination node, $y_{d}^{TS}(k)$, in the TSR scheme is given by [1]

$$y_{d}^{TS}(k) = \frac{1}{\sqrt{d_2^\alpha}} \sqrt{\frac{P_{TS}}{T}} |g| s(k) + n_{a,d}(k) + n_{c,d}(k),$$

where $d_2$ is the relay to destination distance, $g$ is the relay to destination channel gain, $s(k)$ is the decoded version of the signal $s(k)$, and $n_{a,d}(k)$ and $n_{c,d}(k)$ are the antenna and converter AWGNs at the destination node, respectively. Since relay node transmits the decoded signal $s(k)$ using the harvested energy $E_{h}^{TS}$ as a source of power, for $(1-\alpha)T/2$ time, the transmitted power from the relay node, $P_{TS}^{R}$, can be given by

$$P_{TS}^{R} = \frac{E_{h}^{TS}}{(1-\alpha)T/2} = \frac{2\eta P_s |h|^2}{d_1^\alpha (1-\alpha)}.$$
Substituting the value of $P_{TS}$ from (4) into (3), the received signal at the destination, $y_{d}^{TS}(k)$ in terms of $P_{s}, \eta, \alpha, d_{1}$ and $d_{2}$, is given by

$$y_{d}^{TS}(k) = \frac{\sqrt{2n_{d}(k)}}{\sqrt{\sigma_{n}^{2}(1-\alpha)}} + n_{d}(k), \quad (5)$$

where $n_{d}(k) \triangleq n_{a,d}(k) + n_{c,d}(k)$ is the overall AWGN at the destination node.

### C. Throughput Analysis

Using (1), the SNR at the relay node, $\gamma_{r}^{TS}$, in the TSR scheme is given by

$$\gamma_{r}^{TS} = \frac{P_{r}|\mathbf{h}|^{2}}{d_{r}^{m_{r}} \sigma_{n}^{2} + \sigma^{2}_{n_{r,c}}}, \quad (6)$$

where $\sigma_{n}^{2} \triangleq \sigma_{n_{r,c}}^{2} + \sigma_{n_{c,r}}^{2}$ is the variance of overall AWGN at the relay node, $\gamma_{r}^{TS} \triangleq n_{a,d}(k) + n_{a,c}(k)$, in the TSR scheme and $\sigma_{n_{r,c}}^{2}$ and $\sigma_{n_{c,r}}^{2}$ are the noise variances of the AWGNs $n_{a,d}(k)$ and $n_{c,d}(k)$. Using (5), the SNR at the destination, $\gamma_{d}^{TS}$, in the TSR scheme is given by

$$\gamma_{d}^{TS} = \frac{2n_{d}(k)}{d_{r}^{m_{r}} \sigma^{2}_{n_{r,c}} (1-\alpha)}, \quad (7)$$

where $\sigma_{d}^{2} \triangleq \sigma_{n_{a,d}}^{2} + \sigma_{n_{c,d}}^{2}$. In order to determine the throughput, we need to evaluate the ergodic capacity for source to relay link, $C_{TS}^{TS}$, and for relay to destination link, $C_{TS}^{TS}$. Using the received SNRs, $\gamma_{r}^{TS}$ and $\gamma_{d}^{TS}$, defined in (6) and (7), respectively, $C_{TS}^{TS}$ and $C_{TS}^{TS}$ are given by

$$C_{TS}^{TS} = E_h \left\{ \log_{2}(1 + \gamma_{TS}) \right\}, \quad C_{TS}^{TS} = E_{h,a} \left\{ \log_{2}(1 + \gamma_{TS}) \right\}, \quad (8)$$

where $\gamma_{TS}$ depends on $h$ and $\gamma_{TS}$ depends on $h$ and $g$. Note that we need to calculate the ergodic capacities for both source to relay and relay to destination links because the actual ergodic capacity is given by the minimum of $C_{TS}^{TS}$ and $C_{TS}^{TS}$ [17]. The analytical expressions for the ergodic capacities, $C_{r}^{TS}$ and $C_{d}^{TS}$, are given in the following theorem.

**Theorem 1.** The ergodic capacities, $C_{TS}^{TS}$ and $C_{TS}^{TS}$ for the TSR scheme are given by

$$C_{TS}^{TS} = \frac{\lambda_{s}^{0} E_{1} \left( \frac{a}{\lambda_{s}} \right)}{\alpha \log(2)}, \quad (9a)$$

$$C_{TS}^{TS} = \frac{2b}{\lambda_{g}^{0} \lambda_{s} \log(4)} \left[ \sum_{p,q} \left( \frac{m_{p,n}}{n_{s}} \right) \right], \quad (9b)$$

where $a \triangleq \frac{\sigma^{2}_{s}}{\sigma^{2}_{n_{s}}}$ is the exponential integral, $\lambda_{s}$ and $\lambda_{g}$ are the mean values of the exponential random variables $|\mathbf{h}|^{2}$ and $|g|^{2}$, respectively, $\log$ is the natural logarithm, $b \triangleq \frac{\alpha \sigma_{n_{r,c}}^{2} (1-\alpha)}{2\eta P_{s} \alpha}$ and the meijerG function is defined below

$$G_{p,q}^{m_{p,n}} \left( \left[ \{0\}, \{0\} \right], \left[ \{0\}, \{0\} \right] \right) = \frac{1}{2\pi i} \int \prod_{j=m+1}^{n+1} \Gamma(1-\alpha j-s) \prod_{j=m+1}^{n+1} \Gamma(\beta_{j} + s) z^{-s} d\zeta,$$

where $i = \sqrt{-1}$ and $\Gamma(x) = \int_{0}^{\infty} e^{-t} t^{-1} dt$ is the Gamma function.

### IV. POWER SPLITTING-BASED RELAYING (PSR)

Fig. 2 shows the transmission block structure in the PSR scheme for energy harvesting and information processing at the relay, where $P$ is the received signal power and $T$ is the block time. Half of the time, $T/2$ is used for the source to relay information transmission and the remaining half, $T/2$ is used for the relay to destination information transmission. During the first half, the fraction of the received signal power, $\rho P$, is used for energy harvesting and the remaining received power, $(1-\rho) P$ is used for transmitting source information to the relay node, where $\rho \in (0,1)$. In PSR scheme, the choice of the power fraction, $\rho$, controls the achievable throughput at the destination. The following subsections analyze the energy harvesting and information processing at the relay for the PSR scheme.

#### A. S → R Energy Harvesting and Information Transmission

In the PSR scheme, the power splitter at the relay node splits the received signal in $\rho : 1-\rho$ proportion, such that the portion of the received signal, $\sqrt{\rho} y_{r}(k)$ is sent to the energy harvesting receiver and the remaining signal strength, $\sqrt{1-\rho} y_{r}(k)$ drives the information receiver, where $y_{r}(k)$ is defined in (1). Using the signal received at the input of the energy harvesting receiver, $\sqrt{\rho} y_{r}(k) = \frac{\sqrt{\sigma_{n}^{2}}}{\sqrt{1-\rho}} \sqrt{\rho} \mathbf{h}_{s}(k) + \sqrt{\rho} \sqrt{\sigma_{n_{s}}^{2}}(k)$, the harvested energy, $E_{h}^{TS}$ at the relay in the PSR scheme is given by [1]
\[ E_{h}^{PS} = \frac{\eta P_s |h|^2}{d_1^m} (T/2), \]  
(13)
where the energy is harvested at the relay during half of the block time, \( T/2 \), as shown in Fig. 2, and \( 0 < \eta < 1 \) is the energy conversion efficiency. After power splitting, the baseband signal at the input of the information receiver, \( \sqrt{1-\rho}y_r(k) \), in the PSR scheme is given by
\[ \sqrt{1-\rho}y_r(k) = \frac{1}{\sqrt{d_1^m}} \sqrt{(1-\rho)P_s h \sigma(k) + \sqrt{(1-\rho)}n_{a,r}(k)} + n_{c,r}(k). \]  
(14)

B. \( \mathbb{R} \rightarrow \mathbb{D} \) Information Transmission

Using the received signal, \( \sqrt{1-\rho}y_r(k) \) in (14), the DF relay decodes the source signal and forward it to the destination node with the power \( P_{s}^{PS} \), which is available from the energy harvested during the first half of the block time \( T \). The received signal at the destination node, \( y_d^{PS}(k) \), in the PSR scheme is given by
\[ y_d^{PS}(k) = \frac{1}{\sqrt{d_2^m}} \sqrt{P_r^PS g \sigma(k) + n_{a,d}(k) + n_{c,d}(k)}, \]  
(15)
where the transmitted power from the relay node, \( P_{r}^{PS} \), can be given by
\[ P_{r}^{PS} = \frac{E_{h}^{PS}}{T/2} = \frac{\eta P_s |h|^2 \rho}{d_1^m}. \]  
(16)
Substituting the value of \( P_{r}^{PS} \) from (16) into (15), the received signal at the destination, \( y_d^{PS}(k) \) in terms of \( P_s, \eta, \alpha, d_1 \) and \( d_2 \), is given by
\[ y_d^{PS}(k) = \frac{\sqrt{\eta P_s |h|^2 g \sigma(k)}}{\sqrt{d_1^m d_2^m}} + n_d(k). \]  
(17)

C. Throughput Analysis

Given the received signal at the input of the information receiver at the relay node, \( \sqrt{1-\rho}y_r(k) \) in (14), the SNR at the relay node, \( \gamma_r^{PS} \), in the PSR scheme is given by
\[ \gamma_r^{PS} = \frac{P_{r}^{PS} |h|^2 (1-\rho)}{d_1^m \sigma_{a,r}^{PS}}, \]  
(18)
where \( \sigma_{a,r}^{PS} = \sqrt{(1-\rho)\sigma_{a,r}^2 + \sigma_{n,r}^2} \) is the variance of overall AWGN at the relay node, \( \alpha_r^{PS} = \sqrt{(1-\rho)\sigma_{n,a}(k) + \sigma_{n,r}(k)} \). Using (17), the SNR at the destination, \( \gamma_d^{PS} \), in the PSR scheme is given by
\[ \gamma_d^{PS} = \frac{P_{r}^{PS} |h|^2 |g|^2 \rho}{d_1^m d_2^m \sigma_{n,d}^{PS}}. \]  
(19)

In order to determine the throughput for the PSR scheme, we need to evaluate the ergodic capacity for source to relay link, \( C_r^{PS} \), and for relay to destination link, \( C_d^{PS} \). Using the received SNRs, \( \gamma_r^{PS} \) and \( \gamma_d^{PS} \), defined in (18) and (19), respectively, \( C_r^{PS} \) and \( C_d^{PS} \) are given by
\[ C_r^{PS} = E_h \{ \log_2 (1 + \gamma_r^{PS}) \}, \quad C_d^{PS} = E_{h,g} \{ \log_2 (1 + \gamma_d^{PS}) \}, \]  
(20)
where \( \gamma_r^{PS} \) depends on \( h \) and \( \gamma_d^{PS} \) depends on \( h \) and \( g \). The analytical expressions for the ergodic capacities, \( C_r^{PS} \) and \( C_d^{PS} \), are given in the following theorem.

**Theorem 2.** The ergodic capacities, \( C_r^{PS} \) and \( C_d^{PS} \) for the PSR scheme are given by
\[ C_r^{PS} = \frac{\lambda_{h} \xi P_s}{c \log(2)} \]  
(21a)
\[ C_d^{PS} = \frac{2d}{\lambda_{h} \log(4)} \]  
(21b)
where \( c \equiv d_1^m \sigma_{n,r}^2 P_r^{PS} (1-\rho) \), \( d \equiv d_1^m d_2^m \sigma_{n,d}^2 \gamma_r^{PS} \), and \( E_1(x) \) and meijerG functions are defined in Theorem 1.

**Proof.** Theorem 2 can be proved by following the same steps as given in Appendix A. The details are omitted here for the sake of brevity.

Using (21), the ergodic capacity of the PSR scheme, \( C_r^{PS} \), is given by
\[ C_r^{PS} = \min \{ C_r^{PS}, C_d^{PS} \} \]  
(22)
Given that \( T/2 \) is the effective communication time from the source node to the destination node in the block of time \( T \) seconds and if source transmits at a fixed rate equal to the ergodic capacity, i.e., \( C_r^{PS} \) bits/sec/Hz, the throughput, \( \tau^{PS} \) of the PSR scheme is given by
\[ \tau^{PS} = \frac{T/2}{C_r^{PS}} = \frac{1}{2} \min \{ C_r^{PS}, C_d^{PS} \}, \]  
(23)
where the throughput, \( \tau^{PS} \) in (23) depends on \( P_s, \eta, \rho, d_1, d_2, \sigma_{n,r}^2 \) and \( \sigma_{n,d}^2 \). It is desirable to find the optimal value of \( \rho \), that result in the maximum value of throughput, \( \tau^{PS} \). Since, the analytical expression of \( \tau^{PS} \) again involves complex meijerG function, which in turn depends on \( \rho \), it seems intractable to evaluate the closed-form expressions for the optimal value of \( \rho \) in terms of \( \tau \). However, the optimization can be done offline by numerically evaluating the optimal values of \( \alpha \) for the given system parameters, \( P_s, \eta, d_1, d_2, \sigma_{n,r}^2 \) and \( \sigma_{n,d}^2 \).
Fig. 4: Optimal throughput $\tau$ for the TSR and the PSR schemes for different values of antenna noise variance $\sigma_n^2$ and $\sigma_n^2 = 0.01$ (fixed)

V. NUMERICAL RESULTS

This section numerically analyzes the throughput performance of a DF relaying network in the presence of energy harvesting. We also use the derived analytical results to provide insights into the various design choices. The optimal value of throughput $\tau$, optimal value of energy harvesting time $\alpha$ in the TSR scheme, and optimal value of power splitting ratio $\rho$ in the PSR scheme are investigated for different values of the noise variances and the source to relay and the relay to destination distances, $d_1$ and $d_2$, respectively. Note that the optimal values of $\alpha$ and $\rho$ are defined as the values, which result in the maximum throughput, $\tau^{TS}$ and $\tau^{PS}$, respectively, at the destination node.

Unless otherwise stated, we set the energy harvesting efficiency, $\eta = 1$, source transmission power, $P_s = 1$ Joules/sec and path loss exponent $m = 2.7$. The distances $d_1$ and $d_2$ are normalized to unit value. For simplicity, similar noise variances at the relay and the destination nodes are assumed, i.e., antenna noise variance, $\sigma_n^2 = \sigma_n^2$ and conversion noise variance, $\sigma_k^2 = \sigma_k^2$. The mean values, $\lambda_h$ and $\lambda_y$, of the exponential random variables $|h|^2$ and $|g|^2$, respectively, are set to 1.

Fig. 3 plots the analytical and simulation based results of throughput, $\tau^{TS}$ and $\tau^{PS}$, with respect to $\alpha$ and $\rho$, for the TSR and PSR schemes, respectively. Note that the analytical throughput results depend on the analytical expressions of ergodic capacity in (9) and (21). However, the simulation results depend on the simulation based expressions for ergodic capacity in (8) and (20), which are evaluated by averaging these expressions over $10^5$ random realizations of the Rayleigh fading channels $h$ and $g$.

Fig. 3 shows that analytical results of throughput perfectly matches with the simulation results for the different values of $\alpha$ and $\rho$ for the TSR and PSR schemes, respectively. This verifies our analysis in Theorem 1 and Theorem 2. Fig. 3 also shows that for the chosen system parameters, the maximum throughput in the PSR scheme is greater than the maximum throughput achievable in the TSR scheme.

Fig. 5 plots the optimal throughput $\tau$ for the TSR and the PSR schemes for different values of conversion noise variance $\sigma_k^2$ and $\sigma_k^2 = 0.01$ (fixed)

VI. CONCLUSIONS

In this paper, a DF relaying network has been considered, where an energy constrained relay node harvests energy from the received RF signal and uses that harvested energy to forward the source signal to the destination node. To enable wireless energy harvesting and information processing at the relay, both TSR and PSR schemes have been considered. The exact achievable throughput at the destination is determined by deriving the ergodic
capacity of a DF relaying network in the presence of wireless energy harvesting constraint at the relay node. The optimal value of energy harvesting time in the TSR protocol and the optimal value of power splitting ratio in the PSR protocol are numerically investigated. The numerical analysis in this paper has provided practical insights into the effect of various system parameters on the performance of wireless energy harvesting and information processing using DF relay nodes.

**APPENDIX A**

**PROOF OF THEOREM 1 IN (9)**

Let us first find the analytical expression for $C_r^{TS}$. Using (8), $C_r^{TS}$ is given by

$$C_r^{TS} = \frac{A}{\gamma_0} \int_0^\infty f_{\lambda_0}^{TS} (\gamma) \log_2(1 + \gamma) d\gamma \quad (A.1)$$

where $f_{\lambda_0}^{TS} (\gamma)$ is the probability density function (PDF) of $\lambda_0^{TS}$ in (6). Since $|h|^2$ is exponentially distributed, $\gamma_r^{TS}$ is also exponentially distributed, i.e., $f_{\gamma_r}^{TS} (\gamma) = \frac{1}{b} e^{-\frac{\gamma}{b}}$, where $b \triangleq \frac{d_l^{TS} d_r^{TS}}{\gamma_0}$. Substituting the value of $f_{\gamma_r}^{TS} (\gamma)$ into (A.1), $C_r^{TS}$ is given by

$$C_r^{TS} = \frac{A}{\gamma_0} \int_0^\infty \frac{a}{\lambda_h} e^{-\frac{\lambda_0^2}{\lambda_h}} \log_2(1 + \gamma) d\gamma = \frac{\lambda_0 e^{-\frac{\lambda_0^2}{\lambda_h}} E_1 \left( \frac{\lambda_0^2}{\lambda_h} \right)}{a \log(2)}, \quad (A.2)$$

where $E_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt$ is the exponential integral. Next, using (8), $C_d^{TS}$ is given by

$$C_d^{TS} = \frac{C_d}{\gamma_0} \int_0^\infty f_{\lambda_0}^{TS} (\gamma) \log_2(1 + \gamma) d\gamma \quad (A.3)$$

where $f_{\lambda_0}^{TS} (\gamma)$ is the PDF of $\gamma_0^{TS}$ in (7). In order to find the PDF of $\gamma_0^{TS}$, let us first find the cumulative distribution function (CDF), $F_{\lambda_0}^{TS} (\gamma)$ of $\gamma_0^{TS}$, which is given by

$$F_{\lambda_0}^{TS} (\gamma) = p \left( \frac{\gamma_0^{TS}}{b} < \gamma \right) = p \left( |g|^2 < \frac{b^2}{|h|^2} \right) \quad (A.4a)$$

$$= \int_0^{\infty} f_{|h|^2} (x) \left( 1 - e^{-\frac{\lambda_0^2}{\lambda_h}} \right) dx \quad (A.4b)$$

$$= 1 - \frac{4b^2}{\lambda_0^2 \lambda_h} K_1 \left( \frac{4b^2}{\lambda_0^2 \lambda_h} \right) \quad (A.4d)$$

where $b \triangleq \frac{d_l^{TS} d_r^{TS}}{\gamma_0}$ is the PDF of exponential random variable $|h|^2$. (A.4c) follows from (A.4b) because $|g|^2$ is exponential random variable with mean $\lambda_0$, $K_1(x)$ is the first-order modified Bessel function of the second kind [18], and (A.4d) follows from (A.4c) by using the formula

$$\int_0^\infty e^{-\frac{x}{\lambda_0^2 \lambda_h}} dx = \frac{\sqrt{\pi}}{2\sqrt{\lambda_h}} K_1 \left( \sqrt{\lambda_h} \right) \quad [18, \S 3.324.1].$$

By taking the derivative of the CDF function, $F_{\lambda_0}^{TS} (\gamma)$, $f_{\lambda_0}^{TS} (\gamma)$ is given by

$$f_{\lambda_0}^{TS} (\gamma) = \frac{\partial F_{\lambda_0}^{TS} (\gamma)}{\partial \gamma} = \frac{2b}{\lambda_0^2 \lambda_h} K_0 \left( \frac{4b^2}{\lambda_0^2 \lambda_h} \right) \quad (A.5)$$

where we used the property of Bessel function, $\frac{d}{d\gamma} (z^{\mu} K_\mu (z)) = -z^{\mu} K_{\mu + 1} (z)$ [18, §8.46.18]. Substituting the value of $f_{\lambda_0}^{TS} (\gamma)$ from (A.5) into (A.3), $C_d^{TS}$ is given by

$$C_d^{TS} = \int_0^\infty \frac{2b}{\lambda_0^2 \lambda_h} K_0 \left( \frac{4b^2}{\lambda_0^2 \lambda_h} \right) \log_2(1 + \gamma) d\gamma = \frac{2b}{\lambda_0^2 \lambda_h} \log(4)^{\gamma_0} \left( \sum_{l=0}^{\infty} \left( \frac{1}{\lambda_0^2 \lambda_h} \right)^l \right) \quad (A.6)$$

where the meijer $G$ function $G^{m,n}_{p,q} \left( \begin{array}{c} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{array} | z \right)$ is defined in (10). This ends the proof of Theorem 1.

**REFERENCES**


