PERFORMANCE STUDY OF COMPRESSIVE SAMPLING FOR ECG SIGNAL COMPRESSION IN NOISY AND VARYING SPARSITY ACQUISITION

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ABSTRACT

In this paper, we investigate the performance of compressive sampling (CS) for ECG compression in telecardiology, when the signal acquisition is noisy and unavoidable body movements lead to varying heartbeat rate and sparsity of the signal. We show analytically that CS recovery noise does not scale linearly with the input noise. Hence, it is not easy to reduce the adverse impact of noise in CS. Additionally, any variation in the heartbeat rate changes the sparsity and can adversely affect compression. We compare the performance of CS with thresholding discrete wavelet transform (TH-DWT), which is the best technique for real-time ECG compression. We show that CS is quite sensitive to sparsity and compression ratio, while the reconstruction quality of TH-DWT is quite stable. Our results suggest that while CS is an attractive option for telecardiology due to its encoder simplicity, caution should be exercised in applying it for ECG signal compression.

***Index Terms*—** Electrocardiogram (ECG), compression, compressive sampling, discrete wavelet transform.

1. INTRODUCTION

ECG telemonitoring (i.e., telecardiology) via wireless body area networks is an attractive solution to the important problem of long-term ambulatory monitoring of chronic cardiovascular disease patients [1]. Traditionally, chronic cardiovascular disease patients are required to wear an ECG data logging device for 2 days (Holter monitoring) or up to a week (event monitoring). These devices are bulky and obtrusive and can limit patient autonomy and mobility. With the advent of wireless body area networks [1], it becomes possible to transmit the ECG signal to a patient's smart phone using low-profile wireless sensors [2] and to send this information to the cardiologist over the wireless/cellular network. In order to limit the cost and to maximize the life of the sensors, it is desirable to limit the amount of ECG data to be sent over the bandwidth constrained wireless/cellular networks.

Compressive sampling (CS) [3,4] has recently emerged as an attractive solution for compressing signals, and it has been applied to the real-time energy-efficient compression for ECG telemonitoring systems [5–8]. Current state-of-the-art real-time ECG compression techniques are based on the digital wavelet transform (DWT) and achieve higher compression at the cost of increased complexity [9]. In particular, thresholding discrete wavelet transform (TH-DWT) is shown to outperform other techniques for real-time ECG compression [10]. By contrast, compressed sensing is a low complexity technique which exploits the sparsity in a signal to compress it. Since an ECG signal is sparse in the wavelet domain, it is suitable for compression using CS. Using dynamic thresholding for sparsity control, it has been shown that CS of ECG in time domain can achieve $16\times$ compression [5]. Further CS based ECG compression can extend sensor life by 37% compared to wavelet based compression [6]. A hardware test bed that can achieve $10\times$ or more CS based ECG compression, without the need for any general purpose memory or processing at the sensor nodes, is reported in [7]. Compressed sensing for fetal ECG telemonitoring is investigated in [8].

Relationship to prior work: Most of the above studies assume no noise [6–8] or extremely good signal-to-noise ratio (SNR), e.g., 80 dB SNR [5]. They also assume a fixed sparsity level of the signal. However, these assumptions cannot be guaranteed in mobile or future personal/wearable telecardiology where perturbations of the ECG signal are unavoidable due to body movements and the exact sparsity level of the signal may be time-variant. These important issues are addressed in this paper.

In this paper, we evaluate the performance of ECG compression using CS from the perspective of telecardiology, where the ECG signal acquisition is noisy and the heartbeat rate may vary due to body movements. This is crucial because noise will adversely affect the signal reconstruction and the heartbeat rate will influence the sparsity of the ECG signal. We compare the compression performance of CS and TH-DWT methods for ECG signals of different sparsity and noise levels, subject to the percentage root-mean-square difference (PRD) constraint which links the signal reconstruction quality to diagnostic distortion [11]. The major contributions of this paper are as follows:

- We consider the effect of imperfect ECG signal acquisition due to noise and perturbations that may occur with body movements. We show analytically that CS recovery noise does not scale linearly with the input noise. Hence, it is not easy to reduce the adverse impact of noise in CS.
- We show that subject to PRD $< 9\%$ constraint, which is regarded as good or very good diagnostic quality ECG signal, TH-DWT achieves twice the compression as CS. In addition, while the reconstruction quality of TH-DWT is quite stable, for CS it is quite sensitive to noise, sparsity level and compression ratio.
- Our results suggest that while CS is an attractive option for telecardiology due to its encoder simplicity, caution should be exercised in applying it for ECG compression where the acquired ECG may be affected by noise and the sparsity level of the signal may be time-variant.

The remainder of this paper is organized as follows. Section 2 briefly reviews the ECG signal compression using TH-DWT and CS techniques. Section 3 discusses the effect of noise and sparsity of the ECG signal on CS signal recovery. Section 4 presents the simulation results comparing the performance of TH-DWT and CS for ECG compression. Finally, conclusions are presented in Section 5.

2. ECG SIGNAL COMPRESSION

2.1. Thresholding Discrete Wavelet Transform (TH-DWT)

Using wavelet basis functions, an ECG signal can be expressed as [9, 10]

$$\boldsymbol{y} = \boldsymbol{\Psi}\boldsymbol{x}, \tag{1}$$

where $\boldsymbol{\Psi} \in \mathbb{R}^{N\times N}$ is wavelet basis matrix defined as $\boldsymbol{\Psi} = [\boldsymbol{\psi}_1|\boldsymbol{\psi}_2|\cdots|\boldsymbol{\psi}_N]$, and $\boldsymbol{x} \in \mathbb{R}^N$ represents the coefficient vector. We are only required to find $\boldsymbol{x}$ to recover the ECG signal as

$$\tilde{\boldsymbol{y}} = \boldsymbol{\Psi}\tilde{\boldsymbol{x}}, \tag{2}$$

where $\tilde{\boldsymbol{x}}$ is the estimation of $\boldsymbol{x}$. The wavelet coefficients of the ECG signal can be estimated as

$$\tilde{\boldsymbol{x}} = \boldsymbol{\Psi}^\dagger \boldsymbol{y}, \tag{3}$$

where $\boldsymbol{\Psi}^\dagger$ is the Hermitian matrix of $\boldsymbol{\Psi}$. Since an ECG signal is sparse in wavelet basis domain, i.e., most coefficients are close to zero, we can compress the ECG signal by choosing the coefficients above some threshold with discrete wavelet transform. This is the basic principle of the TH-DWT technique [12] which is used as benchmark in ECG signal compression [10].

2.2. Compressive Sampling

According to the theory of CS [3, 4], the number of measurements can be much smaller than that of bases to recover the coefficient vector of a sparse signal. Thus, we can apply CS in finding the sparse ECG coefficients of wavelet bases. By applying the CS sensing matrix $\boldsymbol{\Phi} \in \mathbb{R}^{M\times N}$ to the ECG signal $\boldsymbol{y} = \boldsymbol{\Psi}\boldsymbol{x}$, where $M \ll N$, we get the CS ECG signal $\boldsymbol{b} \in \mathbb{R}^M$ as

$$\boldsymbol{b} = \boldsymbol{A}\boldsymbol{x}, \tag{4}$$

where $\boldsymbol{A} \in \mathbb{R}^{M\times N}$ is the CS measurement matrix defined as $\boldsymbol{A} = \boldsymbol{\Phi}\boldsymbol{\Psi}$. If the vector $\boldsymbol{x}$ is K-sparse, i.e.,

$$K = \big|\{j : x_j \neq 0\}\big|, \tag{5}$$

and any $2K$ columns of $\boldsymbol{A}$ are nearly orthogonal, then the vector $\boldsymbol{x}$ can be estimated as

$$\tilde{\boldsymbol{x}} = \arg\min_{\hat{\boldsymbol{x}}} \|\hat{\boldsymbol{x}}\|_1, \quad \text{s.t. } \boldsymbol{A}\hat{\boldsymbol{x}} = \boldsymbol{b}, \tag{6}$$

where $\|\hat{\boldsymbol{x}}\|_1 := \sum_{i=1}^N |\hat{x}_i|$. Candès and Tao [13] explain the possibility of this recovery with the concept of restricted isometry property (RIP). In particular, if there exists δ_{2K} called restricted isometry constant (RIC) satisfying

$$(1-\delta_{2K}) \leq \frac{\|\boldsymbol{A}\boldsymbol{x}_{2K}\|_2^2}{\|\boldsymbol{x}_{2K}\|_2^2} \leq (1+\delta_{2K}), \tag{7}$$

where $\|\hat{\boldsymbol{x}}\|_2 := \sqrt{\sum_{i=1}^N |\hat{x}_i|^2}$, and it is quite smaller than one for any $2K$-sparse vector $\boldsymbol{x}_{2K}$, recovery is possible with high probability even when M is much smaller than N. Since we only use M measurements in reconstructing the N-dimensional ECG signal, it is possible to compress the ECG signal with CS.

Remark 1. In terms of complexity, the CS encoder is much simpler than TH-DWT encoder since the size of CS observation matrix $\boldsymbol{\Phi}$ is much smaller than that of the Hermitian matrix of wavelet bases $\boldsymbol{\Psi}^\dagger$. Another reason why CS is particularly attractive for ECG telemonitoring is that the computational burden of the technique lies in the decoding (ℓ_1-minimization), which can be done accurately at the remote hospital side.

3. EFFECT OF NOISE ON CS ECG SIGNAL

If the ECG signal is perturbed by body movements and any noise while it is being measured, then the perturbed signal can be expressed in wavelet domain as

$$\boldsymbol{y}^p = \boldsymbol{\Psi}(\boldsymbol{x}+\boldsymbol{n}) + \boldsymbol{e} = \boldsymbol{y} + \boldsymbol{y}_n, \tag{8}$$

where $\boldsymbol{n}$ is the noise on the coefficient $\boldsymbol{x}$, $\boldsymbol{e}$ is the noise on the measurement, and $\boldsymbol{y}_n$ is the total noise on $\boldsymbol{y}$ expressed as

$$\boldsymbol{y}_n = \boldsymbol{\Psi}\boldsymbol{n} + \boldsymbol{e}. \tag{9}$$

In order to characterize the effect of perturbations analytically, we assume that $\boldsymbol{n}$ and $\boldsymbol{e}$ are independent additive white Gaussian noise (AWGN) with covariances $\sigma_n^2\boldsymbol{I}$ and $\sigma_e^2\boldsymbol{I}$, respectively. Note that this is a simple way to model the perturbations arising from body movement, and further investigations are required to model the perturbations more accurately. Given this assumption, we can characterize the effect of noise for both TH-DWT and CS methods.

For TH-DWT, since the wavelet basis matrix $\boldsymbol{\Psi}$ is orthonormal, the variance of $\boldsymbol{y}_n$ will be $\sigma_y^2 = \sigma_n^2 + \sigma_e^2$. Since the TH-DWT method sets the negligible coefficients to zero, we can see that this helps to reduce the adverse effects of signal perturbation. We can, therefore, expect ECG compression using TH-DWT to be robust to noise [12].

For CS, the situation is more complicated. The CS ECG signal which is contaminated with perturbation can be expressed as

$$\boldsymbol{b}^p = \boldsymbol{b} + \boldsymbol{b}_n, \tag{10}$$

where $\boldsymbol{b}_n$ is the CS noise observed as $\boldsymbol{b}_n = \boldsymbol{\Phi}\boldsymbol{y}_n$. The variance of the CS noise $\boldsymbol{b}_n$, if $\boldsymbol{A}$ satisfies the RIP, can be approximated through noise folding effect of CS [14] as

$$\sigma_b^2 \approx \sigma_n^2 \frac{N}{M} + c_0\sigma_e^2, \tag{11}$$

where c_0 is a constant determined by CS sensing matrix $\boldsymbol{\Phi}$. From (11), we can see that the noise on the wavelet coefficient $\boldsymbol{x}$ is amplified by the factor of N/M. This could adversely affect the signal recovery.

The vector $\boldsymbol{x}$ can be found as

$$\tilde{\boldsymbol{x}} = \arg\min_{\hat{\boldsymbol{x}}} \|\hat{\boldsymbol{x}}\|_1, \quad \text{s.t. } \|\boldsymbol{A}\hat{\boldsymbol{x}} - \boldsymbol{b}^p\|_2 \leq \varepsilon, \tag{12}$$

where ε is an upper bound on $\|\boldsymbol{b}_n\|_2$. Using Markov inequality, the tail probability of the Gaussian noise $\boldsymbol{b}_n$ can be calculated as

$$\mathbb{P}\big((1+\epsilon)\sqrt{M}\sigma_b \leq \|\boldsymbol{b}_n\|_2\big) \leq \exp(-c_1\epsilon^2 M), \tag{13}$$

where $\mathbb{P}(E)$ denotes the probability that the event E occurs, $\epsilon > 0$ is a constant and c_1 is a constant (Ch. 1 of [15]). If we set ϵ to one, $\boldsymbol{b}_n$ can be bounded as

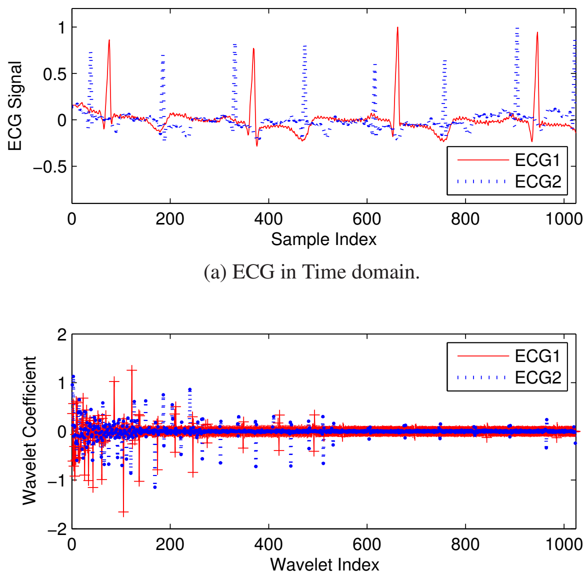
$$\|\boldsymbol{b}_n\|_2 < 2\sqrt{N\sigma_n^2 + c_0 M\sigma_e^2} = \varepsilon \tag{14}$$

with high probability. If $\delta_{2K} < \sqrt{2}-1$, the recovery noise is known to obey

$$\|\boldsymbol{x} - \tilde{\boldsymbol{x}}\|_2 \leq C\varepsilon, \tag{15}$$

where C is a constant which increases as the number of measurement M decreases or the number of nonzero coefficients K increases. Then, we can reconstruct the ECG signal as

$$\tilde{\boldsymbol{y}^p} = \boldsymbol{\Psi}\tilde{\boldsymbol{x}}. \tag{16}$$



(a) ECG in Time domain.

(b) ECG in Wavelet domain.

Fig. 1: ECG signals having different heartbeat rate and sparsity. The beat rate of ECG2 is higher than that of ECG1, and ECG1 is more sparse in wavelet domain.

Remark 2. The above analysis shows that the noise of CS recovery is related to the error bound constant C, the number of wavelet bases N, and the number of measurements M. If M,N and the noise are fixed, the CS recovery is affected by the sparsity K of the ECG signal. This shows that the recovery noise of CS is more complicated than that of DWT, and it is not easy to reduce the noise since the CS recovery noise does not scale linearly with the input noise.

4. SIMULATION RESULTS

In this section, we compare the performance of ECG compression using TH-DWT and CS methods, respectively, in various scenarios. We use the ECG signal record number 100 from the MIT-BIH database [16] as the test signal. For TH-DWT, the ECG signal is captured with the length of $L = 512$ samples, and we use 512 bases of D4 Daubechies wavelet in representing ECG signal length of 512 samples. The size of the wavelet basis matrix $\mathbb{R}^{N\times N}$ is $N = 512$. For CS, we use random Bernoulli matrix whose entries are $\pm 1/\sqrt{N}$ as CS sensing matrix $\mathbf{\Phi} \in \mathbb{R}^{M\times N}$, where M is the number of CS measurements varying according to compression ratio. For a fair comparison between TH-DWT and CS methods, we compare the ECG recovery with the same amount of transmitted signals, e.g., if the compression ratio is 50 %, then the highest 50 % wavelet coefficients are transmitted to the receiver with TH-DWT method, while CS measurements of $M = 0.5 \times N$ are transmitted with CS method.

4.1. Recovery of ECG signals without noise

First we examine the recovery performance of ECG signals with different heartbeat rate and no noise (as a perturbation). The heartbeat rate can change with physical activity and also in case of certain medical conditions such as arrhythmia (irregular heartbeat). Thus any change in the heartbeat rate will affect the signal sparsity. For the simple emulation of different sparsity of an ECG signal, we just change the speed rate with the same signal. Fig. 1 shows the plot of first 1000 samples of ECG signal record number 100 from the MIT-BIH database (labelled ECG1). In Fig. 1(a), ECG2 is the twice speeded up version of ECG1. We can see from Fig. 1(b) that since the beat rate of ECG2 is higher than ECG 1, ECG2 has lower sparsity of the wavelet coefficients than ECG1.

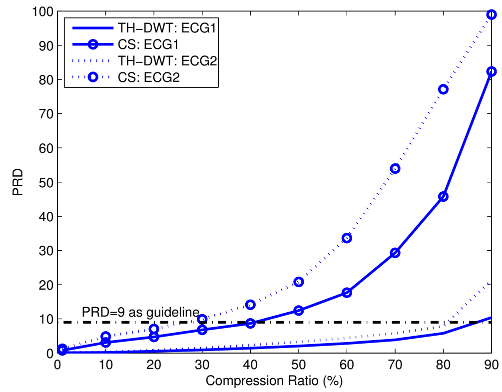


Fig. 2: ECG recovery performance versus compression ratio.

We compare the recovery performance subject to the percentage root-mean-square difference (PRD), which is defined as

$$\text{PRD} = \frac{\|\boldsymbol{y} - \tilde{\boldsymbol{y}}\|_2}{\|\boldsymbol{y}\|_2} \times 100\%, \tag{17}$$

where $\boldsymbol{y}$ is the received signal $\tilde{\boldsymbol{y}}$ is the reconstructed signal. Note that PRD < 9 is regarded as good or very good quality of the reconstructed ECG signal in terms of diagnostic distortion [11].

Fig. 2 shows the PRD of the recovered signal versus compression ratio, averaged over 20 trials, with both TH-DWT and CS methods for ECG1 and ECG2. We can see that the recovery quality of TH-DWT method is quite stable regardless of the sparsity and the compression ratio. For PRD$=9$, TH-DWT method achieves a maximum compression of around 80% for both ECG1 and ECG2. However, the recovery quality of CS method varies greatly according to the compression ratio and the signal sparsity. For example, for PRD$=9$, a compression ratio of 40 % is achieved for ECG1 signal and a compression ratio of 30 % is achieved for ECG2 signal. This result shows that TH-DWT method is better than CS method in terms of PRD and CS cannot guarantee the same recovery quality with different sparsity.

4.2. Recovery of ECG signals in noisy environments

For the emulation of mobile cardiology, where different kinds of perturbations are expected, we intentionally add some noise $\boldsymbol{e}$ to the ECG signal before compression. For simplicity, we do not add the noise $\boldsymbol{n}$ on the wavelet coefficients. To measure the adverse effect of perturbation, we observe the signal quality of the reconstructed signal with signal-to-noise ratio (SNR) defined as

$$\text{SNR} = 20\log_{10}\frac{\|\boldsymbol{y}\|_2}{\|\boldsymbol{y} - \tilde{\boldsymbol{y}}^p\|_2}. \tag{18}$$

Fig. 3 and Fig. 4 show the SNR of the recovered ECG signal with TH-DWT and CS methods. Fig. 3 shows that the reconstructed SNR is relatively flat for TH-DWT method up to about 60 % compression ratio for all input SNRs. Additionally, the reconstructed SNR is getting improved slightly due to the noise suppression ability of

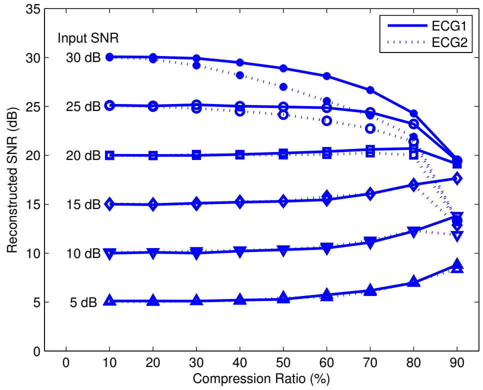


Fig. 3: TH-DWT Compression Under Noise : ECG recovery performance versus compression ratio.

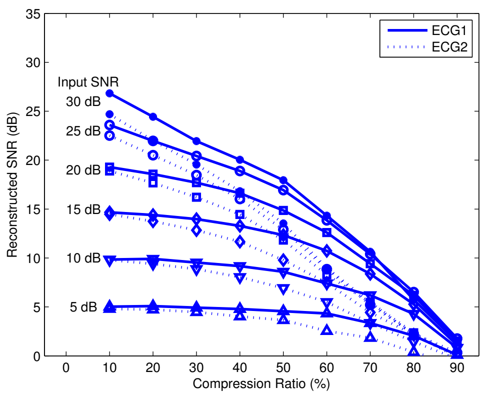


Fig. 4: CS Compression Under Noise : ECG recovery performance versus compression ratio.

TH-DWT if the input SNR is under 20 dB. However, for CS, the reconstructed SNR falls sharply with compression ratio. This loss in reconstructed SNR is more serious with higher SNR of input signal and the less sparse signal. The SNR loss of CS against TH-DWT is less than 3 dB under the compression ratio of 40 % if input SNR is below 20 dB. These results show TH-DWT method should be used if higher SNR of the reconstructed signal is required. However, we can use CS method if higher SNR is not required for general ECG monitoring.

5. CONCLUSION

In this paper, we have assessed the impact of noisy ECG signal acquisition and varying heartbeat rate on the ECG signal compression with CS. We have showed that the effect of noise on CS is not easy to reduce since the CS recovery noise is not linear to the input noise. We have compared the performance of CS to that of TH-DWT for real-time ECG compression. We have showed that TH-DWT outperforms CS in terms of compression ratio and CS is very sensitive to noise and sparsity level of the signal. Our analysis and simulation results suggest that while CS is still an attractive solution for telecardiology considering its encoder simplicity, caution should be exercised in applying it for ECG compression in the presence of noise and time-variant sparsity level of the signal.

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