Stability and throughput of FAST transfer control protocol traffic in bi-directional connections

F. Ge\textsuperscript{1,2} L. Tan\textsuperscript{1,3} R.A. Kennedy\textsuperscript{3}

\textsuperscript{1}Department of Computer Science, Central China Normal University, Wuhan 430079, People's Republic of China
\textsuperscript{2}Department of Electronic Engineering, City University of Hong Kong, Hong Kong
\textsuperscript{3}Research School of Information Sciences and Engineering, The Australian National University, Canberra ACT 0200, Australia
E-mail: l.tan@mail.ccnu.edu.cn

Abstract: FAST TCP is a promising new transfer control protocol developed for high-speed long-latency networks, whose performance has previously only been studied for data traffic sent in one direction. In this study, the authors propose a mathematical model for bi-directional connections using the FAST TCP protocol, which captures the asymmetric bandwidth dynamics in duplex dumbbell networks, prevalent in ADSL, satellite and other high-speed technologies. Using this model, the authors obtain a powerful result that the queue delays observed by opposite FAST TCP flows only have a time difference in a dumbbell network. Furthermore, the authors establish the conditions under which the bi-directional FAST TCP flows achieve stability, and on this basis the throughput rates of the forward and backward flows at steady-state are deduced. The authors find that, in the case of bandwidth asymmetry and one flow in each direction, in equilibrium the throughput of the bi-directional FAST TCP flows can only achieve the smaller link capacity of the duplex links, and the link with larger capacity is not fully occupied. These theoretical findings are corroborated by NS2 simulations.

1 Introduction

FAST TCP is a new transfer control protocol, designed to improve performance for high Internet bandwidth-delay product (BDP) links [1]. FAST TCP uses queuing delay as a main measure, rather than packet loss, when detecting congestion. To guarantee satisfactory performance, a congestion control system with active queue management (AQM) should be stable and the dynamics brought to equilibrium. Driven by this need, recent theoretical and experimental studies have investigated the stability of FAST TCP [2–7]. These researches pay attention to FAST [2–7] TCP stability only in one-way traffic, whereas the stability issue of FAST TCP in a two-way traffic scenario has not been studied yet. In recent years, two-way traditional TCP traffic issues have been widely studied in [8, 9–12]. Although the performance of two-way traditional TCP and the stability effects of two-way traffic in a TCP/AQM system are investigated, the conclusions may be not directly related to delay-based FAST TCP. Therefore the performance of two-way FAST TCP flows should be investigated for the sake of the future use of FAST TCP.

Two-way traffic or bi-directional traffic is the traffic pattern resulting from two or more TCP connections transferring data segments simultaneously in opposite directions between the same pair of end nodes in the IP layer over a network path [8]. Asymmetry and ACK compression are the principal characteristics of such transmission, which may arise in the network [9, 13]. Another significant characteristic of such transmission is the competition among data packets and ACK packets in both connections, as the data packets in one direction share the same path bandwidth with the ACK packets of the opposite direction. The queue delay of flows in one direction can be disturbed, prolonged or even shortened, by extra delay of flows in the opposite direction, but the effect of this on the ACK streams is often ignored in simulations. The alterability of queue delay resulting from the opposite...
flows will accordingly influence the stability and the performance. The performance of this situation is not the same as that of [6], which considers only flows in one direction. Hence, it is necessary to investigate the dynamics of FAST TCP in a two-way traffic scenario as the transmission of ACK packets in both directions are influenced by the data packets in the corresponding opposite direction. This paper proposes a mathematical model, in which the congestion window is formulated for FAST TCP flows transferring packets in the two opposite directions in a full duplex (FDX) link. Based on this model, the stability of the FAST TCP flows is analysed in dumbbell networks and the throughput of the bi-directional FAST TCP flows is evaluated.

2 A bi-directional FAST TCP flow model

We consider a scenario as shown in Fig. 1, in which an FDX link carries several flows in opposite directions. The capacity of one direction of the link \((c_{12})\) may have the same capacity with or be different from that of the opposite direction of the link \((c_{21})\). The \(m\) FlowPs (FlowP 1, \ldots, FlowP \(m\)) send data packets from \(m\) SourcePs to \(m\) DestinationPs, whereas the \(n\) FlowNs (FlowN 1, \ldots, FlowN \(n\)) send data packets from \(n\) SourceNs to \(n\) DestinationNs, where \(m \geq 1\) and \(n \geq 1\). FlowP \(i\) represents one FAST TCP flow transmitting data packets through Router 1 firstly, and FlowP \(j\) represents one FAST TCP flow transmitting data packets through Router 2 firstly. The conditions \(m \geq 1\) and \(n \geq 1\) make this scenario different from that of [6] where one of \(m\) or \(n\) is 0. This difference gives rise to different stability conditions and throughput.

The FDX link cannot be simply considered as two FDX links for FAST TCP flows in two different directions, in which we can analyse the flows’ performance according to [2–7]. Two FDX links mean that each FDX link has an independent sending buffer to forward data packets and an independent receiving buffer to receive ACK packets. In the current scenario, the data packets and the ACK packets share the common buffer in each FDX link. That is to say, the ACK packets of all FlowNs and the data packets of all FlowPs are put in a single queue at Router 1 and vice versa in the opposite direction. The two types of packets cowork or interact to change the queue delay of the FAST TCP flows in the two directions of each FDX link, while the phenomena does not appear in two FDX links with independent buffers.

The constraint condition of the utility maximisation problem defined in [14] has its characteristic in this situation. Let \(x_{P,j}\) denote the throughput of FlowP \(i\) and \(x_{N,j}\) denote the throughput of FlowN \(j\). Let the vector \(R_p = (1 \cdots 1)^T\) represents that the FlowPs are connected to the link from Router 1 to Router 2. Let \(x\) denote the vector \((x_{P,1}, \ldots, x_{P,n}, x_{N,1}, \ldots, x_{N,n})^T\). Then we can obtain the constraint \(R_p x \leq c_{12}\). In the same way we obtain \(R_n x_N \leq c_{21}\). Hence, the two constraints can be rewritten as

\[
\begin{pmatrix}
R_p & 0 \\
0 & R_N
\end{pmatrix}
\begin{pmatrix}
x_P \\
x_N
\end{pmatrix} \leq \begin{pmatrix} c_{12} \\ c_{21} \end{pmatrix}
\]

(1)

The constraint has the same form as defined in [14], but the link utilisation cannot easily achieve the approximate link capacity. A continuous model will be built in the following.

A FAST TCP source periodically updates its congestion window \(w(t)\) based on its average round-trip time (RTT) and its estimated queueing delay [1]. The window updating algorithms for FAST TCP FlowP \(i\) and FlowN \(j\) congestion windows are

\[
\dot{w}_{P,j}(t) = \gamma \left( \frac{d w_{P,j}(t)}{d + q_{P,j}(t)} - \alpha - \dot{w}_{P,j}(t) \right)
\]

(2)

\[
\dot{w}_{N,j}(t) = \gamma \left( \frac{d w_{N,j}(t)}{d + q_{N,j}(t)} - \alpha - \dot{w}_{N,j}(t) \right)
\]

(3)

where \(d\) is the round-trip propagation delay, \(\gamma \in (0, 1]\), and \(\alpha\) is a constant. The sending rates of flows, in terms of the queueing delays at time \(t\), are defined as [4]

\[
x_{P,j}(t) = \frac{w_{P,j}(t)}{d + q_{P,j}(t)}, \quad x_{N,j}(t) = \frac{w_{N,j}(t)}{d + q_{N,j}(t)}
\]

(4)

The aggregate rate after time \(T_P\), which denotes the duration between the time that the packet from SourceP \(i\) is put in the queue and the time that it arrives the destination, in the FlowP \(i\) forward direction is

\[
y_P(t + T_P) = \sum_{t} x_{P,j}(t)
\]

(5)

Note that at time \(t + T_P\), the sending rate of FlowN \(j\) is \(x_{N,j}(t + T_P)\). The aggregate rate after time \(T_P\) in the
FlowP of the backward direction is

\[ y_p^b(t + \tau_p^b + \tau_p^f) = \sum_j x_{N,j}(t + \tau_p^f) \]  

(6)

Let \( p_p^b(t) \) be the queueing delay in the forward link, \( p_p^f(t) \) be the queueing delay in the backward link of FlowP. The queueing delay has been traditionally modelled [11] by

\[ \dot{p}_p^b(t) = \frac{1}{\zeta_{12}}(y_p^b(t) - \zeta_{12}), \quad \dot{p}_p^f(t) = \frac{1}{\zeta_{21}}(y_p^b(t) - \zeta_{21}) \]  

(7)

The end-to-end queue \( q_p(t + \tau_p^b + \tau_p^f) \), observed by Source 1, consists of the backward and forward queueing delay

\[ q_p(t + \tau_p^b + \tau_p^f) = p_p^b(t + \tau_p^b) + p_p^f(t + \tau_p^f + \tau_p^b) \]  

(8)

Note that the forward queueing delay \( p_p^f(t) \) is observed after the duration \( \tau_p^b \). Therefore we obtain the queueing delay observed by FlowP

\[ \dot{q}_p(t + \tau_p^b + \tau_p^f) = \frac{1}{\zeta_{12}} \left( \sum_i \frac{w_{p,i}(t)}{d + q_p^f(t)} - \zeta_{12} \right) + \frac{1}{\zeta_{21}} \left( \sum_j \frac{w_{N,j}(t + \tau_p^f)}{d + q_{N,j}(t + \tau_p^f)} - \zeta_{21} \right) \]  

(9)

Let \( \tau_p = \tau_p^b + \tau_p^f \) and replace \( t + \tau_p \) with \( t \) in (9), thus

\[ \dot{q}_p(t) = \frac{1}{\zeta_{12}} \left( \sum_i \frac{w_{p,i}(t)}{d + q_p^f(t - \tau_p)} - \zeta_{12} \right) + \frac{1}{\zeta_{21}} \left( \sum_j \frac{w_{N,j}(t - \tau_p^f)}{d + q_{N,j}(t - \tau_p^f)} - \zeta_{21} \right) \]  

(10)

Similarly, we obtain the queueing delay observed by FlowN

\[ \dot{q}_{N,j}(t) = \frac{1}{\zeta_{21}} \left( \sum_j \frac{w_{N,j}(t - \tau_{N,j}^f)}{d + q_{N,j}(t - \tau_{N,j}^f)} - \zeta_{21} \right) + \frac{1}{\zeta_{12}} \left( \sum_i \frac{w_{p,i}(t - \tau_{N,j}^b)}{d + q_p^f(t - \tau_{N,j}^b)} - \zeta_{12} \right) \]  

(11)

Then (2), (3), (10) and (11) can model dynamic of bi-directional FAST TCP flows.

### 3 Stability of bi-directional FAST TCP flows

Linearising (2) and (11) at the equilibrium point \( (q_p^*, w_p^*) \) and omitting the subscript \( i \) in \( q_p(t) \), we obtain the updating algorithms for the window size and end-to-end

queueing delay for FlowP

\[ \Delta \dot{w}_{p,j}(t) = -\gamma \left( \frac{q_{p,j}}{d + q_{p,j}^*} \Delta w_{p,j}(t) + \frac{w_{p,j}^*}{d + q_{p,j}^*} \Delta q_p(t) \right) \]  

(12)

\[ \Delta \dot{q}_p(t) = \frac{1}{\zeta_{12}} \left( \frac{1}{d + q_p} \Delta w_{p,j}(t - \tau_p) - \frac{\Delta q_p(t - \tau_p)}{(d + q_p^*)} \right) + \frac{n}{\zeta_{21}} \left( \frac{1}{d + q_{N,j}} \Delta w_{N,j}(t - \tau_p) - \frac{\Delta q_N(t - \tau_p)}{(d + q_{N,j}^*)} \right) \]  

(13)

Formula (13) indicates that the dynamic behaviour of FlowP is queueing delay is related to not only the congestion window and the queueing delay of FlowP but also those of FlowN.

From (8), we obtain the queueing delay of FlowP

\[ \Delta q_p(t) = \Delta p_p^f(t - \tau_p) + \Delta p_p^f(t) \]  

(14)

and similarly for FlowN

\[ \Delta q_N(t) = \Delta p_N^f(t) + \Delta p_N^f(t) \]  

(15)

In the dumbbell network, we have \( \Delta p_p^f(t) = \Delta p_p^b(t - \tau_p) \), \( \Delta p_N^f(t) = \Delta p_N^b(t - \tau_p) \), \( \tau_p = \tau_p^b = \tau_p^f \). Substituting the above into (15), we finally have the relation of the queue delays

\[ \Delta q_N(t) = \Delta p_p(t - \tau_p) \]  

(16)

The above relationship reveals that the queueing delays, in a dumbbell network as seen by different directional flows, only have a time difference. Linearising (3) and substituting (16) into (3), we arrive at the window updating algorithm for FlowN

\[ \Delta w_{N,j}(t) = -\gamma \left( \frac{q_{N,j}}{d + q_{N,j}^*} \Delta w_{N,j}(t) + \frac{w_{N,j}^*}{d + q_{N,j}^*} \Delta q_p(t - \tau_p^f) \right) \]  

(17)

In a dumbbell network at the equilibrium point, we have \( \dot{q}_{N,j} = \dot{q}_p \). Comparing (12) with (17), we obtain the relation between opposite FAST TCP congestion windows

\[ \Delta w_{N,j}(t) = \Delta w_{p,j}(t - \tau_p^f) \]  

(18)

Formulae (16) and (18) shed light into the delay relationship between FlowP and FlowN and enable us to obtain stability conditions on the flows by only analysing the dynamics of one flow. Therefore from now on we focus on
FlowP i. Substituting (16) and (18) into (13), we have

\[ \Delta q_p(t) = \left( \frac{m}{c_{12}} + \frac{n}{c_{21}} \right) \left( \frac{1}{d + q^e} \Delta w_{p,i}(t - \tau_p) - \frac{w^e}{(d + q^e)} \Delta q_p(t - \tau_p) \right) \quad (19) \]

Taking the Laplace transforms of (12), (19), and letting \( \frac{1}{c_0} = \frac{1}{c_{12}} + \frac{1}{c_{21}} \), we obtain

\[ sW_{p,i}(s) = -\gamma \left( \frac{q^e}{\tau} W_{p,i}(s) + \frac{w^e d}{\tau^2} Q_p(s) \right) \quad (20) \]

\[ sQ_p(s) = \frac{1}{c_0} \left( \frac{1}{\tau} W_{p,i}(s)e^{-\tau} - \frac{w^e}{\tau} Q_p(s)e^{-\tau} \right) \quad (21) \]

Thus, we can obtain the characteristic equation as follows

\[ s^2 + \left( \frac{\gamma q^e}{\tau} + \frac{w^e}{\tau c_0} e^{-\tau} \right) s + \frac{\gamma w^e}{\tau c_0} e^{-\tau} = 0 \quad (22) \]

Using the approximation, \( e^{-\tau} \approx 1/1 + \tau, \) the characteristic equation approximates to

\[ a_0 s^3 + a_1 s^2 + a_2 s + a_3 = 0 \quad (23) \]

where \( a_0 = \tau, a_1 = 1 + \gamma q^e, a_2 = \gamma q^e / \tau + \frac{w^e}{\tau^2 c_0} \) and \( a_3 = \gamma w^e / (\tau^2 c_0) \). According to Routh criteria, the system is stable if and only if all the values of the first column in the Routh table are positive. That is, \( a_1 a_2 - a_0 a_3 > 0 \). Note \( w^e = x^e = 2c_0 \tau \), we thus have

\[ (1 + \gamma q^e) \left( \frac{\gamma q^e + 2}{\tau} \right) - 2\gamma > 0 \quad (24) \]

Solving the above inequality we obtain the following stability condition:

**Theorem 1:** In a duplex dumbbell network with bi-directional FAST TCP flows, the congestion windows of flows of one direction are stable when the following quadratic inequality is satisfied

\[ f(\gamma q^e) \triangleq \gamma^2 (q^e)^2 + \gamma q^e + 2 - 2\gamma d > 0 \quad (25) \]

The discriminant, \( \Delta_n \), of the single-variable quadratic inequality (25) is \( 8\gamma d - 7 \). Then one has the following two situations: (1) \( f(\gamma q^e) \) is always positive when \( \Delta_n < 0 \), that is, \( \gamma d < 7/8 \). If \( \gamma \) is set to 0.5 as usual [15], the propagation delay can be up to 1.75 s. That corresponds to a free space link distance of 262,500 km without relaying. (2) When \( \Delta_n \geq 0 \), to force \( f(\gamma q^e) \) positive, we have

\[ 2\gamma q^e \geq -1 + \sqrt{8\gamma d - 7} \quad (26) \]

Substituting \( q^e = \alpha/2c_0 \) and after some algebra, we obtain

\[ \alpha \geq -1 + \sqrt{8\gamma d - 7} / c_0 \quad (27) \]

From (27), it is observed that when \( 7/8 \leq \gamma d \leq 1 \), \( \alpha \) can be set at any positive value.

Also it can be observed that (10) and (11) are symmetric, which implies FlowN j has the same dynamic behaviour as FlowP i. This condition is different from, and more complex than, the one-way FAST TCP flow model [6] because of the coupling effect between ACKs and data packets.

In the scenario, the throughput in equilibrium can be obtained by solving

\[ m x_p' \leq c_{12}, \quad m x_N' \leq c_{21}, \quad x_p' = x_N' \quad (28) \]

Then we obtain the theoretical results of throughput in equilibrium

\[ x_p' = x_N' = \min \left[ \frac{c_{12}}{m}, \frac{c_{21}}{n} \right] \quad (29) \]

Also we obtain

\[ \sum x_p' = m \times \min \left[ \frac{c_{12}}{m}, \frac{c_{21}}{n} \right], \quad \sum x_N' = n \times \min \left[ \frac{c_{12}}{m}, \frac{c_{21}}{n} \right] \quad (30) \]

So we can conclude that the FAST TCP throughput is related to both the link capacity and the number of FAST TCP flows. The utilisation ratio of link capacity when stable is \( \sum x_p' / \epsilon_T \) and \( \sum x_N' / \epsilon_T \). When \( \epsilon_T / m = \epsilon_T / n \), the maximum utilisation of the duplex link can be reached, which reveals that a symmetric link is the most beneficial for capacity utilisation of FAST TCP flows. However, in most cases \( \epsilon_T / m \) is not equal to \( \epsilon_T / n \), there is one direction whose capacity is not fully utilised.

The results offer insight when using FAST TCP with bi-directional traffic. For example, in an ADSL link more upload FAST TCP sessions can dramatically decrease the download speed.

### 4 Simulation results

We demonstrate the stability of two-way FAST TCP flows in a dumbbell network using NS2 [15, 16]. Consider a single link network with the following settings: \( \epsilon_T = 100 \text{ Mb/s}, \quad \epsilon_T = 50 \text{ Mb/s}, \quad d = 20 \text{ ms}, \quad \alpha = 1000, \quad \gamma = 0.5, m = 2 \) and \( n = 1 \). The size of data packet and ACK packet are 1000 bytes and 40 bytes. Hence, we can easily calculate that \( \gamma d = 0.01 \), and by Theorem 1 we obtain that the FAST TCP flows are stable. By (29) we obtain the theoretical throughput of the FAST TCP flows in
equilibrium $x_N = x_P = c_{12}/m = c_{21}/n = 50 \text{ Mb/s}$, and the throughput of all flows: $\sum x_f = 100 \text{ Mb/s}$, $\sum x_i = 50 \text{ Mb/s}$. The congestion window results with NS2 simulations are shown in Fig. 2. The values presented are the average throughput measured in consecutive 0.001 s intervals. From Fig. 2 we observe that FAST TCP congestion window is stable. The throughput of FlowN and FlowP is shown in Fig. 3, which is 5580 packets/s or 46.43 Mbps.

Let $m = 3$, $n = 2$, $\alpha = 100$ in the above scenario, then we obtain the theoretical throughput of the FAST TCP flows in equilibrium $x_N = x_P = \min \{c_{12}/m, c_{21}/n\} = 25 \text{ Mb/s}$, and the throughput of FlowNs $\sum x_N = 50 \text{ Mb/s}$, the throughput of FlowPs $\sum x_P = 75 \text{ Mb/s}$. The average throughput 8 of FlowP 1 and FlowN 1 are plotted in Fig. 4, which are 2800 packets/s and 2782 packets/s (23.3 and 23.15 Mbps), respectively.

5 Conclusion
In a two-way traffic scenario, the bi-directional flows may interact with each other and lead to different stability conditions and different throughput results. Regarding this, we have proposed a model describing the bi-directional FAST TCP flows in a dumbbell network, and have analysed the stability and throughput of the congestion window of the bi-directional FAST TCP flows. Our analysis, which considers the forward queueing delay, the backward queueing delay and their interaction for bi-directional FAST TCP flows, has shown that all the bi-directional flows achieve stability according to Theorem 1. The throughput of the bidirectional FAST TCP flows is studied and we reveal that, in the case of bi-directional FAST TCP traffic in equilibrium, the maximum throughput of any flow is the minimum fair share capacity of either of the duplex links. Hence, multiple upload FAST TCP sessions may dramatically decrease the download speed or asymmetric links with one flow in each direction will lead to the link with larger capacity not being fully occupied. The results have been validated by NS2 simulations.

6 Acknowledgments
This research was partially supported by the Program NCET-05-0673, the key project (no 108166) from Chinese Ministry of Education and partially by the Australian Research Council (ARC).

7 References


