New Improved Decision Aided Turbo Equalization

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Abstract—In this paper we propose a new Turbo Equalization algorithm with Decision Aided Equalizer (DAE). The algorithm takes into account that the soft feedback decisions from the previous iteration contain errors that cannot be neglected. The proposed algorithm finds the error variance and recalculate DAE coefficients at each turbo iteration. The algorithm shows Bit Error Rate (BER) performance improvement relative to the conventional Turbo DAE for severe frequency-selective channels. The achieved improvement is 0.8 dB at BER of $10^{-5}$.

I. INTRODUCTION

Turbo equalization combining Maximum Likelihood Sequence Estimator (MLSE) as the channel equalizer and Soft Input Soft Output (SISO) decoding, has been proposed in [1]. The computational complexity of such a scheme is very high because the MLSE requires complexity that grows exponentially with the length of the channel impulse response. A simplified scheme that employs Decision Feedback Equalization (DFE) instead of MLSE was proposed in [2]. However, severe error propagation happens at the first turbo iteration especially for highly frequency selective channels. A hybrid turbo equalization scheme, employing MLSE at the first turbo iteration and Decision Aided Equalization (DAE) at higher turbo iterations, proposed in [3], reduces the complexity significantly relative to the MLSE and solves the problem of severe error propagation after the first turbo iteration in [2]. The DAE coefficients are determined according to the Minimum Mean Squared Error (MMSE) criterion assuming perfect outputs from the previous iteration. The same coefficients are used at each iteration. A Turbo equalization algorithm with imperfect decision feedback has been proposed recently in [4]. Turbo equalization combining a linear equalizer and soft output decoder has been proposed in [5] (also [6]). In the scheme in [5], extrinsic information from the decoder at one iteration is used at the following iteration in order to reduce the influence of Inter-Symbol Interference (ISI). This approach requires high computational complexity since one matrix inversion per every coded bit is needed. In this paper we improve the Turbo DAE approach proposed in [3].

The main contributions of this paper are:

1) We propose a new turbo equalization algorithm employing DAE from the previous iteration contain error. DAE coefficients in the proposed algorithm are recalculate at each turbo iteration taking into account this error. This is in contrast to MMSE DAE in [3] where the error is not considered.

2) We show that the proposed algorithm outperforms the conventional MMSE DAE algorithm in [3]. The achieved SNR gain is 0.8 dB at BER=$10^{-5}$ for severe frequency-selective channels. We also show that the proposed algorithm approaches the ISI-free bound of the coded system within 0.4 dB at BER=$10^{-5}$.

3) We provide a theoretical analysis of the proposed algorithm and show that when the error is very small ($\rightarrow 0$), the proposed algorithm becomes equivalent to MMSE DAE in [3]. For very large error ($\rightarrow \infty$), the algorithm “disconnects” the feedback part of the DAE to prevent the performance degradation.

The rest of the paper is organized as follows: In Section II we explain the system model, in Section III we analyze the DAE with imperfect decisions. In Section IV we present Simulation Results and Section V concludes the paper.

II. SYSTEM MODEL

The block diagram of the system model is shown in Fig. 1. The binary information bits are encoded with Recursive Systematic Convolutional (RSC) channel encoder and code rate 1/2. The interleaved coded bits are BPSK modulated and the symbols are transmitted through the ISI channel. At the receiver, the received sequence at the time instant $k$ is given by

$$ r_k = Hx_k + n_k $$

where $H$ is the Toeplitz channel impulse response matrix, $x_k$ is the input vector of transmitted symbols, and $(\cdot)$ denotes interleaving, $n_k$ is the vector of Additive White Gaussian Noise (AWGN) samples with covariance matrix $\sigma_n^2I$. The received sequence is equalized first. At the first turbo iteration, the MLSE is used as the channel equalizer. The MLSE delivers soft outputs in the form of Log Likelihood Ratios (LLR) as

$$ z_k^{(1)} = \log \frac{p(x_k = 1 | r)}{p(x_k = -1 | r)} $$

where superscript $(^1)$ denotes the first turbo iteration. At higher iterations the DAE is used as the channel equalizer, which will be described later. After equalization, the signal is deinterleaved, and decoded using the SISO decoder. LLRs delivered by the SISO decoder at iteration $n$ are defined as [7]

$$ \Lambda^{(n)}(x_k) = \log \frac{p(x_k = 1 | z^{(n)})}{p(x_k = -1 | z^{(n)})}. $$

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For the purpose of the SISO decoding we use the Maximum a Posteriori Probability algorithm (MAP) as described in [7]. The optimal rule to find the soft decisions, that are used at the following iteration, is [8, pp.10]

$$\hat{x}_k^{(n)} = \tanh \frac{A^{(n)}(x_k)}{2}. \quad (4)$$

In our analysis we assume that for long sequences the LLRs at the output of the MAP decoder can be approximated by i.i.d. Gaussian random variables [9]. Consequently, the pdf of the soft decision feedback symbols (4) at iteration $n + 1$ can be expressed as [4]

$$p(\hat{x}_k^{(n)}|x_k = +1) = \frac{2}{\pi \sigma^2_{L(n)}} \left( 1 - (\hat{x}_k^{(n)})^2 \right) \times \exp \left( -\frac{\left( \log \frac{1+\hat{x}_k^{(n)}}{1-\hat{x}_k^{(n)}} \right) - \bar{L}(n)^2}{2\sigma^2_{L(n)}} \right) \quad (5)$$

given that the coded symbol $x_k = +1$, $\bar{L}(n)$ and $\sigma^2_{L(n)}$ are the mean and the variance of the LLRs (3) at iteration $n$, respectively. The feedback error is defined as the difference between the correct and the estimated symbols, i.e., $x_k - \hat{x}_k^{(n)}$. The variance of the feedback error $\sigma_{(n)}^2$ is obtained by numerical evaluation of the second central moment of $x_k - \hat{x}_k^{(n)}$ for which purpose we use the pdf in (5)

$$\sigma^2_{(n)} = \frac{2}{\pi \sigma^2_{L(n)}} \int_{-1}^{1} \frac{1 - \hat{x}_k^{(n)}}{1 + \hat{x}_k^{(n)}} \times \exp \left( -\frac{\left( \log \frac{1+\hat{x}_k^{(n)}}{1-\hat{x}_k^{(n)}} \right) - \bar{L}(n)^2}{2\sigma^2_{L(n)}} \right) d\hat{x}_k^{(n)} \quad (6)$$

III. TURBO DAE WITH IMPERFECT DECISIONS

The DAE can be implemented using two finite-impulse response (FIR) filters; namely a Feed-Forward Filter (FFF) and a Feed-Back Filter (FBF). The DAE is designed to cancel both pre- and post-cursor interference. This is in contrast to DFE, which cancels precursor interference only. We refer to the sets of uncancelled symbols as $U$, and cancelled symbols as $\bar{U}$. For the DAE, $U=\{k\}$ and $U = \{1, \ldots, k-1, k+1, \ldots, K\}$, where $K$ is the number of columns in the channel matrix $H$. Furthermore, we define a matrix related to the symbol currently being detected as $H_U = \begin{bmatrix} 0 & 0 & \cdots & 0 & | & h_k & | & 0 & \cdots & 0 \end{bmatrix}$, and a matrix related to decided symbols as $H_{\bar{U}} = H - H_U = \begin{bmatrix} h_1 & h_2 & \cdots & h_{k-1} & | & 0 & | & h_{k+1} & \cdots & h_K \end{bmatrix}$.

We also define $R_U = H_U H_{\bar{U}}^H + \sigma^2_{L} I_L$ and $R_{\bar{U}} = H_{\bar{U}} H_U^H$. A general expression for the output of the DAE at iteration $n > 1$ is given by

$$z_k^{(n)} = \left( f_1^{(n)} \right)^H R_k - \left( f_2^{(n)} \right)^H x_k^{(n)}$$

where $f_1^{(n)}$ and $f_2^{(n)}$ are FFF and FBF coefficients, respectively, and $x_k^{(n)}$ is the vector containing expectations (4) from the previous iteration (n-1). In [3] $f_1^{(n)}$ and $f_2^{(n)}$ are calculated using the MMSE criterion, with the assumption that already detected symbols in the feedback are correct (perfect feedback). The expression for MMSE DAE coefficients [3] can be written as

$$f_1^{(n)} = f_1 = R_{U}^{-1} h_k \quad (8)$$

$$f_2^{(n)} = f_2 = H_{\bar{U}} f_1 \quad (9)$$

i.e., $f_1$ and $f_2$ are calculated only once, and the same coefficients are used at all iterations. $h_k$ is the $k$-th column of the channel matrix $H$. For low SNRs, feedback error cannot be neglected. Consequently, (8) and (9) become inaccurate due to feedback error propagation and a new set of FFF/FBF coefficients has to be determined. In what follows we omit the time index $k$, the iteration index $(n)$ and the interleaving sign $(\hat{)}$ for simplicity of notation. From (7), the error at the equalizer output is

$$e_k = x_k - f_1^{H} r + f_2^{H} \hat{x} \quad (10)$$

Since $H = H_U + H_{\bar{U}}$, combining (35) and (10) we get the expression for Mean Squared Error (MSE) as

$$\varepsilon = E||e_k||^2 = \sigma^2_{\delta_1} + f_1^{H} R_{\delta_1} f_1 + f_2^{H} R_{\delta_2} f_1 - 2f_1^{H} H_{\bar{U}} f_2 + f_2^{H} R_{\delta_2} f_2 - 2f_1^{H} h_k \quad (11)$$

where $\sigma^2_{\delta_1} = E||x_k||^2$ and $R_{\delta_1} = I_N + E n_n n_n^H$. $n_n$ is the vector containing the feedback errors $x_k - \hat{x}_k^{(n)}$, and $I_N$ is a $N \times N$ identity matrix. Furthermore, we make the assumption that the feedback errors are i.i.d. random variables with variance $\sigma^2_{\delta_1}$, which gives $R_{\delta_1} = (1 + \sigma^2_{\delta_1}) I_N$. This assumption is reasonable if the channel interleaver is pseudo-random with a large block size (e.g., $> 1000$) [3]. If we find the following gradients and set them to 0 we get

$$\nabla_{f_1} \varepsilon = \frac{\partial \varepsilon}{\partial f_1} = 2 R_{\delta_1} f_1 + 2 R_{\delta_2} f_1 - 2 H_{\bar{U}}^H f_1 - 2 h_k = 0 \quad (12)$$

and

$$\nabla_{f_2} \varepsilon = \frac{\partial \varepsilon}{\partial f_2} = 2 R_{\delta_2} f_2 - 2 H_{\bar{U}}^H f_1 = 0. \quad (13)$$

From (12) and (13) a new set of FFF and FBF coefficients that takes into account the error propagation is

$$f_1^{(n)} = (R_U + \frac{\sigma^2_{\delta_1}}{\bar{1} + \sigma^2_{\delta_1}} R_{\delta_1})^{-1} h_k \quad (14)$$

and

$$f_2^{(n)} = (1 + \sigma^2_{\delta_1})^{-1} H_{\bar{U}} H_U^{H} f_1^{(n)}. \quad (15)$$

The complete flowchart summarizing the proposed algorithm is shown in Fig. 2. Furthermore, we analyze two cases. First,
feedback is always taken into account independently on the feedback noise level. (14) and (15) are the MMSE solution for the system in which feedback error is modelled according to (6). This model is showed to be very close to a realistic scenario, which is confirmed by comparison of the distribution of the soft feedback symbols obtained analytically based on (6) and simulations, as showed in Fig. 3. The obtained MMSE solution minimizes the Mean Squared Error (MSE) and all other solutions (including the conventional MMSE DAE) with the coefficients obtained according to (8) and (9) produce MSE which is always \( \epsilon \geq \epsilon_{MMSE} \). The equality holds if and only if \( \sigma_{k}^{2} = 0 \).

IV. SIMULATION RESULTS

We apply the proposed Turbo algorithm to the MEEPR4 channel \( h_1=[5 \ 4 \ -4 \ -2]^T \) (also used in [3]) and to the channel (c) [10, p.631] \( h_2=[0.227 \ 0.460 \ 0.688 \ 0.460 \ 0.227]^T \). The results in both Figs. 4 and 5 are for BPSK. The parameters used to obtain the results in Fig. 4 are identical to those in [3]. Information bits are encoded by a sixteen-state RSC channel encoder with the generator polynomials given by \( G = [31 \ 35] \) and punctured to obtain an overall code rate 8/9. The number of FFF taps is \( N_1 = \mu + 1 = 5 \) and FBFs are \( N_2 = N_1 + \mu - 1 = 8 \) taps at all turbo iterations \( n > 1, \) where \( \mu + 1 \) is the length of channel impulse response. All curves in Fig. 4 show BER performance of a MLSE Turbo equalizer, the proposed algorithm and the conventional approach used in [3] after \( n_{max} = 8 \) iterations. The performance of MLSE equalizer is used as a reference for the SNR gain, i.e., the same as in [3]. The results show that the proposed detector slightly outperforms the conventional one for low SNRs while for higher SNRs both detectors exhibit identical performance. Both algorithms reach the performance of the MLSE turbo equalizer at SNR=5.5 dB and BER slightly below \( 10^{-5} \). Simulation results showed in Fig. 5 are obtained for the following parameters. The generator polynomial is \( G = [7 \ 5] \). The same code has been used in [3] as the constituent code in Turbo coded scheme. Fig. 5 compares the BER performance of the coded system without ISI, the MLSE
Turbo equalizer, the proposed algorithm and the conventional Turbo DAE. The results shows that the proposed algorithm achieves better BER performance delivering SNR gain of 0.8 dB relative to the conventional MMSE DAE after \( n_{\text{max}} = 15 \) iterations. It also approaches the performance of ISI-free system within 0.4 dB at BER=10\(^{-5}\). The results in Fig. 5 are obtained for different filter lengths from those in Fig. 4, i.e. \( N_1 = 15 \) and \( N_2 = 18 \). The filter length is found to be an important parameter that significantly influences the performance of the proposed algorithm. As discussed before in Section III, when the feedback error becomes very high, the proposed algorithm “disconnects” the feedback part and the DAE becomes equivalent to the linear equalizer with the performance dependent on the filter length \( N_1 \). However, the filter length is irrelevant to the performance of the conventional DAE because its coefficients are determined according to the perfect feedback assumption and the number of non-zero FFF coefficients is always equivalent to \( \mu + 1 \) regardless of the filter length \( N_1 \). Consequently, the BER performance of the conventional DAE does not change with the filter lengths \( N_1 \) and \( N_2 \). The results in both Figs. 4 and 5 are obtained using random interleavers. The number of information bits is identical to that in [3] and it is 4000 in both Figures. All simulations were performed until at least 1000 erroneous bits were collected.

V. CONCLUSION

In this paper we have proposed a new Turbo DAE algorithm that does not use the conventional assumption about perfect feedback and takes into account the feedback error propagation. The algorithm “decides” how much soft decisions from the previous iteration will be taken into account depending on the feedback noise level. BER performance comparison shows that the new turbo equalizer outperforms the conventional Turbo DAE in [3] delivering SNR gain of 0.8 dB at BER=10\(^{-5}\). The proposed algorithm also approaches the BER performance of the coded system within 0.4 dB at BER=10\(^{-5}\).

REFERENCES