On the Feedback Error in Adaptive Turbo Equalization

Vladimir D. Trajkovic, Predrag B. Rapajic and Rodney A. Kennedy

***Abstract*— In this paper we analyze the decision feedback error propagation in the adaptive turbo equalization with a decision feedback loop. We derive the exact mathematical expression for the feedback error probability density function (pdf) with the assumption that the soft outputs of channel decoder are identical independent distributed (i.i.d) Gaussian random variables with known mean value and variance. We also find a new set of turbo equalizer coefficients based on the feedback error pdf and MMSE criterion. New turbo equalizer is shown to outperform the conventional one (assuming no feedback error propagation) in terms of Bit Error Rate (BER). The achieved improvement is up to 4 dB for severe frequency-selective channels. The analysis is applicable to other turbo detection methods employing the feedback loop.**

I. Introduction

TURBO-coding [1] offers a significant coding gain for communication over memoryless Additive White Gaussian Noise channel delivering the Bit Error Performance within 0.5 dB of Shannon theoretical limit at BER of 10^{-5} [2]. The turbo principle has been applied to a variety of detection/decoding problems such as equalization, multiuser detection (MUD), channel estimation, phase recovery etc. However, the computational complexity of turbo detectors is often a prohibitive parameter for practical design and implementation of such systems. For example, the computational complexity of a turbo equalizer applying Maximum Likelihood (Viterbi) channel equalization as proposed in [3] grows exponentially with the number of discrete-channel coefficients. The solution for the complexity problem is found in use of sub-optimum detectors implemented in turbo schemes which gives a significant reduction of complexity and maintains BER performances still close to those of the optimal detectors. Low complexity turbo equalization combining adaptive Decision Feedback Equalizer (DFE) and Soft Input Soft Output (SISO) decoding has been proposed by Glavieux *et al.* in [4] and later in [5]. Further analysis has been provided in [6] and [7]. In [8] a hybrid turbo equalization scheme combining Maximum Likelihood Sequence Estimation (MLSE) based equalizer in the first turbo iteration and Interference Cancellers (IC) in higher turbo iterations has been analyzed. The ICs are designed so that the outputs from the previous iteration are used to cancel pre- and post-cursor interference. ICs are determined according to MMSE criterion assuming the perfect outputs from the previous iteration. For low SNRs perfect feedback assumption becomes invalid and the feedback error propagation has to be taken into account. In [9] it was shown that the adaptive turbo equalizer employing Least Mean Square (LMS) adaptive algorithm outperforms the MMSE one where the coefficients are obtained according to perfect decision feedback assumption.

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In this paper we analyze the decision feedback error propagation in adaptive turbo equalization.

The main contributions of the paper are:

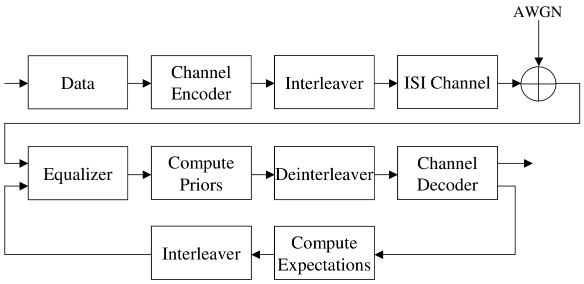
1) We obtain the feedback error (noise) probability density function for the adaptive turbo equalizer assuming that soft outputs from SISO channel decoder are i.i.d. Gaussian random variables. The function is obtained analytically and confirmed by simulations.
2) We propose a new turbo equalizer where a new set of MMSE DFE and MMSE IC coefficients are obtained based on the derived pdf function without assuming perfect decision feedback.
3) We show that for time invariant communication channels, which exhibit sever Inter-Symbol Interference (ISI) the proposed turbo equalizer outperforms one that is obtained according to perfect decision feedback assumption delivering SNR gain of up to 4 dB, and up to 2 dB relative to the adaptive LMS turbo equalizer.

This paper is organized as following: In section II we present the system model. In section III we introduce a SISO decoding algorithm and analyze feedback error propagation. In section IV we analyze MMSE decision feedback structures used in turbo equalization and their dependence on the error propagation. The simulation results and conclusion are presented in sections V and VI, respectively.

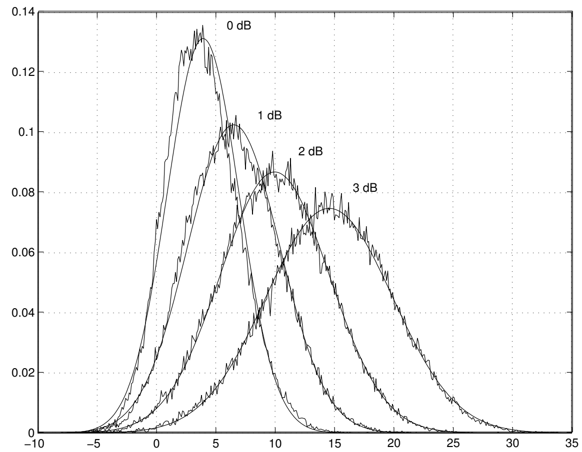
II. System Model

The communication system model applying turbo equalization is shown in Fig. 1. At the transmitter side the information data is encoded using a channel encoder. Interleaved signal is than modulated and transmitted over discrete-coefficient ISI channel. The received signal is

$$\mathbf{r}_k = \mathbf{H}\mathbf{x}_k + \mathbf{n}_k \tag{1}$$



1: System Model employing Turbo Equalization



2: Pdf of LLR at the output of the SISO decoder for different SNRs

where $\mathbf{H}$ is the channel impulse response matrix, $\mathbf{x}_k$ is the vector of transmitted symbols and k is a time instant. $\mathbf{n}_k$ is the vector of Additive White Gaussian Noise (AWGN) samples, i.e.

$$\mathbf{H} = \begin{bmatrix} h_L & h_{L-1} & \cdots & h_0 & 0 & 0 & \cdots \\ 0 & h_L & h_{L-1} & \cdots & h_0 & 0 & \cdots \\ & & \ddots & & & & \\ 0 & \cdots & 0 & h_L & h_{L-1} & \cdots & h_0 \end{bmatrix} \tag{2}$$

$$\mathbf{x}_k = \begin{bmatrix} x_{k-L} & \cdots & x_{k-1} & x_k & x_{k+1} & \cdots & x_{k+L} \end{bmatrix}^T \tag{3}$$

and

$$\mathbf{n}_k = \begin{bmatrix} n_{k+L} & n_{k+L-1} & \cdots & n_k \end{bmatrix}^T \tag{4}$$

At the receiver side, the received sequence is equalized, deinterleaved and decoded using a SISO channel decoder which is designed to deliver soft outputs in the form of the Log-Likelihood Ratios (LLR) [10]. LLRs are calculated for all coded bits are then used in the next turbo iteration in order to improve further detection. The detection is repeated in the same fashion several times. After a certain number of repetitions (turbo iterations) the turbo equalizer BER performance reaches the limit after which it cannot be improved anymore by further increase of the number of iterations. The optimal channel equalization [11] is MLSE where the sequence detection is performed by searching for the path with the smallest metric through the trellis diagram describing the discrete channel. However, the MLSE detection has a complexity proportional to m^L, where m is the size of the input alphabet and L is the length of the channel impulse response. For long channels the computational complexity becomes too high so that the alternative methods for channel equalization have to be applied. Reduced complexity equalization methods used in this paper employ DFE and IC.

III. SISO Decoding Algorithms and Feedback Error Propagation

In this section we briefly present a SISO decoding algorithms named Soft Output Viterbi Algorithm (SOVA). Soft value of the k-th bit (for binary signaling) is defined as the following LLR [1], [10]

$$\Lambda(x_k) = \log \frac{p(x_k = +1|\mathbf{r})}{p(x_k = -1|\mathbf{r})} = \log \frac{\sum_{(s',s),x_k=+1} p(s', s, \mathbf{r})}{\sum_{(s',s),x_k=-1} p(s', s, \mathbf{r})} \tag{5}$$

where s' and s are trellis state indexes at time instants $k-1$ and k, respectively. The summations in numerator and denominator in (5) are over all transitions from state s' to state s for which $x_k = +1$ and $x_k = -1$, respectively. SOVA calculates the approximate values for the probabilities of (5). It is less accurate but also significantly less complex than Maximum A Posteriori Probability (MAP) algorithm (MAP calculates the exact probabilities). However, it was already shown in [12] that almost identical BER performance are obtained when applied in the adaptive turbo equalizer. The soft values using SOVA can be calculated as [10], [13, p. 129]

$$\Lambda(x_k) = \log \frac{p(x_k = +1|\mathbf{r})}{p(x_k = -1|\mathbf{r})} \sim d_k(M_{k,c} - M_{min}) \tag{6}$$

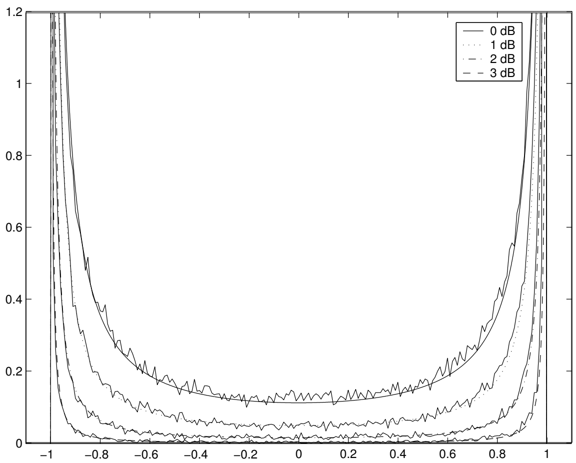
where M_{min} is the path with the minimum metric through trellis, d_k is the k-th symbol in the path with the minimum metric and $M_{k,c}$ is the path with the minimum metric among all paths with the k-th symbol having the value complementary to d_k . Generally, LLRs at the decoders outputs are very large values and they are not suitable to be used in the feedback directly so that they have to be normalized. One possibility is to calculate expectations of 1s and -1s as it was shown in [5], i.e.

$$\bar{a}_k = 1 \cdot p\{x_k = +1|\mathbf{r}\} + (-1) \cdot p\{x_k = -1|\mathbf{r}\} \tag{7}$$

Combining (5) and (7) we get

$$\bar{a}_k = \tanh \frac{\Lambda(x_k)}{2} \tag{8}$$

It was already shown in [14] that for relatively long sequences the LLRs at the output of the SISO decoders



3: Pdf of expectations calculated from the output of the SISO decoder for different SNRs: Analytical and simulation results

could be considered to be random variables with Gaussian pdf. This statement is supported by the simulation results of Fig. 2 where the LLRs from the SISO decoder output are compared with the Gaussian pdf for SNRs from 0 to 3 dB. Combining (8) and the Gaussian assumption about feedback error it is shown that the error pdf is of the form

$$p(x) = \sqrt{\frac{2}{\pi\sigma_L^2}}\frac{1}{1-x^2}\exp(-\frac{(\log(\frac{1+x}{1-x}) - \bar{L})^2}{2\sigma_L^2}) \quad (9)$$

where $\bar{L}$ and σ_L^2 are the mean value and the variance of LLRs obtained according to (5). This result has been supported by simulations of a turbo equalizer where the expectation calculation after SISO decoding has been performed according to (8). The comparison of the analytically obtained expectation (9) and simulation results is shown in Fig. 3. Simulations show exact match between real system and the analytical pdf of the feedback error for SNRs from 0 to 3 dB. The feedback error variance can be obtained by numerical evaluation of the second central moment of pdf in (9), which yields

$$\sigma_b^2 = \sqrt{\frac{2}{\pi\sigma_L^2}}\int_{-1}^{1}\frac{1-x}{1+x}\exp(-\frac{(\log(\frac{1+x}{1-x}) - \bar{L})^2}{2\sigma_L^2})dx \quad (10)$$

IV. MMSE DFE and IC with imperfect feedback

DFE has been shown to be a promising structure that by combination with coding can achieve the capacity of ISI-free AWGN channel [15], [16]. DFE consists of two linear filters namely Feed-Forward Filter (FFF) and Feed-Back Filter (FBF). In previous work on the DFE [17] constant-coefficient MMSE criterion has been considered as the optimization criterion when determining the equalizer coefficients. Based on the assumption that already detected symbols in the feedback are correct, a general expression for both MMSE DFE and MMSE IC coefficients can be written as

$$\mathbf{w} = \mathbf{R}_U^{-1}\mathbf{s}_k \quad (11)$$

$$\mathbf{b} = \mathbf{H}_D^T\mathbf{w} \quad (12)$$

where $\mathbf{w}$ and $\mathbf{b}$ are FFF and FBF coefficients, respectively, $\mathbf{R}_U$ and $\mathbf{H}_D$ are the correlation matrices related to decided (cancelled) and undecided (uncancelled) symbols. $\mathbf{s}_k$ is k-th column of the channel matrix $\mathbf{H}$ related to the symbol currently being detected. $\mathbf{R}_U$, $\mathbf{H}_D$ and $\mathbf{s}$ are explained in more details in Appendix II. The equations (11) and (12) represent the general solution for both DFE and IC. However, for different types of feedback detectors, the matrices $\mathbf{R}_U$, $\mathbf{H}_D$ are different (Appendix II). The equations (11) and (12) are derived assuming the perfect (error free) feedback. From practical point of view, the perfect feedback assumption is valid for moderate to high SNRs,i.e. the number of the erroneous bits used in the feedback is relatively small. However, for low SNRs feedback error cannot be neglected so that perfect feedback assumption is not valid anymore. The new set of equations assuming no perfect feedback can be expressed as

$$\mathbf{b} = (1+\sigma_b^2)^{-1}\mathbf{H}_D^T\mathbf{w}. \quad (13)$$

and

$$\mathbf{w} = (\mathbf{R}_U + \frac{\sigma_b^2}{1+\sigma_b^2}\mathbf{R}_D)^{-1}\mathbf{s} \quad (14)$$

where σ_b^2 is the variance of the feedback error. This is in contrast to [8] where the MMSE IC coefficients are obtained with the perfect feedback assumption. It is clear that when the feedback is perfect (noise-free), the equations (13) and (14) are equivalent to (11) and (12). The proof of (13) and (14) can be found in Appendix II.

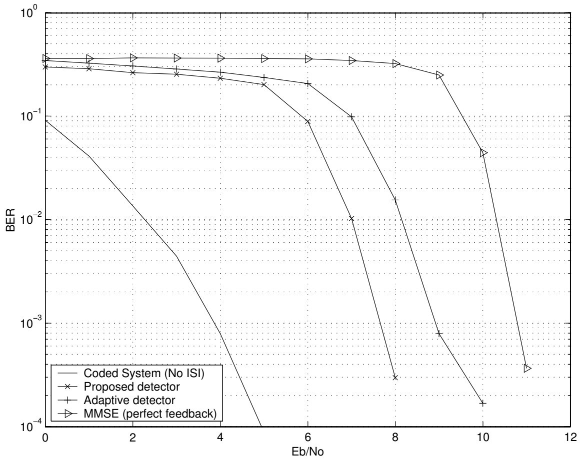
A. Adaptive Implementation

Since the communication channel impulse response is usually not known the adaptive implementation becomes particulary important. In this paper we use LMS adaptive algorithm due to its simplicity of implementation [18]. For decision feedback type of detectors the algorithm is usually expressed by the following pair of equation related to FFF and FBF coefficients [19], i.e.

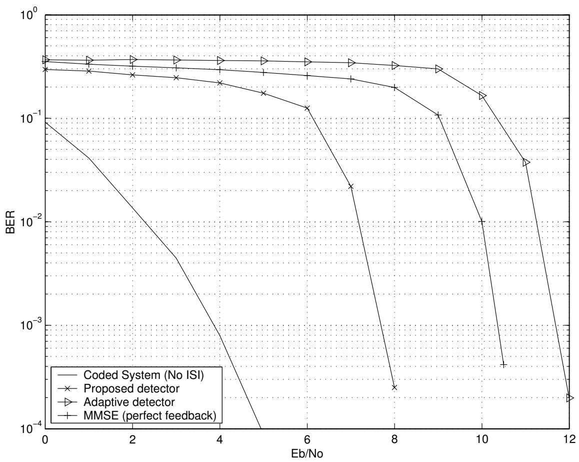
$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu_1 e_k \mathbf{r}_k \quad (15)$$

$$\mathbf{b}_{k+1} = \mathbf{b}_k - \mu_2 e_k \mathbf{d}_k \quad (16)$$

where $\mathbf{w}_k$ and $\mathbf{b}_k$ are the sets of the adaptive FFF and FBF coefficients respectively, μ_1 and μ_2 are appropriate adaptation constants, e_k is the error at the equalizer output which is obtained as the difference between the equalizer output and a known training bit. In the tracking (information) period the error is obtained as a difference of the symbol values after and before the decision element. $\mathbf{r}_k$ is the received vector. Vector $\mathbf{d}_k$ is a vector containing previous decisions (estimates).



4: Turbo equalization over the channel 1 [20]: Adaptive and MMSE results



5: Turbo equalization over the channel 2 [21]: Adaptive and MMSE results

V. Simulation Results

The simulations are performed for two different channels proposed in [20], [21] that exhibit severe ISI. The channels are represented by vectors of discrete coefficients as $\mathbf{h}_1$=[0.227 0.46 0.688 0.46 0.227]^T and $\mathbf{h}_2$=[0.31 0.493 0.562 0.493 0.31]^T. Information bits are encoded by sixteen-state Recursive Systematic Convolutional (RSC) [1] channel encoder with the generator polynomials given as g1=23 and g2=35, in octals. Classical Non-Systematic Convolutional (NSC) codes can be also used since it was already shown in [22] that when applied in the adaptive turbo equalizer both RSC and NSC codes using same generator polynomials show identical BER performance. In the first iteration, DFE is fed by known bits in training period and estimated hard decisions from its own output in tracking period. FFF and FBF are of the length 20 and 10 taps, respectively. In higher iterations ICs employ 20 taps long FFFs and FBFs. FFFs in higher iterations $(n > 1)$ are still fed by received sequence $\mathbf{r}_k$, while FBF of n-th iteration is fed by the output from the previous n-1st iteration obtained using (8). The adaptation constants μ_1 and μ_2 in (15) and (16)are chosen to be 0.01 and 0.0008 in the first iteration, in training and tracking period respectively, and 0.0004 for the other iterations in tracking period. The encoded bits are interleaved by random interleaver of size L=2048 bits. Figs. 4 and 5 show BER performance comparison of turbo equalizer where its DFE coefficients are determined using proposed (imperfect MMSE) method, the adaptive LMS algorithm and conventional MMSE (perfect feedback). The results of both Figs. show that the proposed detector outperforms both adaptive and MMSE detectors after certain number of turbo iterations delivering SNR gain of up to 2 dB and 4 dB at BER of 10^{-4}. The BER results show that the feedback error is better evaluated by using (10) than it was achieved by the adaptive LMS algorithm. The reason for this is relatively slow convergence speed of the LMS algorithm especially in the tracking period when the adaptation rule is not based on known training bits but on the LLRs, which for low SNRs can be unreliable. Since the adaptive LMS turbo equalizer has been already shown to deliver better BER performance relatively to the convectional MMSE one [9], the significant SNR gain of up to 4 dB between the proposed and the conventional MMSE turbo equalizer is not surprising. For all detectors, simulations in terms of number of turbo iterations are performed until the point after which no more improvement can be achieved by increasing the number of iterations.

VI. Conclusion

In this paper we analyzed the decision feedback error propagation in an adaptive turbo equalization scheme. Our analysis shows that the exact mathematical expression can be obtained if the LLRs (soft outputs) from a SISO decoding algorithm are assumed to be i.i.d. Gaussian random variables. For this case, it is possible to calculate a new set of MMSE coefficients without assumption about perfect decision feedback. If the decision feedback error variance is evaluated properly the BER performance of the new turbo equalizer can be improved significantly delivering SNR gain from 2-4 dB relative to adaptive and the conventional MMSE detectors. The slow convergence speed of the adaptive LMS algorithm (especially in the tracking period) is a prohibitive parameter which prevents the adaptive turbo equalizer to approach the BER performance of the proposed detector.

Appendix I
Proof of the equation (9)

Starting from the equation (8) and introducing new random variables x and y instead of $L(x_k)$ and $\bar{a}$ respectively, we get

$$y = \tanh(\frac{x}{2}) \tag{17}$$

where x is assumed to comply to the Gaussian probability distribution. Then, from (17)

$$x = \log \frac{1+y}{1-y} \tag{18}$$

From the Fundamental theorem [23] pdf of y can be found as

$$f_y(y) = \frac{f_x(x_1)}{|g'(x_1)|} + \cdots + \frac{f_x(x_n)}{|g'(x_n)|} + \cdots \tag{19}$$

where x_n are real roots of the equation $y = g(x)$ and $g'(x)$ is the derivative of $g(x)$ and it is

$$g'(x) = \frac{d}{dx}(\tanh \frac{x}{2}) = \frac{2e^x}{(1+e^x)^2} \tag{20}$$

Combining (18), (19), (20), and Gaussian assumption about random variable x the pdf of the random variable y is

$$f_y(y) = \sqrt{\frac{2}{\pi\sigma_x^2}} \frac{1}{1-y^2} \exp(-\frac{(\log\frac{1+y}{1-y} - \bar{x})^2}{2\sigma_x^2}) \tag{21}$$

where $\bar{x}$ and σ_x^2 are mean and variance of the Gaussian random variable x.

Appendix II
Proof of the equations (11) and (12)

The equation (1) can be expressed as

$$\mathbf{r} = \mathbf{H}_U\mathbf{x} + \mathbf{H}_D\mathbf{x} + \mathbf{n} \tag{22}$$

where $\mathbf{H}_U$ and $\mathbf{H}_D$ are referred to undecided (uncancelled) and decided (cancelled) symbols respectively. Here we omit to use the time index k due to simplicity reasons. For DFE the matrices are

$$\mathbf{H}_U = \left[\begin{array}{cccc|ccc} h_L & h_{L-1} & \cdots & h_0 & 0 & \cdots & 0 \\ 0 & h_L & \cdots & h_1 & 0 & \cdots & 0 \\ & & \ddots & & & & \\ 0 & 0 & \cdots & h_L & 0 & \cdots & 0 \end{array}\right] \tag{23}$$

and

$$\mathbf{H}_D = \left[\begin{array}{cccc|ccc} 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & h_0 & \cdots & 0 \\ & & \ddots & & & & \\ 0 & 0 & \cdots & 0 & h_{L-1} & \cdots & h_0 \end{array}\right]. \tag{24}$$

For ICs the matrices take the following forms

$$\mathbf{H}_U = \left[\begin{array}{ccc|c|ccc} 0 & 0 & \cdots & h_0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & h_1 & 0 & \cdots & 0 \\ & & \ddots & & & & \\ 0 & 0 & \cdots & h_L & 0 & \cdots & 0 \end{array}\right] \tag{25}$$

$$\mathbf{H}_D = \left[\begin{array}{ccc|c|ccc} h_L & h_{L-1} & \cdots & 0 & 0 & \cdots & 0 \\ 0 & h_L & \cdots & 0 & h_0 & \cdots & 0 \\ & & \ddots & & & & \\ 0 & 0 & \cdots & 0 & h_{L-1} & \cdots & h_0 \end{array}\right] \tag{26}$$

$$e_k = \mathbf{w}^T\mathbf{r} - \mathbf{b}^T\hat{\mathbf{x}} - x_k \tag{27}$$

Combining (22) and (27)

$$e_k = \mathbf{w}^T\mathbf{H}_U\mathbf{x} + \mathbf{w}^T\mathbf{H}_D\mathbf{x} + \mathbf{w}^T\mathbf{n} - \mathbf{b}^T\mathbf{x} - \mathbf{b}^T\mathbf{n}_b - x_k \tag{28}$$

where $\mathbf{n}_b$ is a vector of decision feedback error samples. Taking the expectation we get the expression for Mean Squared Error (MSE)

$$\begin{aligned} \varepsilon = E[|e_k|^2] = \mathbf{w}^T\mathbf{R}_U\mathbf{w} + \mathbf{w}^T\mathbf{R}_D\mathbf{w} - \\ -2\mathbf{w}^T\mathbf{H}_D\mathbf{b} + \mathbf{b}^T\mathbf{R}_b\mathbf{b} - 2\mathbf{w}^T\mathbf{s} + \sigma_x^2 \end{aligned} \tag{29}$$

$\mathbf{R}_U$, $\mathbf{R}_D$ and $\mathbf{R}_b$ are defined as following

$$\mathbf{R}_U = \mathbf{H}_U\mathbf{H}_U^T + \sigma_n^2\mathbf{I}_L \tag{30}$$

$$\mathbf{R}_D = \mathbf{H}_D\mathbf{H}_D^T \tag{31}$$

and

$$\mathbf{R}_b = \mathbf{I}_L + E[\mathbf{n}_b\mathbf{n}_b^T] \tag{32}$$

and $\mathbf{s}$ is the k-th column of the matrix $\mathbf{H}$ related to the symbol detected at the time instant k. $\mathbf{I}_L$ is a $L \times L$ identity matrix and σ_x^2 is the average power of the received symbols and it can be normalized to 1 without the loss of generality. σ_n^2 is the variance of the random noise process. Finding the gradients and setting them to 0 we get

$$\nabla_{\mathbf{w}}\varepsilon = \frac{\partial\varepsilon}{\partial\mathbf{w}} = 2\mathbf{R}_U\mathbf{w} + 2\mathbf{R}_D\mathbf{w} - 2\mathbf{H}_D\mathbf{b} - 2\mathbf{s} = \mathbf{0} \tag{33}$$

and

$$\nabla_{\mathbf{b}}\varepsilon = \frac{\partial\varepsilon}{\partial\mathbf{b}} = 2\mathbf{R}_b\mathbf{b} - 2\mathbf{H}_D^T\mathbf{w} = \mathbf{0}. \tag{34}$$

From (34)

$$\mathbf{b} = \mathbf{R}_b^{-1}\mathbf{H}_D^T\mathbf{w}. \tag{35}$$

Substituting (35) in (33) we get the expression for $\mathbf{w}$ as

$$\mathbf{w} = (\mathbf{R}_U + \mathbf{R}_D - \mathbf{H}_D\mathbf{R}_b^{-1}\mathbf{H}_D^T)^{-1}\mathbf{s} \tag{36}$$

If feedback errors are assumed to be *i.i.d.* random variables with variance σ_b^2 than $\mathbf{R}_b$ takes the following form

$$\mathbf{R}_b = (1+\sigma_b^2)\mathbf{I}_L \tag{37}$$

Replacing (37) in (35) and (36) gives

$$\mathbf{b} = (1+\sigma_b^2)^{-1}\mathbf{H}_D^T\mathbf{w}. \tag{38}$$

and

$$\mathbf{w} = (\mathbf{R}_U + \frac{\sigma_b^2}{1+\sigma_b^2}\mathbf{R}_D)^{-1}\mathbf{s} \tag{39}$$

If the feedback is perfect than $\sigma_b^2 = 0$ and $\mathbf{R}_b = \mathbf{I}_L$ which gives the following solutions for $\mathbf{w}$ and $\mathbf{b}$

$$\mathbf{b} = \mathbf{H}_D^T\mathbf{w}. \tag{40}$$

and

$$\mathbf{w} = \mathbf{R}_U^{-1}\mathbf{s} \tag{41}$$

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