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Novel Receiver Structure for Joint Timing Recovery and Equalization in Frequency Selective Channels

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***Abstract*— This paper presents a new receiver structure that addresses the difficult problem of achieving symbol synchronization given a frequency-selective channel. The new joint timing recovery and equalizer structure is partitioned into a number of component parts. The most critical of these component parts, a magnitude equalizing portion which is insensitive to timing errors, is positioned prior to timing recovery, effectively creating an all-pass channel between the transmitter and timing recovery portion of the receiver. Such a structure gives rise to gains in both receiver performance and robustness, and allows the use of less complex symbol synchronization schemes. It is also shown that the magnitude equalizing portion of the equalizer can be retro-fitted to existing receiver designs for corresponding gains.**

I. Introduction

Receivers in digital synchronous communication systems must sample the received analog signal in order to obtain the encoded digital information by means of a decision device. The decision device operates at the symbol rate $1/T$, where T is the symbol period. For maximum noise immunity and minimum intersymbol interference (ISI) the received signal should be sampled at the instants of maximum eye opening.

Typically, one samples at a rate of $1/T_s$ where, ideally, $T_s = T/n$, and n is an integer. Due to the inherent inaccuracies in realizable local oscillators, T and T_s are incommensurate, so one must either interpolate the sampled signal or alter the sampling phase to sample at the optimal point decision instants. However, these decision instants are a priori unknown because of the unknown propagation delay t_d between the transmitter and receiver and inaccurate oscillators (phase noise).

In the case of a slowly-varying (or static) channel and stable local oscillators, it is possible for the equalizer to compensate for a non-optimal timing phase. This is achieved by incorporating a fractional delay filter into the equalizer's impulse response. In this case, a separate timing recovery system is not necessary. However, while it is possible for an equalizer to perform timing recovery for time-varying channels, it is unwise. This arizes from the fact that equalization uses a multi-dimensional parameter space, whereas timing recovery only requires, in principle, the adaptation of one parameter, the sampling phase. Hence, timing recovery is best done by a specialized timing recovery system that is only required to estimate the optimal sampling phase.

There exists a chicken and egg problem between timing recovery and equalization—It is unclear whether timing recovery or equalization should be performed first in the receiver. By preceding equalization by timing recovery, the time variability of the channel (as seen by the equalizer) is reduced (as the effect of the time-varying timing error will have been compensated for). The timing recovery system, however, is presented with a possibly time-varying and frequency selective channel. Such channels can present difficulties for timing recovery systems, degrading their performance.

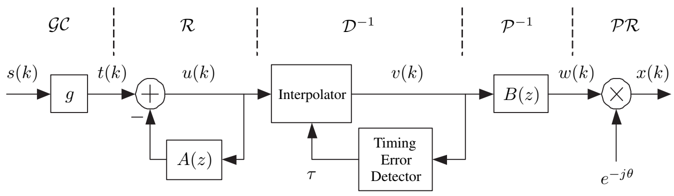


Fig. 1. Preferred structure of the new joint timing recovery and equalizer scheme.

Ideally, frequency-selective channels would be compensated for by an equalizer prior to timing recovery. But we have already mentioned that it is generally better to perform timing recovery before equalization. This is the chicken and egg problem.

Instead of choosing whether to perform timing recovery or equalization first, we identify which parts of the equalization process can be done prior to timing recovery. In turn, we present a five-part receiver structure, as illustrated in Fig. 1.

The structure includes a symbol synchronizer as well as an equalizer (based on [4]) that is partitioned into four separate parts. The result is a joint timing recovery and equalizer system that performs part of the equalization before timing recovery.

This paper expands on the work done by [2], [3], [4] who decomposed the equalizer into a cascade of linear filters. The structure presented here most closely resembles that of [4], which uses a cascade of a purely recursive whitening filter ($\mathcal{R}$), gain control ($\mathcal{GC}$), phase rotator ($\mathcal{PR}$), and a purely transversal filter ($\mathcal{T}$). The authors just listed showed that separating the tasks of the equalizer into four separate, but easier, subtasks improved performance. We show that timing recovery can be introduced into this cascade after the whitening filter to further improve performance. The resulting cascade will produce a signal for the timing recovery system that has a constant power spectral density (PSD), with the remaining phase distortion being cleaned up by the transversal filter. The outcome is a timing recovery unit, and hence receiver, that is more robust towards multipath and exhibits better performance under the effects of a time-varying frequency-selective channel. Furthermore, the new structure may avoid the catastrophic failure of timing recovery in the case of a particularly difficult channel.

II. Problem formulation

The overall goal of this work is to provide the timing recovery unit with a signal that is uncorrupted by the magnitude effects of the channel between the transmitter and receiver. This section details the partitioning of the equalizer into five component parts, one of which represents timing recovery. It is shown that some parts of the equalizer can be placed in front of the timing recovery unit. In doing so, the channel is effectively reduced to an all-pass channel, which is a significantly easier problem for timing recovery.

The transfer function (TF) of a non-minimum phase FIR channel $\mathcal{F}$ can be written

$$F(z) = f\prod_{i=1}^{N_1}\left(1 - z_{\mathrm{I},i}z^{-1}\right)\prod_{j=1}^{N_2}\left(z_{\mathrm{O},j}^{-1} - z^{-1}\right) \tag{1}$$

where $N = N_1 + N_2$ is the order of the channel, f can be complex, and where $|z_{\mathrm{I},i}| < 1$ and $|z_{\mathrm{O},i}| > 1$ correspond to the zeros inside and outside of the unit circle, respectively. The channel TF in (1) can be written as $F(z) = fA(z)B(z)$, where

$$A(z) = \prod_{j=1}^{N_2}\frac{z_{\mathrm{O},j}^{-1} - z^{-1}}{1 - \left(z_{\mathrm{O},j}z\right)^{-1}} \tag{2}$$

is the transfer function of an all-pass filter and

$$B(z) = \prod_{i=1}^{N_1}\left(1 - z_{\mathrm{I},i}z^{-1}\right)\prod_{j=1}^{N_2}\left(1 - \left(z_{\mathrm{O},j}z\right)^{-1}\right) \tag{3}$$

is the transfer function of a minimum phase filter. In the absence of noise, the optimal linear equalizer $E(z)$ (in the MSE sense) should implement the inverse of $F(z)$, up to a delay δ to maintain causality. The equalizer TF is clearly

$$E(z) = z^{-\delta}F^{-1}(z) = f^{-1}T(z)R(z) \tag{4}$$

where $T(z) = z^{-\delta}A^{-1}(z)$, $R(z) = B^{-1}(z)$, and $f^{-1} = ge^{-j\theta}$. Note that $B(z)$ is causally invertible since it has no zeros outside of the unit circle and thus requires no introduced delay. Furthermore, its inverse $R(z)$ is a whitening filter (also innovator or predictive filter). The effect of $\mathcal{R}$ is to flatten the PSD of a signal that is passed through it to a constant value. Hence, the cascade $\mathcal{R}\circ\mathcal{F}$ results in the all-pass channel $\mathcal{A}$ which has no amplitude distortion. The filter $\mathcal{T}$ should compensate (with a delay) the phase distortion caused by $\mathcal{A}$.

Equation (4) decomposes the equalizer $\mathcal{E}$ into a cascade of four linear filters $\mathcal{GC}$ (g), $\mathcal{PR}$ $(e^{-j\theta})$, $\mathcal{R}$ and $\mathcal{T}$. The original contribution of this paper is based on the further decomposition of the all-pass component of the channel $\mathcal{A}$ into two separate all-pass filters $\mathcal{D}$ and $\mathcal{P}$ such that

$$A(z) = P(z)D(z) \tag{5}$$

where $\mathcal{D}$ is a fractional delay filter that represents bulk delays (linear phase distortions), such as the propagation delay t_d, in the signal. $\mathcal{P}$ represents the remaining nonlinear phase distortions of the channel.

The TF $T(z)$, which represents the inverse of $\mathcal{A}$, can now be written as

$$T(z) = z^{-\delta}A^{-1}(z) = z^{-\varepsilon}D^{-1}(z)P^{-1}(z) \tag{6}$$

where ε is a delay that is introduced to maintain causality. Hence the transversal part $\mathcal{T}$ of our equalizer may be replaced by $\mathcal{D}^{-1}$ and $\mathcal{P}^{-1}$. It is important to note that the order of the linear transformations $\mathcal{GC}, \mathcal{R}, \mathcal{D}^{-1}, \mathcal{P}^{-1}$ and $\mathcal{PR}$ is irrelevant in steady state, but for reasons which will be developed later, it is recommended to position them in the order just listed.

Since the order of components in the cascade is irrelevent, we are free to place $\mathcal{R}$ before $\mathcal{D}^{-1}$. In doing so we have effectively moved the magnitude compensating portion of the equalizer before the timing recovery unit, and, as a result, provided a signal with constant PSD for timing recovery. More specifically, the timing recovery unit $\mathcal{D}^{-1}$ directly follows the cascade $\mathcal{R}\circ\mathcal{F}$ which is an all-pass channel $\mathcal{A}$.

III. Structure

Fig. 1 illustrates the preferred structure for the joint equalizer and timing recovery scheme. The following section will describe the structure of each of the equalizer units and explain why the configuration of Fig. 1 was chosen.

A. $\mathcal{GC}$ — *Gain Control*

The purpose of gain control is to provide the filters that follow it with a signal of reduced dynamical range. It is a one-coefficient real equalizer that scales its input by a gain g. It generates the output

$$t(k) = gs(k). \tag{7}$$

We choose to perform gain control prior to equalization by the whitening filter, but it may be placed after it, with comparable performance.

B. $\mathcal{R}$ — *Whitening Filter*

The whitening filter $\mathcal{R}$ is an autoregressive filter that predicts the value of $t(k)$ from past samples and determines the prediction error $u(k)$. This gives rise to its alternate names of predictor and innovator. The output is

$$u(k) = t(k) - \hat{t}(k) \tag{8}$$

with

$$\hat{t}(k) = \sum_{l=1}^{N} a_l u(k-l) = \mathbf{A}^T\mathbf{U}_N(k-1) \tag{9}$$

$$\mathbf{A} = [a_1, \cdots, a_{N-1}, a_N]^T \tag{10}$$

$$\mathbf{U}_N = \left(u(k-1), \cdots, u(k-N)\right)^T. \tag{11}$$

where $\mathbf{A}$ is called the "prediction vector". The cost function to be minimised is

$$J_p = E\left\{|u(k)|^2\right\}. \tag{12}$$

This criterion attempts to decorrelate the output, thus making it white. Note that the cost function relies on second order statistics, and as such, cannot correct phase distortions in the input $t(k)$. It is for this reason that filtering by $\mathcal{R}$ may be performed before timing recovery.

C. $\mathcal{D}^{-1}$ — Timing Recovery

As discussed in section II, by having $\mathcal{D}^{-1}$ directly follow $\mathcal{R} \circ \mathcal{F}$, then the timing recovery unit $\mathcal{D}^{-1}$ is presented with an all-pass channel $\mathcal{A}$. This opens up the number of available timing recovery schemes for consideration since the candidate schemes no longer have to deal with frequency-selective fading. In this study the Mueller and Müller scheme [1] is used because it is well known and has relatively low complexity.

The Mueller and Müller scheme provides an estimate of the timing error τ that is used to control an interpolator filter. The interpolator output is

$$v(k) = u(k-\tau). \tag{13}$$

D. $\mathcal{P}^{-1}$ — Phase Correction

The all-pass filter $\mathcal{P}^{-1}$ is responsible for correcting the remaining signal phase distortions that were not corrected by the timing recovery unit. Note that $\mathcal{P}^{-1}$ need not have an all-pass structure, it may actually be advantageous to employ a filter that is able to correct residual magnitude distortions that were not corrected by $\mathcal{R}$. Such an equalizer will be referred to as a *full equalizer* henceforth. In this case, the receiver has the same structure as a conventional receiver (which has timing recovery followed by full equalization) except for the addition of the whitening filter $\mathcal{R}$ before the timing recovery unit.

A transversal filter that employs the Godard criterion [5] is used in this study because of its popularity and simplicity, but other schemes may be used for their own desirable properties. Note that an equalizer employing the Godard criterion will correct for residual magnitude errors.

The filter output is

$$w(k) = \sum_{l=0}^{L} b_l v(k-l) = \mathbf{B}^T \mathbf{V}_{L+1}(k) \tag{14}$$

where

$$\mathbf{B} = [b_0, b_1, \ldots, b_L]^T \tag{15}$$

$$\mathbf{V}_{L+1} = \big(v(k), v(k-1), \ldots, v(k-L)\big). \tag{16}$$

E. $\mathcal{PR}$ — Phase Recovery

The input is $w(k)$ and the output is

$$x(k) = w(k)e^{-j\theta} \tag{17}$$

where θ is an estimate of the carrier phase. It is also possible to place the $\mathcal{PR}$ unit at any other suitable point in the cascade. For example, phase recovery could be done prior to timing recovery if the symbol synchroniser was especially susceptible to the carrier phase.

IV. Adaptation

A. $\mathcal{GC}$ — Gain Control

The power level of the samples are fixed to a particular value P_s

$$E\big\{|u(k)|^2\big\} = P_s. \tag{18}$$

An adaptive algorithm that controls g in such as way is presented below

$$G(k) = G(k-1) + \mu_G\big[1 - |u(k)|^2\big] \tag{19}$$

$$g(k) = \sqrt{|G(k)|} \tag{20}$$

with $G(0) = 1$, and μ_G is a suitable small positive step size.

B. $\mathcal{R}$ — Whitening Filter

Using a stochastic gradient descent algorithm with the cost function of (12) we arrive at the adaptation rule

$$\mathbf{A}(k) = \mathbf{A}(k-1) + \mu_A u(k)\mathbf{U}_N^*(k-1) \tag{21}$$

$$u(k) = t(k) - \mathbf{A}(k-1)^T \mathbf{U}_N(k-1) \tag{22}$$

where the superscript * stands for complex conjugatation and $\mathbf{A}(0) = [0, 0, \ldots, 0]^T$. Note that this adaptation rule is approximate since the filter is recursive, however, it is sufficiently accurate for our purposes.

C. $\mathcal{D}^{-1}$ — Timing Recovery

The Mueller and Müller scheme that was chosen for timing recovery is an error-tracking style synchroniser; it derives an indication about the current timing error τ from discrete symbol samples. The timing error detector produces the sequence

$$y(k) = \hat{d}(k-1)v(k) - \hat{d}(k)v(k-1) \tag{23}$$

where $\hat{d}(k)$ is the timing error detector's decision about $v(k)$. The statistical average of $y(k)$ gives an indication of the sign and magnitude of the timing error τ. This error is used to control an interpolator that effectively advances/delays the sampling phase.

D. $\mathcal{P}^{-1}$ — Phase Correction

The Godard algorithm is used to correct the residual distortions in the magnitude and phase of the signal after timing recovery. The cost function for the Godard criterion is

$$J_G(\mathbf{B}) = E\Big\{\big[|w(k)|^p - R_p\big]^2\Big\} \tag{24}$$

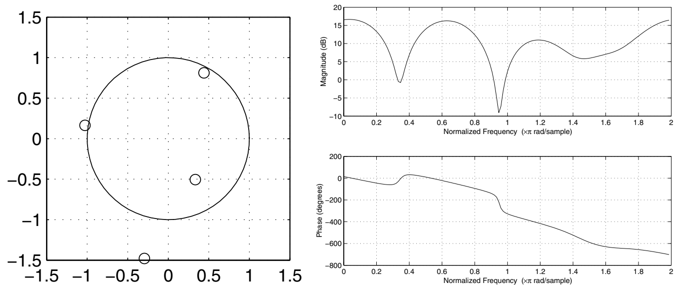
with

$$R_p = \frac{E\big\{|d(k)|^{2p}\big\}}{E\big\{|d(k)|^p\big\}} \tag{25}$$

where $p = 2$. The stochastic gradient adaptation rule to minimize the cost function of (24) is

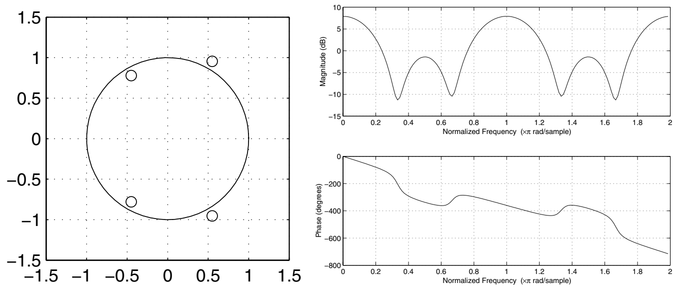
$$\mathbf{B}(k) = \mathbf{B}(k-1) + \mu_B w(k)\big(R_2 - |w(k)|^2\big)\mathbf{V}_{L+1}^*(k) \tag{26}$$

$$w(k) = \mathbf{B}^T(k-1)\mathbf{V}_{L+1}(k) \tag{27}$$



(a) (b)

Fig. 2. (a) Zeros of $\mathbf{f}_1$, (b) Magnitude and phase responses of $\mathbf{f}_1$.



(a) (b)

Fig. 3. (a) Zeros of $\mathbf{f}_2$, (b) Magnitude and phase responses of $\mathbf{f}_2$.

where $\mathbf{B}$ is initialised with a center tap strategy i.e., $\mathbf{B}(0) = [0, 0, \ldots, 0, 1, 0, \ldots, 0]^T$, and μ_B is a suitable small positive step size.

In the noiseless case, or with a sufficiently high signal-to-noise ratio, the receiver up to this point is able to recover the transmitted data symbols $d(k)$ up to a delay δ and arbitrary phase θ. Consequently, in steady state

$$w(k) \approx d(k - \delta)\exp(j\theta) \tag{28}$$

E. $\mathcal{PR}$ — Phase Recovery

One is free to choose any appropriate scheme for phase recovery as it has no effect on timing recovery and equalization in the setup of Fig. 1. Consequently, perfect carrier recovery is assumed for the studies conducted in Section V.

V. Results and Conclusions

Simulation results are presented that illustrate the behaviour of the new receiver structure shown in Fig.1. Results were obtained via Monte Carlo simulations of 50 runs using severe channels, $\mathbf{f}_1$ and $\mathbf{f}_2$, that were proposed in [6] and [7], respectively. The impulse responses are

$$\mathbf{f}_1 = [2 - 0.4j, 1.5 + 1.8j, 1, 1.2 - 1.3j, 0.8 + 1.6j] \tag{29}$$

$$\mathbf{f}_2 = [0.8264, -0.1653, 0.8512, 0.1636, 0.81]. \tag{30}$$

Figs. 2 and 3 depict the location of the channel zeros and magnitude and phase responses of channels $\mathbf{f}_1$ and $\mathbf{f}_2$, respectively. Note that the channels are non-minimum phase with deep spectral nulls and and nonlinear phase distortions. The transmitted signal is BPSK $\{-1, 1\}$ and the SNR at the receiver is 15dB. The $\mathcal{R}$ and $\mathcal{T}$ equalizers have 10 and 20 taps, respectively. A timing error of $0.4T$ is introduced in the channel and perfect carrier recovery is assumed.

A decision directed mean square error (DDMSE) performance measure is used. It is calculated using the following recursive formula

$$M_{DD}(k) = \lambda M_{DD}(k-1) + (1-\lambda)|\hat{d}(k) - w(k)|^2 \tag{31}$$

where $\lambda = 0.99$ and $\hat{d}(k)$ is the receiver's decision about $w(k)$. The eye is open when M_{DD} is small, and in such cases, the DDMSE is equivalent to the true MSE.

Fig. 4 illustrates the performance of the receiver with and without $\mathcal{D}^{-1}$ and $\mathcal{R}$ present in the cascade of Fig. 1 for channels $\mathbf{f}_1$ and $\mathbf{f}_2$. Note that the use of the whitening filter results in a lower DDMSE for both channels. The DDMSE for channel $\mathbf{f}_2$ when the whitening filter is off is constant since the receiver does not manage to open the eye at all.

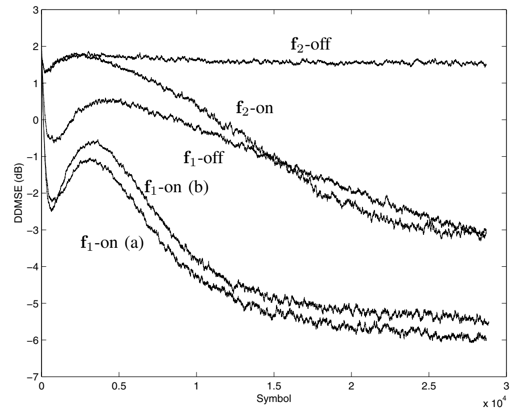


Fig. 4. Performance comparison of the proposed receiver setup with and without timing recovery $\mathcal{D}^{-1}$ and whitening filter $\mathcal{R}$. Naming convention for labels is <channel>–< $\mathcal{R}$ on/off>

As was mentioned in Section III-D, the use of the Godard criterion for $\mathcal{P}^{-1}$ results in a conventional receiver[1], but with the addition of a whitening filter before timing recovery. Consequently, the results that compare the operation of the receiver with the whitening filter on and off are equivalent to comparing the new receiver structure to a conventional structure, respectively. From the simulations, it is clear that the new receiver structure performs better than a conventional one.

The two curves "$\mathbf{f}_1$-on (a)" and "$\mathbf{f}_1$-on (b)" represent runs with channel $\mathbf{f}_1$, and the whitening filter turned on, but the timing recovery unit is turned off in the "(b) case". Note that the inclusion of timing recovery in the "(a) case" results a lower DDMSE and faster convergence. This indicates that the joint timing recovery and equalizer system is compensating for the timing error faster than the equalizer alone.

Fig. 5 depicts the estimated value of the timing error over time for the Mueller and Müller scheme with and without the whitening filter included in the receiver for the channel $\mathbf{f}_2$.

[1]With a conventional receiver structure being one which employs full equalization after timing recovery, but without a separate whitening filter.

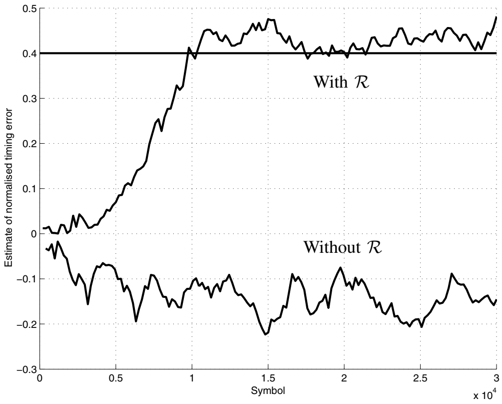


Fig. 5. Comparison of timing recovery performance with and without whitening filter $\mathcal{R}$. Actual timing error is $0.4T$.

Observe that the timing recovery scheme estimates the error of $0.4T$ more precisely when the whitening filter is included.

VI. Conclusion

A novel receiver structure has been presented that performs joint timing recovery and equalization. The new structure employs a whitening filter (that is insensitive to timing errors) prior to equalization, effectively presenting an all-pass channel between the transmitter and timing recovery portion of the receiver. This allows for less complex (and cheaper) timing recovery schemes to be employed in receivers that use such a structure.

Through simulation, this new structure has been shown to be better suited to frequency selective channels than its counterpart that conducts timing recovery before any equalization.

It has also been shown that the new receiver structure is equivalent to a conventional receiver structure (that performs full equalization after timing recovery), except that it employs a whitening filter prior to timing recovery. This suggests that a whitening filter can be added to conventional receiver designs prior to timing recovery, thus combating the magnitude effects of the channel, and resulting in structures that are more robust to frequency selective channels.

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