Exact Error Probability Expressions for Arbitrary Two-Dimensional Signaling with I/Q Unbalances over Nakagami-\(m\) Fading Channels

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Abstract—Recently, we provided closed-form expressions involving two-dimensional (2-D) joint Gaussian Q-function for the symbol error rate (SER) and bit error rate (BER) of an arbitrary 2-D signal with I/Q unbalances over an additive white Gaussian noise (AWGN) channel [1]. In this paper, we extend the expressions to Nakagami-\(m\) fading channels. Using Craig’s representation of the 2-D Gaussian Q-function, we derive an exact and general expression for the error probabilities of arbitrary 2-D signaling with I/Q phase and amplitude unbalances over Nakagami-\(m\) fading channels.

Index Terms—error probability, Nakagami-\(m\) fading, two-dimensional modulation, I/Q unbalance

I. INTRODUCTION

In a practical coherent two-dimensional (2-D) modulation scheme, the performance of the receiver is less than ideal: imperfect components create I/Q phase and gain unbalances, and there are severe small- and large-scale variations in the received signal strength. The I/Q unbalances arise from an imperfect 90-degree phase shifter, and from mixers or filters with different losses, and the variations of the received signal strength are caused by multi-path and shadow fading.

A number of recent studies [1]-[2] have analyzed the effects of these impediments on the error performances of 2-D signaling. The exact expression for the error probability of arbitrary 2-D signaling with I/Q unbalances over an AWGN channel was reported in [1]. Error probabilities of 2-D M-ary signaling signal with I/Q balance over fading channels for Rayleigh, Nakagami-\(m\), and Ricean distributions were presented in [2].

In this paper, as an extension of our previous work [1], we provide a new closed-form expression involving the 2-D Gaussian Q-function for the error probability of arbitrary 2-D signaling with I/Q phase and amplitude unbalances over a Nakagami-\(m\) fading channel. For this purpose, we first transform the 2-D Gaussian Q-function into Craig’s form [3]. Then, using the moment generating function (MGF) of the Nakagami-\(m\) distribution, we obtain the error probability expression for a 2-D signal over the Nakagami-\(m\) fading channel. Finally, we verify the provided expression through comparison with the previous results of [2] and [4] in Nakagami-\(m\) fading, and analyze the effect of I/Q unbalances on the performance.

II. SYSTEM MODEL

We assume that the received signal envelope \(A\) has a Nakagami-\(m\) distribution with the probability density function (pdf) given as [5]

\[
f_A(a) = \frac{2^m a^{2m-1}}{\Omega^2 \Gamma(m)} \exp\left(-\frac{ma^2}{\Omega}\right), \quad a \geq 0
\]  

(1)

where \(m = \Omega^2 / E[(A^2 - \Omega^2)^2]\) and \(\Omega = E[A^2]\) are fading and power-scaling parameters, respectively. The PDF of the instantaneous signal-to-noise ratio (SNR), \(\gamma = a^2 E_n / N_0\) can be expressed as

\[
f_\gamma(\gamma) = \frac{m \gamma^{m-1}}{\bar{\gamma} \Gamma(m)} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \quad \gamma \geq 0
\]  

(2)

where \(\bar{\gamma} = \Omega E_n / N_0\) is the average SNR per bit and \(\Gamma(x)\) is the Gamma function.

The 2-D joint Gaussian Q-function is defined by [6]

\[
Q(x, y; \rho) = \frac{1}{2\pi \sqrt{1 - \rho^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2 + y^2 - 2\rho xy}{2(1 - \rho^2)}\right) dx dy.
\]  

(3)

From the Craig representation in [7], (3) is rewritten as

\[
Q(x, y; \rho) = \frac{1}{2\pi} \int_{0}^{\sqrt{\rho}} \exp\left(-\frac{x^2}{2\sin^2 \theta}\right) d\theta + \frac{1}{2\pi} \int_{\sqrt{\rho}}^{\infty} \exp\left(-\frac{y^2}{2\sin^2 \theta}\right) d\theta,
\]  

(4)

where

\[
w_i = \tan^{-1}\left(\frac{1 - \rho^2 x_i / y_i}{(1 - \rho x_i / y_i)}\right)
\]  

and

\[
w_i = \tan^{-1}\left(\frac{1 - \rho^2 y_i / x_i}{(1 - \rho y_i / x_i)}\right).
\]  

(5)

The error probability \(P_{\text{err}}(E)\) in a flat fading channel can be obtained as
From [1], when a signal point $S_i^c$ that has a closed decision region $(R_{c,n})$ with $n$-sided polygonal shape is transmitted, we can find that the SER for the signal point $S_i^c$ is

$$P_{S_i^c} (E | γ) = 1 - P(S_{o,n} | S_i^c) \cdot P(S_{o,n} | S_i^c)$$

Similarly, the SER for a signal point $S_i^o$ that has an open decision region $(R_{o,n})$ with $q$ sides is

$$P_{S_i^o} (E | γ) = 1 - P(S_{o,q} | S_i^c) \cdot P(S_{o,q} | S_i^c)$$

In (12) and (13), $z_i = E[Y_i] / \sqrt{\text{Var}[Y_i]}$ and $Y_i$ is a random variable on the perpendicular axis to the decision boundary of the closed or open regions. Then, $Y_i$ and $Y_{i+1}$ have the joint Gaussian distribution with

$$E[Y_i] = \sqrt{E[(\beta \zeta, \cos \theta \sin (\psi_i + \phi_i) - \alpha \zeta, \sin \theta \cos \psi_i) - d_i]}$$

$$\text{Var}[Y_i] = \sigma^2 (\alpha^2 \sin^2 \theta + \beta^2 \cos^2 \theta - \alpha \sigma \sin \phi_i \sin (2 \theta))$$

$$\text{COV}(Y_i, Y_{i+1}) = \frac{\alpha \sin \theta \sin \theta_{i+1} + \beta \cos \theta \cos \theta_{i+1} - \alpha \beta \sin \phi_i \sin (\theta + \theta_{i+1})}{\sqrt{\text{Var}[Y_i] \text{Var}[Y_{i+1}]}},$$

where $\alpha$ and $\beta$ are the gains of filters or mixers which represent the amplitude unbalance; $\zeta$ is a scale factor which varies with the position of the signal point; $\psi_i$ is the phase of the transmitted signal; $d_i$ is a distance between the origin and the $i$-th decision boundary of the closed or open regions; $\theta$ is the slope of the $i$-th decision boundary, respectively [1].

By combining (12) and (13), the average SER of an arbitrary 2-D signaling with I/Q unbalances can be obtained [1]:
average symbol error probability of 16-star-QAM with I/Q unbalances over Nakagami-m fading channels is rewritten as

$$P_{\text{SER}, \text{fading}}(E) = \int_0^\infty P_{\beta}(E|\gamma)p_\beta(\gamma)d\gamma$$

where

$$P_{\beta}(E|\gamma) = \sum_{j=1}^{V} \left[ \sum_{i=1}^{M} \left( I[\pi, z_i] - I[w_{z_i, z_{i+1}}, z_i] + I[w_{z_i, z_{i+1}}, z_{i+1}] \right) \right]$$

and

$$P_{\beta}(E|\gamma) = \sum_{j=1}^{V} \left[ \sum_{i=1}^{M} \left( I[\pi, z_i] - I[w_{z_i, z_{i+1}}, z_i] - I[w_{z_i, z_{i+1}}, z_{i+1}] \right) \right]$$

To compare the results in this paper with the results in previous researches [2] and [4], we assume 8-PSK and 16-star-QAM in Nakagami-m fading channel, and show the effect of I/Q unbalances on the error performance. For this end, the values of parameters, $\phi_\alpha = 0^\circ$, $\beta$ and $\alpha = 1, 1.1$, are used since typical values achievable with careful design are $\phi_\alpha = 5^\circ$ and $\alpha = 1.1 [10]$.

In Fig. 1 and Fig. 2, we have plotted the average SER for 8-PSK and 16-star-QAM with I/Q unbalances corresponding to several values of the fading parameter $m$. As shown in Fig. 1 and Fig. 2, the results for $\alpha = \beta = 1$ and $\phi_\alpha = 0^\circ$ in eq. (18) and (19) of this paper are exactly the same as the results of [4, Fig. 8.4] and [2, eq.(12)], respectively. Fig. 3 shows the effects of I/Q unbalances on the SER performance of MPSK, and in Fig 4, for $m = 1, \alpha = 1, \beta = 1.1$ and $\phi_\alpha = 5^\circ$ the SER of several 16-APSK modulation schemes is depicted.

As shown in Fig. 1 and Fig. 2, as $m$ increases, we can confirm that the severity of fading decreases, but the effect of I/Q unbalances on the SER performance becomes serious. And, in Fig. 3, we can see that the SER performances are more sensitive to the effect of I/Q unbalances according to increasing $M$. Also, we can observe that the 1+5+10 APSK outperform the other 16-APSK modulation schemes through Fig. 4. Consequently, for fading channel, one of the dominant causes of performance degradation is multi-path fading rather than I/Q unbalances of the components.

V. CONCLUSIONS

In this paper, we have provided a new closed-form expression involving the 2-D Gaussian Q-function for the SER of arbitrary 2-D signaling with I/Q unbalances over a Nakagami-m fading channel. The BER is also obtained from the provided result by using [11, eq. (14)]. We first transformed the 2-D joint Gaussian Q-function into Craig’s form. Then, using the MGF of the Nakagami-m distribution we provided the error probability of arbitrary 2-D signaling with I/Q unbalances over a Nakagami-m fading channel. Finally, from the provided result, we analyzed that one of the dominant causes of performance degradation is multi-path fading rather than I/Q unbalances of the components. The result can be readily applied to numerical evaluation for various cases of
practical interest involving unbalanced I/Q modulation systems operating in a wide range of fading environments.

REFERENCES


