Abstract—This paper proposes a spectrum shaping technique exploiting frequency clipping for frequency domain soft canceller with minimum mean square error (FD-SC/MMSE) based turbo equalization systems, of which aims to achieve high spectral efficiency and efficient energy transfer from a transmitter to a receiver, as well as to provide a novel spectrum division multiplexing strategy. In the proposed scheme, the water-filling strategy is modified to clip more spectrum components with lower signal to noise power ratio, and transmit power allocated to the clipped spectrum components are evenly distributed to the non-clipped components thereby concentrates transmit power on spectrum with higher channel gain. Furthermore, the proposed spectrum shaping technique is applied to user multiplexing in the frequency domain with higher spectral efficiency. Computer simulation including extrinsic information transfer (EXIT) analysis confirms that the proposed scheme is effective in enhancement of spectral efficiency with low power penalty.

I. INTRODUCTION

In broadband wireless access systems, compensation for frequency selective fading and efficient energy transfer from a transmitter to a receiver are the most important issues for enhancement of sum-rate capacity as well as the average user rate under transmit power limited conditions. Although orthogonal frequency division multiplexing (OFDM) has been extensively developed as a suitable wireless transmission technique under frequency selective fading conditions, single carrier broadband transmission is also becoming a realistic technique for broadband wireless access system due to development of frequency domain equalizer (FDE).

In the linear FDE, because its frequency transfer function is determined by the tradeoff between residual inter-symbol interference (ISI) after equalization and noise enhancement due to large weight multiplication at deeply faded frequency components, channel frequency transfer functions after equalization are still deviated. One of its solutions is pre-coding [1]-[3] in which residual ISI after equalization is reduced not by the receiver signal processing but by the transmitter encoding process. However, the pre-coding technique does not guarantee an efficient energy transfer from a transmitter to a receiver.

Theoretically, water-filling spectrum shaping [4] in which more energy is allocated to spectrum with higher channel gain and less or no energy is allocated to spectrum with lower channel gain, is optimum to promise the efficient energy transfer. However, it would cause extra ISI in the transmitter, thereby residual ISI after equalization is potentially enhanced in the case of linear FDE.

Frequency domain soft-canceller with minimum mean square error (FD-SC/MMSE) based turbo equalizer [5] is another equalization technique suitable for broadband single carrier transmission. The most important features of the FD-SC/MMSE turbo equalizer is

- ISI after equalization is gradually suppressed in the iterative manner using a soft canceller, and if the turbo equalization process is successfully converged, residual ISI becomes negligible [6]-[9].
- All the effective desired signal components dispersed in the time domain can be efficiently combined by following matched filter combining basis even though the channel memory length is large [10].

These features suggest us that water-filling spectrum shaping is applicable when an FD-SC/MMSE turbo equalizer is equipped in the receiver, thereby requirements for broadband transmission, compensation of frequency selective fading and efficient energy transfer from a transmitter to a receiver, can be jointly satisfied. In this paper, no energy allocation to specific spectrum components will be called "clipping" in the following.

This spectrum shaping process is also considered as one of the pre-coding techniques. However, this concept can be considered as an approach contrary to the conventional pre-coding techniques aiming at suppressing residual ISI at the equalizer output in that it intentionally emphasizes ISI in the transmitter side for efficient energy transfer from a transmitter to a receiver.

In this paper, we will utilize such clipping process more actively to enhance spectral efficiency while minimizing degradation of power efficiency. In the proposed scheme, in addition to the frequency components clipped according to the water-filling process, a certain amount of spectrum components with lower channel gain is intentionally clipped to create vacancy in spectrum. With this process, spectral efficiency for a user is enhanced. For example, when 50% of spectrum is clipped, spectral efficiency for the user can be doubled. Moreover, if
is zero or an weighting factor for k-th spectrum component in the transmitted signal. In this process, the receiver should notify information about an optimum spectrum shape to the transmitter via feedback channels.

Assuming data symbol energy is normalized as one, the covariance matrix after spectrum shaping $\mathbf{K}_u$ is given by

$$
\mathbf{K}_u = \mathbb{E}\{\mathbf{M}_u s_u^2 \mathbf{M}_u^H\} = \mathbf{M}_u^2
$$

where $\mathbb{E}\{\cdot\}$ and $^H$ denote expectation and conjugate transpose, respectively. Thus, the total transmitted energy $P_u$ is given by

$$
P_u = \text{tr}(\mathbf{K}_u) = \sum_{k=1}^{K} m_u^2(k)
$$

where $\text{tr}[X]$ denotes summation of all elements in a diagonal matrix $X$. In this paper, transmitted energy per symbol $E_s$ is given by $P_u/K$.

Using Eqs. (1) and (4), the received signal energy to noise density ratio $(E_s/N_0)$ matrix $\rho_u$ is expressed as

$$
\rho_u = \frac{\mathbb{E}_u \mathbf{M}_u^2 \mathbf{\Xi}_u}{N_0} = \text{diag}\left[ \frac{m_u^2(1)}{N_0} \mathbf{\Xi}_u(1), \frac{m_u^2(2)}{N_0} \mathbf{\Xi}_u(2), \cdots, \frac{m_u^2(K)}{N_0} \mathbf{\Xi}_u(K) \right]
$$

where $^*$ denotes conjugate, and $\mathbf{\Xi}_u(k)$ is the complex channel weight in frequency domain representation at a discrete frequency position $k$.

Figure 1 shows an example of the proposed spectrum shaping process in the case of two users. In the first step, each user independently exploits its optimum weight to be multiplied to each spectrum component based on water-filling theorem as shown in Fig. 1 (a). The optimum $m_u(k)$ for maximization of $\text{tr}[\rho_u]$ can be obtained using Lagrange’s method of undetermined multipliers as

$$
m_u(k) = \left( \xi - \frac{N_0}{\mathbb{E}_u(k) \mathbf{\Xi}_u(k)} \right)^+
$$

where $(x)^+$ denotes the positive part of $x$ otherwise zero, and $\xi$ is decided in power constraint given by Eq. (4).

In the second step, more number of spectrum components having smaller $m_u(k)$ are clipped. In other words, only a certain number of spectrum components are reserved and the others are clipped as shown in Fig. 1 (b). In this process, the BS alternately selects a spectrum component with higher channel gain for each user among non-selected spectrum components, where clipping rate for each user is assumed to be preliminarily determined. For example, when clipping rate is 0.25, 75% of spectrum components in the assigned bandwidth are allocated to each user. In the case of Fig. 1, clipping rate is 0.5. Due to independency of channel characteristics for each user, user diversity effect can be expected in the spectrum
component selection process. That is, spectrum components with relatively higher channel gain tend to be allocated to each user. Finally, transmitted energy, which is allocated to the clipped spectrum components, is evenly distributed to the selected spectrum components so that total transmitted power given by Eq. (4) is equal to preliminarily determined level. As the result, spectrum shaping matrix \( M_u \) to be applied to each user is obtained as shown in Fig. 1 (c).

B. Transmission process

Figure 2 shows a block diagram of transmitter and receiver. In the transmitter, information data for each user is independently encoded to construct a coded bit sequence of length \( 2K \), interleaved and mapped onto constellation points, according to Gray encoding of quadrature phase shift keying (QPSK). Consequently, the \( u \)-th user’s data symbol vector \( s_u \) having data symbol length \( K \) is generated. Then an fast fourier transform (FFT) is employed to transform the data symbol into the original spectrum \( s_u' \). After multiplying \( M_u \) and \( s_u' \), a partially clipped transmitted spectrum \( \chi_u \) is generated at the spectrum shaping module. After \( \chi_u \) is transformed back into the time domain using an IFFT, a length \( P \)-symbol cyclic prefix (CP) is appended to the head of the transmitted vector to allow frequency domain discrete signal processing at the receiver. As a result, the transmitted spectrum for each user is multiplied over wireless channels.

Figure 3 illustrates the signal processing at the receiver. After CP in the frame is removed, an FFT is employed to transform the received symbol vector \( r \) into the received spectrum vector \( r^f \) depicted in Fig. 3 (a).

Let, \( r^f \) be rewritten as
\[
\begin{align*}
\mathbf{r}^f &= \sum_{u=1}^{U} \mathbf{\Theta}_u \mathbf{s}_u^f + \mathbf{v}^f \\
&= \mathbf{M}_u \mathbf{s}_u^f
\end{align*}
\]

where \( \mathbf{M}_u \) denotes an equivalent complex channel matrix including the effect of spectrum shaping matrix. Then, a received spectrum \( \mathbf{r}_u^f \) illustrated in Fig. 3 (b) is extracted from \( \mathbf{r}^f \), which is defined as
\[
\mathbf{r}_u^f = \mathbf{M}_u \mathbf{s}_u^f
\]

where \( \mathbf{M}_u \) denotes an \( K \times K \) diagonal matrix which aims to extract the \( u \)-th user’s partial spectrum. For example, \( \mathbf{M}_u[i, i] \) is one if \( i \)-th spectrum component is used, otherwise zero.

Then, FD-SC/MMSE turbo equalization is adopted to the received partial spectrum \( \mathbf{r}_u^f \). As described in Section I, turbo equalization is capable of suppressing ISI completely in the case where the behavior of the iterative detection is converged. Therefore, the original spectrum \( \mathbf{s}_u \) can be derived by using turbo equalization as shown in Fig. 3 (c). In other words, turbo equalization can also regenerate the clipped spectrum components. Note that any other modification of FD-SC/MMSE algorithm is not required except that the channel matrix \( \mathbf{\Xi}_u \) is replaced by \( \mathbf{\Theta}_u \).

Hereinafter, the advantage of the proposed spectrum shaping is discussed from the standpoint of the frequency domain MMSE filter output. When \( E_s \) is set at one, the frequency domain MMSE filter output vector \( \mathbf{z}_u = [z_u(1), z_u(2), \cdots, z_u(K)]^T \) is obtained as
\[
\mathbf{z}_u = (1 + \gamma_u \Theta_u)^{-1} \left[ \mathbf{\gamma}_u \mathbf{\hat{s}}_u + \mathbf{F}_u \mathbf{\hat{s}}_f \right] \tag{9}
\]

where
\[
\mathbf{\Phi}_u = \Theta_u^H \left( \Theta_u \mathbf{\Lambda}_u \Theta_u^H + N_0 \mathbf{I}_K \right)^{-1} \tag{10}
\]
\[
\gamma_u = \frac{1}{K} \text{tr} \{ \mathbf{\Phi}_u \mathbf{\Theta}_u \} \tag{11}
\]
\[
\mathbf{\Lambda}_u = (1 - \delta_u) \mathbf{I}_K \tag{12}
\]

and
\[
\delta_u = \frac{1}{K} \sum_{k=1}^{K} |\hat{s}_u(k)|^2. \tag{13}
\]

The vector \( \mathbf{\hat{r}}_u^f = [\hat{r}_u(1), \hat{r}_u(2), \cdots, \hat{r}_u(K)]^T \) is referred to as an interference residual vector, which is given by
\[
\mathbf{\hat{r}}_u^f = \mathbf{r}_u^f - \mathbf{\Theta}_u \mathbf{\hat{s}}_u^f \tag{14}
\]

\( \mathbf{\hat{s}}_u^f = [\hat{s}_u^f(1), \hat{s}_u^f(2), \cdots, \hat{s}_u^f(K)]^T \) denotes an expected spectrum vector derived by a priori log-likelihood ratio (LLR) [11] fed back from the soft-input soft-output (SISO) channel decoder.
TABLE I
SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Modulation</th>
<th>QPSK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel coding</td>
<td>Convolutional Code</td>
</tr>
<tr>
<td>Constraint length</td>
<td>4</td>
</tr>
<tr>
<td>Coding rate</td>
<td>1/2</td>
</tr>
<tr>
<td>Cyclic prefix length :</td>
<td>64 symbols</td>
</tr>
<tr>
<td>Data symbols length :</td>
<td>2048 symbols</td>
</tr>
<tr>
<td>Coded bit length :</td>
<td>4096 bits</td>
</tr>
<tr>
<td>Interleaver</td>
<td>Random</td>
</tr>
<tr>
<td>Num. of antennas</td>
<td>Tx : 1, Rx : 1</td>
</tr>
<tr>
<td>Symbol rate</td>
<td>20 Msymbols / sec</td>
</tr>
<tr>
<td>Channel model</td>
<td>24-path exponentially decaying Rayleigh model</td>
</tr>
<tr>
<td>Channel estimation</td>
<td>Perfect</td>
</tr>
<tr>
<td>Feedback</td>
<td>Perfect</td>
</tr>
<tr>
<td>Equalizer</td>
<td>Frequency domain SC/MMSE</td>
</tr>
<tr>
<td>Decoder</td>
<td>Max/Log-MAP</td>
</tr>
<tr>
<td>w/ shaping</td>
<td></td>
</tr>
</tbody>
</table>

[13], which is expressed as
\[
\hat{\delta}_u = F\delta_u
\]
\[
\hat{\delta}_u = F[\delta_u(1), \delta_u(2), \cdots, \delta_u(K)]^T
\]  
(15)
where
\[
\delta_u(k) = \mathbb{E}\{z_u(k)\}.
\]  
(16)
Note that \(\delta_u(k)\) in Eq. (16) can be obtained by following Refs. [11], [14]. Assuming that the MMSE filter output follows a Gaussian distribution, the equalizer output can be rewritten as
\[
z_u(k) = \mu_u s_u(k) + \psi_u
\]  
(17)
where \(\mu_u\) and \(\psi_u\) denote equivalent amplitude level and zero mean independent complex Gaussian noise with variance \(N_u\), and \(\mu_u\) and \(N_u\) are given by
\[
\mu_u = (1 + \gamma_u \delta_u)^{-1} \gamma_u.
\]  
(18)
\[
N_u = \mu_u - \mu_u^2.
\]  
(19)
Therefore, the averaged signal to noise power rate (SNR) after MMSE filtering \(\Upsilon_u\) are expressed by
\[
\Upsilon_u = \frac{\mu_u^2}{N_u} = \frac{\mu_u}{1 - \mu_u}.
\]  
(20)

Let us assume the two specific cases. The first one is the case when the turbo algorithm is successfully converged, which corresponds to the case of \(\delta_u = 1\). The second one is the case for the first iteration. Since there are no a priori information, this corresponds to the case of \(\delta_u = 0\). The first and second cases are hereinafter called end-point and start-point, respectively. In these two cases, substituting the value of \(\delta_u\) to Eq. (18), \(\Upsilon_u\) given by Eq. (20) is expressed as
\[
\Upsilon_u = \left\{ \begin{array}{ll}
\gamma_u & (\delta_u = 1 \text{ : end-point}) \\
\frac{1}{\gamma_u} & (\delta_u = 0 \text{: start-point}).
\end{array} \right.
\]  
(21)

Figure 4 shows that \(\Upsilon_u\) at the cumulative distribution function (C.D.F.) of one percent for both end-point and the start-point cases as a function of average transmitted \(E_s/N_0\). \(\alpha\) denotes the frequency clipping rate. Parameters for this evaluation is summarized in Tab. 1. Note that parameters for all simulation results through this paper are based on Tab. 1. At the end-point, \(\Upsilon_u\) with clipping rate \(\alpha = 0.25\) and \(0.5\) achieve higher SNR at the FD-SC/MMSE filter output compared to the performance without the spectrum shaping. The performances without the spectrum shaping is, hereinafter, called the conventional. In contrast, \(\Upsilon_u\) at the start-point case becomes lower. These phenomena indicate that, although the proposed clipping technique lowers equalizer output SNR at the start-point due to ISI by the clipping, it is effective in enhancement of the SNR at the end-point compared to the non spectrum shaping scheme. This tendency can be confirmed at any value of average transmitted \(E_s/N_0\).

III. EXIT ANALYSIS

In this section, EXIT analysis is conducted to verify some advantages of the proposed spectrum shaping. The mutual information (MI) \(I\) between the coded bits \(S \in \pm 1\) with equiprobable occurrence and LLR \(x\) is given by
\[
I = 1 - \int_{-\infty}^{\infty} p_{\mathbb{H}(x|S)} \log_2(1 + e^{-x}) dx
\]  
(22)
where \(p_{\mathbb{H}(x|S)}\) is the probability density function (PDF) of LLR being \(x\) conditioned upon the coded bit \(b\).

Both EXIT properties for the equalizer output and for the SfiSfo channel decoder output are depicted in Fig. 5. The average transmitted \(E_s/N_0\) is set to 7 dB. Let \(I_{\text{Eq}}\) and \(I_{\text{De}}\) be the MI for the equalizer output and decoder output extrinsic LLRs, respectively. The SfiSfo decoder output MI \(I_{\text{De}}\) is calculated by substituting the measured extrinsic LLR histograms to Eq. (22).
As shown in semi-analytical calculation of \( I_{\text{Eq}}^{E} \) in Ref. [15], as long as the equalizer output can be regarded to be subject to a Gaussian random process, the equalizer output MI \( I_{\text{Eq}}^{E} \) can be approximately given by the J-function [16] without LLR measurements as following

\[
I_{\text{Eq}}^{E} = J\left(\frac{1}{\Theta_u}\right) \approx \left(1 - 2^{H_1\left(\frac{1}{\Theta_u}\right)}\right)^{H_1}, \tag{23}
\]

The mapping-specific parameters are \( H_1 = 0.3073, H_2 = 0.8935 \) and \( H_3 = 1.1064 \) that are obtained by least-squared curve fitting [16]. Note that \( \Theta_u \) depends on \( I_{\text{De}}^{E} \), and the relationship is expressed as \( \Theta_u = F(I_{\text{De}}^{E}, \Theta_u, E_s/N_0) \). From Eq. (23), \( I_{\text{Eq}}^{E} \) can be determined only by \( \Theta_u \) shown in Fig. 4. For example, when we focus the end-point with \( \alpha = 0.5 \) at averaged transmitted \( E_s/N_0 = 7 \) dB in Fig. 4, the filter output \( \Theta_u \) is approximately 4.5 dB. Substituting the value 4.5 dB into Eq. (23), the \( I_{\text{Eq}}^{E} \) becomes 0.8. This relationship can be seen at the end-point with \( \alpha = 0.5 \) in the Fig 5.

A remarkable point in Fig. 5 is the MI at the equalizer output for clipping employed cases (both \( \alpha = 0.25 \) and 0.5) is lower than the conventional case at the end-point, although is is lower than the conventional at the start-point. As a result, SNR at the SC/MMSE filter output can also be enhanced as shown in Fig. 4. The most attractive observation in Fig. 5 is that the EXIT curves for \( \alpha = 0.25 \) and 0.5 cross with the curve for decoder at \( I_{\text{De}}^{E} \approx 1 \). This fact indicates that FER is negligibly low since almost perfect knowledge about transmitted information data can be obtained at the decoder. From a different perspective, EXIT curve for the conventional intersects with curve for the decoder at \( I_{\text{De}}^{E} \approx 0.7 \). This means that FER is not 0 since the perfect knowledge cannot be obtained by the iterative detection.

### IV. FER Performances

#### A. Single User Case (\( U = 1 \))

The FER performances of the proposed spectrum shaping under single user environment (\( U = 1 \)) is assessed by computer simulation. Figure 6 shows FER performances with different clipping rates (\( \alpha = 0.25, 0.5 \)) and the conventional as a function of the average transmitted \( E_s/N_0 \). At the \( FER = 10^{-3} \), both \( \alpha = 0.25 \) and 0.5 cases achieve 2.0 dB gain compared to the conventional. In contrast, FDE without iterative detection deteriorates the FER performances for \( \alpha = 0.25 \) and 0.5.

As discussed in section III, cross points of EXIT curves for \( \alpha = 0.5 \) and 0.25 is located at \( I_{\text{De}}^{E} \approx 2 \) at transmitted \( E_s/N_0 = 7 \) dB although those for the conventional is located at \( I_{\text{De}}^{E} \approx 0.7 \). As a result, FER of the spectrum shaping introduced scheme is lower than the conventional. In the case of FDE, the FER performance depends on \( I_{\text{Eq}}^{E} \) at the start-point since no knowledge about transmitted information data is fed back from the decoder. Therefore, FER performances with spectrum shaping is getting worse with higher clipping rate.

#### B. Multi User Case (\( U = 2 \))

The FER performances of a spectrum division multiple access exploiting the proposed spectrum shaping in multiuser environment (\( U = 2 \)) are shown in Fig. 7 as functions of average transmitted \( E_s/N_0 \). Frequency clipping rate is set at \( \alpha = 0.5 \), and performances for the conventional (\( U = 1 \)) discussed in the previous section are depicted in same figure.

As can be seen in the figure, degradation of the FER performance caused by the multiplexing is less than 1 dB compared to the conventional scheme although the spectral efficiency in terms of bit/s/Hz for the proposed scheme is doubled compared to the conventional scheme, because half of the spectrum in each user is clipped. This degradation is due to sub-optimality of spectrum shaping. As explained in section II, the BS allocates available spectrum components for each user.
Therefore, each user cannot select optimal frequency positions. However, 1 dB degradation can be considered to be acceptable because spectrum efficiency can be doubled with only 1 dB of power penalty. When 16QAM is applied instead of QPSK to double spectral efficiency, much larger power penalty is necessary. Thus, the proposed spectrum shaping technique is considered to be effective in enhancement of spectral efficiency in SISO transmission cases.

V. CONCLUSIONS

A spectrum shaping scheme exploiting frequency clipping for turbo equalization system has been proposed in this paper. In the proposed scheme, the transmitted spectrum components with lower channel gain is intentionally clipped to concentrate transmit power on spectrum with higher channel gain, and ISI caused by spectrum shaping in addition to the channel is compensated for using a FD-SC/MMSE turbo equalizer. Computer simulation confirms that the proposed scheme provides high spectrum efficiency with low power penalty.

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REFERENCES


APPENDIX

System Model

In this paper, the channel model is assumed that uncorrelated frequency selective SISO system with U users. At the u-th user’s transmitter, two independent binary coded sequences are input to a QPSK modulator, where the I- and Q-channels are independently modulated by two binary sequences. The channel is comprised of L independent fading path components that are separated in time by the symbol duration. The received signal is sampled once per symbol. The signal received by the received antenna can be expressed as

\[
r = \sum_{u=1}^{U} H_u F_u \tilde{M}_u F_s u + v, \quad (n = 1, 2, \cdots, U)
\]

where \( s_u = [s_u(1), s_u(2), \cdots, s_u(K)]^T \) and \( v = [v(1), v(2), \cdots, v(K)]^T \) denote the u-th user’s transmitted signal vector and the additive white Gaussian noise (AWGN) vector, whose element \( v(k) \) is independent random Gaussian variable with zero mean and variance \( N_0 \). \( F \) is \( K \times K \) discrete Fourier transform (DFT) matrix defined by

\[
F = \frac{1}{\sqrt{K}} \begin{bmatrix}
W(0)(0) & W(0)(1) & \cdots & W(0)(K-1) \\
W(1)(0) & W(1)(1) & \cdots & W(1)(K-1) \\
\vdots & \vdots & \ddots & \vdots \\
W(K-1)(0) & W(K-1)(1) & \cdots & W(K-1)(K-1)
\end{bmatrix}
\]

in which \( W \) denotes a twiddle factor given by \( W = \exp(-j2\pi/k) \). Let a P-length CP be appended to the head of the transmitted
vector, the $K \times K$ channel matrix $H_u$, which is constant over
the frame, is given by

$$H_u = [h_1, h_2, \cdots, h_K]$$

where $H_u$ is a circulant matrix based on a column vectors
given by

$$h_k = [h(1), h(2), \cdots, h(L-1), 0_{K-L}]^T.$$  \hfill (27)

Note that $0_x$ denotes all-zero vector with length $x$.

The transmitted spectrum and noise spectrum vector is given
by

$$s^f_u = Fs_u = [s_u^f(1), s_u^f(2), \cdots, s_u^f(K)]^T,$$  \hfill (28)

$$v^f = Fv = [v^f(1), v^f(2), \cdots, v^f(K)]^T.$$  \hfill (29)

Therefore, the received spectrum vector can be expressed as

$$r^f = Fr$$

where $H_uF^H M_u F s_u + Fv$

$$= \sum_{u=1}^{U} \Xi_u \chi_u + v^f$$

$$= [r^f(1), r^f(2), \cdots, r^f(K)]^T$$  \hfill (30)

where, $\chi_u$ denotes the shaped spectrum of $u$-th user, is given
by

$$\chi^f_u = M_u s_u^f = [\chi_u(1), \chi_u(2), \cdots, \chi_u(K)]^T.$$  \hfill (31)

Moreover, $\Xi_u$ denotes the $K \times K$ frequency domain channel
matrix, is given by

$$\Xi_u = FH_u F^H = \text{diag} [\Xi_u(1), \Xi_u(2), \cdots, \Xi_u(K)]$$  \hfill (32)

where $\Xi_u(k)$ is the complex channel weight of the frequency
domain at symbol timing $k$. 