MIMO Relay Channels with Partial Channel Knowledge/Estimation Error and Spatial Correlation

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Abstract— **In this paper we show the effect of partial channel knowledge at the receiver which causes estimation error of the channel at the receiver and the spatial correlation on MIMO Relay wireless communication systems. We assume that the transmitter has perfect knowledge of statistical parameters of the channel such as covariance or mean. We show how the topology of the equipment can upgrade or degrade the channel and we generalize a relaying MIMO scenario. We mathematically approach the gain of a system that uses relaying antennas. We assume a Zero Mean Circular Symmetric Complex Gaussian (ZMCSCG) channel elements and ZMCSCG noise elements. As we examine spatial correlation at the transmitter only, we assume that the elements of the rows of the channel matrix of the first hop are correlated. Partial CSI at the receiver causes errors at the estimation of the channel at the receiver which results the channel capacity to degrade and to saturate at certain levels of SNR. Numerical simulations are conducted in order to support the theoretical results used in this paper.**

I. Introduction

Relay channels are going to play a significant role in the future of wireless systems. Relays increase the dimensionality of a channel at the same time as it increases the data rates of a channel under certain circumstances. Relay channels can be shown useful in rocky or urban environments where obstacles block or dramatically attenuate the propagation of the signals. Under the traditional SISO (single-input single-output) system configuration, the relay may not always be able to improve the system's performance but it does increase the dimensionality of the system. On the other hand, it is shown that relays can significantly improve MIMO (multiple-input multiple-output) systems but still under certain conditions.

It is shown in [1] and [2] that multiple antennas systems are able to achieve extremely good performances when compared with the traditional single antennas systems. Furthermore it is shown that the use of a relay in a demodulate-and-forward scheme, seems to improve a system's performance.

Channel capacity is been the basic element for the evaluation of the performance of a system. The capacity of system is greatly effected by a number of different factors. First of all, the performance of a MIMO channel is vulnerable to the spatial correlation of the antenna elements of each node within the system. Spatial correlation is the correlation between the antenna elements of a node in space. The smaller the distance is between the antenna elements of a node, the larger is the effect on the capacity of the channel. Another element that plays an important role in the performance of a system is the channel knowledge at each node of the system. If the transmitter has partial or no knowledge of the statistics of the channel it is impossible to optimally allocate the available power over the transmit antenna elements. But the situation becomes worse when the receiver has partial knowledge of the statistics of the channel. If that is the case, errors appear in the estimation of the channel at the receiver.

A continues knowledge of the CSI at the transmitter is not possible to be achieved but it is shown in [3] that partial channel information can be enough. This information is provided by the receiver by the means of feedback. But often we do not want to waste bandwidth for feedback purposes. On the other hand it is important to investigate what happens if the receiver does not have statistical knowledge of the channel. In this paper we examine a situation where we have knowledge of the CSI statistical parameters of the channel such as covariance or mean at the transmitter but we have imperfect CSI at the receiver. We also examine the spatial correlation at the transmitter in order to produce and analyze some important issues in the field of mobile communications.

In this paper we show the circumstances under which a Relay node is advantageous in a wireless communication system. We assume that the receiver has partial knowledge of the statistics of the channel that causes errors at the estimation of the channel at the receiver, and also that the antenna elements of the transmitter are spatially correlated. We implement a scenario which compares the performances of a simple MIMO versus a Relay MIMO systems and we analyze the cases where Relay MIMO system is preferable to the simple MIMO system.

We assume a Zero Mean Circular Symmetric Complex Gaussian (ZMCSCG) channel elements and ZMCSCG noise elements. We assume the system operates in a uniform environment where noise is identical everywhere in the system. We assume spatial correlation at the transmitter only, thus the elements of the rows of the channel matrix are correlated for the simple MIMO system and only on the first hop of the Relay MIMO system.

In section II we describe the system model used in this paper along with some theoretical approach on the channel capacity as taken from [1] , [2] & [4], on the covariance information as shown in [3] & [5] and finally we approach a system design for the relaying system under the investigation of this paper. For simplicity we analyze a 2hops relaying MIMO system.

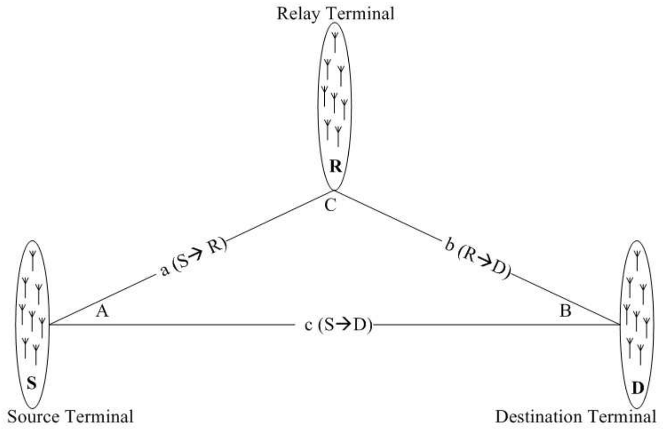

Fig. 1. Schematic representation of the Relay MIMO channel. MIMO antenna systems are placed in the corners of the triangle as shown in [4] and [6].

In section III we derive some numerical results based on our model and we extend our model's details up to the best of our knowledge. In section IV we present our simulation results taken from our simulation and finally in section V we discuss our conclusions derived from this study.

II. General System Model

A. MIMO Relay Systems

In this section we analyze the power distribution over the system's elements based on their geographic position only. We assume that the distances between the nodes of the system are within a range where the communication is achievable.

If a source terminal needs to transmit to a distant receiver, the signal can be forwarded to the destination terminal via intermediate nodes (relay terminals or relays). The use of the relays improves the throughput and the overall performance of the channel.

Obstacles such as mountains or high buildings often interrupt the transmission between two nodes. The signal transmitted attenuates dramatically and as a result, the signal cannot be analyzed at the receiver terminal. The use of the relay nodes, improve the throughput and the overall performance of the channel in this situation too.

Utilizing multiple antennas at the transmitter and/or the receiver of a wireless communication system can result in significant throughput and signal-to-noise ratio (SNR) gains. MIMO (multiple-input multiple-output) systems are shown to offer a significant capacity gain over the traditional SISO (single-input single- output) systems.

Consider the MIMO Relay system shown in Fig.1. Data is to be transmitted from the Source terminal (S) to the Destination terminal (D) through two different paths. In the first case, the signal will be transmitted directly from Source (S) to Destination (D) in one time slot. In the second case, the signal will be transmitted in two time slots: from Source to Relay terminal in one time slot and from Relay to Destination terminal in one time slot.

According to the schematic representation shown in Fig.1 we assume that the Source, the Relay and the Destination terminals are put in a way forming an isosceles triangle, with a $120°$ top angle ($\hat{C}$) and $30°$ side angles ($\hat{A}$ and $\hat{B}$). The SNR of the MIMO channel between the Source and the Destination terminals is given by:

$$SNR_1 = \frac{P_{r1}}{N} \tag{1}$$

$$P_{r1} = P_{t1} K c^{-\gamma} \tag{2}$$

Where P_t is the total power transmitted, P_r is the total power received, K is the total gains and losses of the system and γ is the pathloss index which normally varies between 2 and 6. Note that we assume a uniform environment and identical equipment is used everywhere in the system. That makes K to be a fix value in all the nodes of the system.

For the MIMO Relay system, it is assumed that the Source terminal uses half of the total power to transmit to the Relay terminal in the first time slot, and the Relay terminal uses the other half to transmit to the Destination terminal in the second time slot. So the SNR of each hop of the MIMO Relay channel is calculated by:

$$SNR_2 = \frac{P_{r2}}{N} \tag{3}$$

$$P_{t2} = \frac{P_{t1}}{2} \Longrightarrow P_{r2} = \frac{P_{t1}}{2} K a^{-\gamma} \tag{4}$$

By assuming that identical equipment is used, we compare the two SNRs.

$$\frac{SNR_2}{SNR_1} = \frac{\frac{1}{2} P_{t1} K a^{-\gamma}}{P_{t1} K c^{-\gamma}} \Longrightarrow \frac{SNR_2}{SNR_1} = \frac{1}{2}\left(\frac{a}{c}\right)^{-\gamma} \Longrightarrow \tag{5}$$

$$\frac{SNR_2}{SNR_1} = \frac{1}{2}\left(\frac{c}{a}\right)^{\gamma} \tag{6}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \tag{7}$$

but C is larger than $60°$, so $c^2 \geq a^2$ therefore the ratio $\frac{c}{a}$ is a gain to the system as long as angle C is larger than $60°$.

Since the triangle formed by the 3 nodes of our system is an isosceles triangle, side a is equal to side b. In this case, equation (7) becomes:

$$c^2 = a^2 + a^2 - 2a^2 \cos \hat{C} \Rightarrow c^2 = 2a^2 + a^2 = 3a^2 \Rightarrow$$

$$c = a\sqrt{3} \tag{8}$$

From equations (6 & 8) we get:

$$\frac{SNR_2}{SNR_1} = \frac{1}{2}\left(\frac{a\sqrt{3}}{a}\right)^{\gamma} \Rightarrow \frac{SNR_2}{SNR_1} = \frac{1}{2}\left(\sqrt{3}\right)^{\gamma} \tag{9}$$

For a $\gamma = 4$

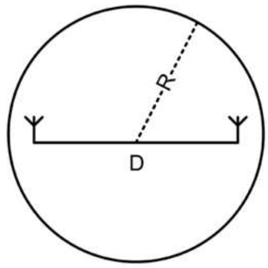

Fig. 2. D is the distance between the transmit and receive sides, R is the radius of the scatter ring and $\Delta \approx \arcsin(R/D)$

$$\frac{SNR_2}{SNR_1} = \frac{1}{2}\left(\sqrt{3}\right)^4 = 4.5 \tag{10}$$

Since double resources are used and the transmission is to be completed in two time slots and two hops, the portion of the total SNR that will be allocated to each hop for each time slot is going to be given by:

$$\frac{SNR_2}{SNR_1} = \frac{4.5}{2} = 2.25 \tag{11}$$

The results shown in equations (10) and (11) are going to be link with the capacity performance of the channel later in this paper.

B. Covariance Information

Under covariance information we assume that all the elements of each row of channel matrix H are correlated and follow the model as shown in [2] and [3] such that

$$\hat{H} = R_r^{\frac{1}{2}} \hat{H}_\omega R_t^{\frac{1}{2}} \tag{12}$$

where R_t and R_r are correlation matrices of sizes $t \times t$ and $r \times r$ respectively.

By $\hat{H}_\omega$ is denoted the white channel matrix detected in error at the receiver and by $\hat{H}$ is denoted the channel matrix of the spatially correlated system.

Based on the model shown in [7] the entries of the transmit correlation matrix are given by

$$R_t(k,l) = J_o\left(2\pi\Delta|k-l|\frac{d_t}{\lambda_c}\right) \tag{13}$$

where J_o is a zeroth-order Bessel function of the first kind and λ_c is the carrier wavelength of the system. The antenna arrays at the transmitter are spaced by a distance of d_t. Δ is defined in Fig.2.

Similarly the entries of the receiver correlation matrix are given by

$$R_r(k,l) = J_o\left(2\pi|k-l|\frac{d_r}{\lambda_c}\right) \tag{14}$$

The antenna arrays at the receiver are spaced by a distance of d_r.

The estimation of the channel matrix $\hat{H}$ at the receiver is shown in [5] to be given by

$$\hat{H} = H_\omega \times \sqrt{1-\sigma_e^2} \tag{15}$$

C. Channel Capacity

The channel capacity of a MIMO system is defined as the amount of information that can be reliably transmitted over a MIMO communication channel. It is shown by [1] and [8] the capacity lower bound of a MIMO system based on it mutual information lower bound is given by

$$C_{lo} = \max_{Q:tr(Q)\leq P}\left[log_2\left|I_r + \frac{\hat{H}Q\hat{H}^{\mathcal{H}}}{\sigma_n^2 + \sigma_e^2 tr(R_tQ)R_r}\right|\right] \tag{16}$$

In practice the mutual information is the instantaneous capacity of a system and cannot be indicated as the system's capacity. For that reason we assume the channel capacity to be the mean value of the mutual information or else the ergodic capacity of the system. Hence the channel capacity of the system in this paper is been determined by

$$C = \mathcal{E}\left[log_2\left|I_r + \frac{\hat{H}Q\hat{H}^{\mathcal{H}}}{\sigma_n^2 + \sigma_e^2 tr(R_tQ)R_r}\right|\right] \tag{17}$$

D. The model under investigation

The model under the investigation of this paper is the one schematically represented in Fig.1 under the assumptions stated below. We compare the performance of the systems ($S \rightarrow D$) and ($S \rightarrow R \rightarrow D$) and we discuss the SNR gain of the Relay MIMO system when compared to the direct simple MIMO system. We show the impact of the SNR gain as derived in equation (11) on the capacity of the two hops system when compared to that of the one direct path.

Throughout this paper we assume Rayleigh fading, no channel knowledge at the systems' transmitters and partial knowledge of the CSI at the receiver. The relay has knowledge of the channel statistical information of the first hop; from the source terminal to the relay terminal ($S \rightarrow R$) and no CSI from the relay to destination ($R \rightarrow D$) node. Error in the estimation of the channel is detected in the Receiver terminal (D) only. The Relay terminal (R) has full knowledge of the statistical parameters of the channel from Source (S) to the Relay (R).

The system capacity of the Relay MIMO channel ($S \rightarrow R \rightarrow D$) will be the worse of the capacities of the two hops ($S \rightarrow R$ and $R \rightarrow D$).

We assume that the noise statistics are common everywhere in the system and they do not change under any assumptions for the channel state information and channel estimation at the receiver.

III. Numerical Results

Since the model under investigation is spatially correlated at the transmitter, the correlation matrix of the receiver (R_r) becomes identity matrix. Under this condition the channel matrix estimation ($\hat{H}$) at the receiver becomes:

$$\hat{H} = \hat{H}_\omega R_t^{\frac{1}{2}} \tag{18}$$

Also, under the condition of the system being spatially correlated at the transmitter, the capacity formula becomes

$$C = \mathcal{E}\left[log_2\left|I_r + \frac{\hat{H}Q\hat{H}^{\mathcal{H}}}{\sigma_n^2 + \sigma_e^2 tr(R_t Q)}\right|\right] \qquad (19)$$

where the channel matrix $\hat{H}$ is given by (18).

For the comparison of the two systems, Relay MIMO and simple MIMO we use the SNR analysis shown in subsection A. For the simple MIMO system formed in the direct path ($S \rightarrow D$), we assume that the system operates with a total signal to noise ratio $SNR_{MIMO} = SNR_1$ while for the Relay MIMO system formed from the path ($S \rightarrow R \rightarrow D$) the total signal to noise ratio used for transmission in each hop is $SNR_{RelayMIMO} = SNR_2 = 2.25 \times SNR_1$ (11).

In order to show what is the effect of the error at the estimation of the system three different variances of error are to be used ($\sigma_e^2 = 0.1, 0.2\&0.5$).

IV. Simulation Results

For simulation purposes the Uniform transmission strategy is used for an SNR range $-10dB \rightarrow +20dB$. Uniform transmission strategy is the one where equal power is allocated to each of the transmit antenna elements.

Fig.3 shows the capacity-wise performance of the direct path system under different variances of error. A system where the receiver has a perfect estimation of the channel can reach capacities of up to 9 $nats/s/Hz$ at an SNR of 20dB. When a small error occurs at the estimation of the channel at the receiver capacity drops to about 5 $nats/s/Hz$ which is a great loss (almost 45%), while for a large error it can be seen that the maximum capacity a system can reach is about 2 $nats/s/Hz$ which is extremely lower than all the cases shown in this paper. In Fig.3 it is also shown how the error at the estimation of the channel at the receiver, causes the system's performance to saturate at certain levels of SNR. This means that even if we had unlimited power available, the capacity of the channel would not increase any further.

Fig.4 shows the capacity-wise performance of the relaying path under different variances of error. From this figure it can be said that a $4 \times 4 \times 4$ Relay MIMO system with perfect knowledge of the statistics of the channel at the receiver can achieve up to 11 $nats/s/Hz$. In the case of the Relay MIMO system too, an error at the estimation of the channel at the receiver can cause the system's performance to saturate at certain SNRs.

The comparison of the two figures shows how Relay MIMO system performs better than a simple MIMO system in general. Under the same conditions and the same environment it is obvious that the Relay MIMO system performs better. In the cases where the receiver has perfect knowledge of the channel we can see a 22% increment of the capacity. From 9 $nats/s/Hz$ at an SNR of 20dB of the simple MIMO system, the capacity reaches 11 $nats/s/Hz$ at the same SNR level.

The error at the estimation of the channel at the receiver, caused by the partial knowledge of the statistics of the channel

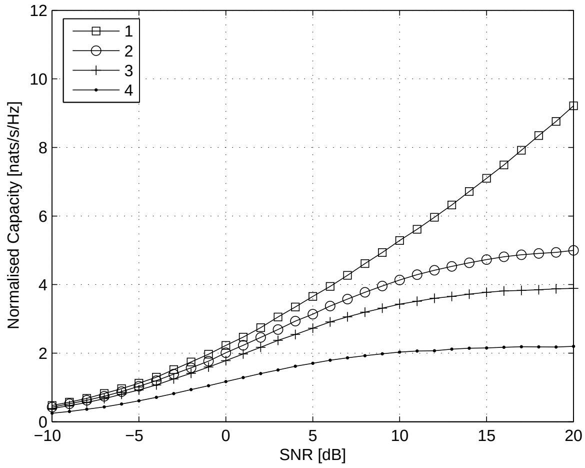

Fig. 3. Capacity lower bound for the 4x4 direct path MIMO system ($S \rightarrow D$) with Spatial Correlation at the transmitter with $d_t/\lambda_c = 0.1$
1. Error free system - Variance of error $\sigma_e^2 = 0.0$
2. Variance of error $\sigma_e^2 = 0.1$
3. Variance of error $\sigma_e^2 = 0.2$
4. Variance of error $\sigma_e^2 = 0.5$

at the receiver, forces the system to saturate at certain capacity levels in both cases. The Relay MIMO saturates sooner than the simple MIMO system which happens because the Relay MIMO system reaches the maximum capacity faster when compared to the simple MIMO system. In other words, maximum capacity can be achieved with less power spent.

For high SNRs (above 20dB) there is not a clear capacity-wise advantage of the Relay MIMO system as the performance is almost the same, and hence it would not be a cost effective solution to invest on extra nodes within the network. The larger the variance of error, the faster the system saturates. From the slopes of the lines on the figures it is clear that the rate of change for the Relay MIMO is much faster than that of the simple MIMO and that is what forces the Relay MIMO system to reach capacity of the system earlier.

The level of spatial correlation is regulated by the d_t/λ_c ratio as shown in section II.B. The closest the antenna elements are, the smaller this ratio is for a specific bandwidth. As long as the distance between the antenna elements of the transmitter increases, for a specific bandwidth, spatial correlation at the transmitter decreases.

A comparison of the simple MIMO versus the Relay MIMO system is indicated in Fig.5 for various d_t/λ_c ratios. It is shown that for a small d_t/λ_c ratio, the capacity achieved cannot exceed 5 $nats/s/Hz$ while for larger ratios, the capacity can reach up to 7 $nats/s/Hz$ which is a 40% increment in the capacity of the system. Fig.5 shows also that Relay MIMO system performs better than the simple MIMO system up to the point of saturation in the increment of the capacity. For the particular scenario, it can be said that, it is worthless to invest for extra infrastructure if the SNR of the system is above 15dB. In general, all the systems will saturate at some level of SNR as long as there is error at the estimation of

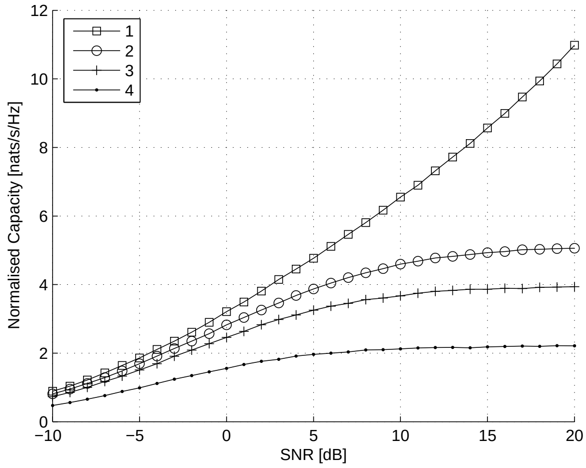

Fig. 4. Capacity lower bound for the 4x4x4 Relay MIMO system ($S \rightarrow R \rightarrow D$) with Spatial Correlation at the transmitter only with $d_t/\lambda_c = 0.1$
1. Error free system - Variance of error $\sigma_e^2 = 0.0$
2. Variance of error $\sigma_e^2 = 0.1$
3. Variance of error $\sigma_e^2 = 0.2$
4. Variance of error $\sigma_e^2 = 0.5$

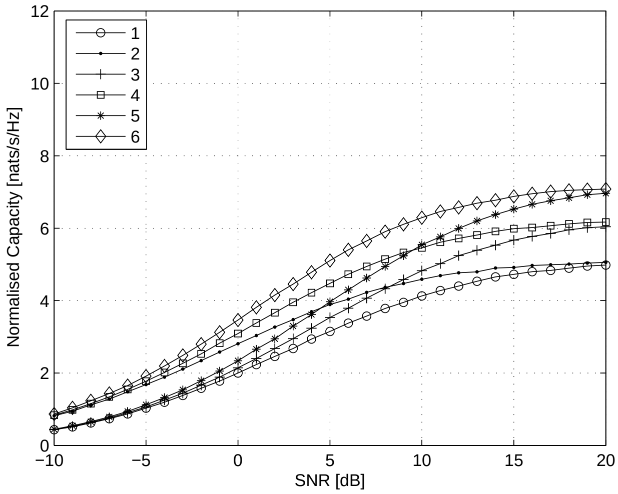

Fig. 5. Capacity lower bound for the direct path 4×4 MIMO system ($S \rightarrow D$) versus the $4 \times 4 \times 4$ Relay MIMO system ($S \rightarrow R \rightarrow D$) with Spatial Correlation at the transmitter and Variance of error $\sigma_e^2 = 0.1$.
1. Direct Path 4×4 MIMO system - $\frac{d_t}{\lambda_c} = 0.1$
2. Relaying $4 \times 4 \times 4$ Relay MIMO system - $\frac{d_t}{\lambda_c} = 0.1$
3. Direct Path 4×4 MIMO system - $\frac{d_t}{\lambda_c} = 0.5$
4. Relaying $4 \times 4 \times 4$ Relay MIMO system - $\frac{d_t}{\lambda_c} = 0.5$
5. Direct Path 4×4 MIMO system - $\frac{d_t}{\lambda_c} = 0.9$
6. Relaying $4 \times 4 \times 4$ Relay MIMO system - $\frac{d_t}{\lambda_c} = 0.9$

the channel at the receiver. For this reason the simple MIMO and the Relay MIMO systems will merge their capacity-wise performance at some SNRs. This logic can indicate when the extra infrastructure is a successful investment for a system operating at a particular SNR level.

V. Conclusions

In this paper we investigate how the use of relays in MIMO antenna systems can change the performance of a channel. The topology of the system also plays a significant role in the performance of the channel. To show the effect of the topology on the performance of a system, we implement a particular scenario of a relay node strategically positioned in such a way to form an isosceles triangle. Under this setup we examine the increment of the capacity of the Relay MIMO system when compared with the direct path simple MIMO system.

We also show how both simple MIMO and Relay MIMO systems saturate when the receiver of the system has only partial statistical knowledge of the channel. It is shown that partial statistical knowledge of the channel at the receiver can cause error at the estimation of the channel at the receiver and that implies the performance of the system to saturate as the SNR of the system increases.

Another important issue examined in this paper, is the effect of spatial correlation at the transmitter on the performance of the channel. It is shown that as the distance between the antenna elements of the transmitter decreases in space, the channel capacity decreases too.

A relay can offer significant improvement at the performance of the system at low SNRs while for higher SNRs, the improvement is negligible. The use of a relay in a system involves higher cost for equipment. That leads to a trade-off between cost and capacity. This trade-off can be stabilized since the Relay MIMO system can achieve capacity in lower SNRs which suggests lower power consumption but on the other hand for operation in higher SNRs, the use of relay is not a necessary investment.

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