Time-Frequency Effects in Microwave and Radio Frequency Electronics

Michael B. Steer , Gregory Mazzaro, Jonathan R. Wilkerson and Kevin G. Gard

North Carolina State University, Department of Electrical and Computer Engineering, Raleigh, NC 27695-7914 U.S.A,

***Abstract*— Co-site interference has been a vexing problem in communications since the origins of radio. The problem occurs when a radio operating at one frequency is jammed by a radio or set of radios operating at other frequencies or time slots. It is well understood that nonlinear mixing can result in the generation of spurious interfering signals but this has not been sufficient to describe all cases of interference. This paper examines long-tail effects as sources of communication system interference. One effect derives from passive intermodulation distortion resulting from electro-thermal effects. A second source of interference derives from long-tail responses of narrowband components. Both effects are described by fractional calculus and circuits having such responses can not be modeled adequately. It is for this reason that long-tail effects have not been suspects as a source of co-site interference.**

I. Introduction

Interesting phenomenology has been discovered recently which, it is believed, will lead to improved understanding of co-site interference and of the response of radio frequency hardware to time-limited radio frequency (RF) signals in general. Traditionally linear and nonlinear RF and microwave design and development have used steady-state conceptualizations, models and simulations. RF and microwave laboratories have also been oriented towards steady-state measurements, both linear and nonlinear. Nearly all specifications for RF systems are in terms of their frequency-domain characteristics which has driven both design and characterization technology. An example of the impact of frequency-domain based specification is the use of spectral regrowth measures with communication systems. Spectral regrowth is a measure that captures the amount of power in a communication channel that bleeds into adjacent channels because of nonlinear distortion. This paper explores time-frequency effects that result from pulsed RF and the interactions of multiple physics.

Time-domain measurements at RF and microwave frequencies have, until recently, relied on sampling oscilloscope-like instruments which in effect capture steady-state waveforms. In the last few years real-time scopes have become available so that the state of the art is 8-bit resolution, 40 megasamples per second, and with analog bandwidth up to 13 GHz. The 8-bit resolution is quite low but has enabled researchers to look at pulsed RF effects. In the past few years our laboratory has focused on developing understanding of the interaction of RF electronics with pulsed RF signals. The work has involved building high power RF hardware and consequently investigations of the fundamentals involved in designing high-power systems to have low levels of interference. We uncovered what first appeared to unexplained phenomena which have pointed to fundamental understanding of how pulsed signals interact with electronics.

II. Electro-Thermal Response

Recently electro-thermal effects have been identified as sources of nonlinear interference [1] resulting from the dynamics linking two different physical domains: the electrical world of circuits, and the temperature/flux relations in thermal physics. The inter-relationship is illustrated in Figure 1. Figure 1 is a plot of the voltage across a platinum resistor excited by a step current. The platinum resistor has a resistance that is proportional to temperature. Physically what is happening is that the resistor dissipates power, and there is a heat flux which is the square of the voltage divided by the resistance. The heat flux then results in a temperature rise which in turn increases the resistance of the platinum resistor. Consequently in the electrical domain the describing differential equation for voltage has a half-derivative and thus description of the circuit requires fractional calculus [2–5].

The thermo-resistance effect, and models the specific resistivity (Ω m) of a material as a function of temperature:

$$\rho(T) = \rho_o(1 + \alpha T + \beta T^2 + ...) . \qquad (1)$$

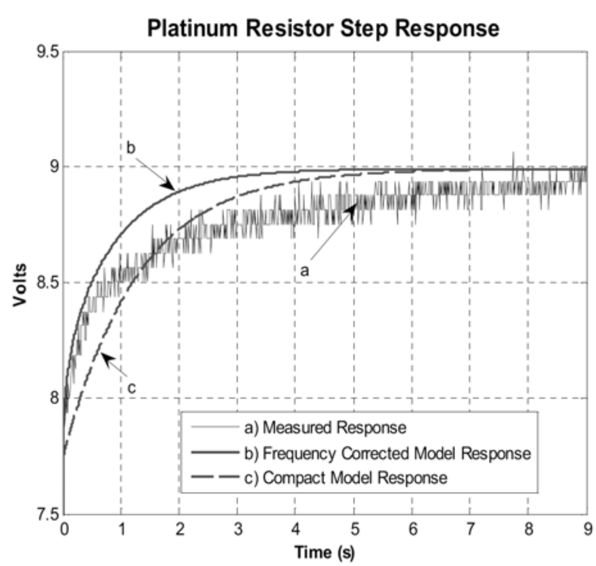

Figure 1. Time response of the voltage across a resistor driven by a current step.

The thermo-resistance effect, or TCR equation, couples the thermal system to the electrical domain. Coupling from the electrical to the thermal domain results from the heat generated by dissipated electrical power, which arises in the material as the electromagnetic field causes increased collisions with the metal lattice. In a resistive element this is termed self heating or joule heating. The generated heat per unit volume (W/m^{-3}) from self heating is defined as

$$Q = J^2 \rho , \qquad (2)$$

where J is the current density vector in A/m^{-2}. The heat produced directly drives the heat conduction equation which is given by

$$\nabla \cdot \left(\frac{\nabla T}{R_{th}} \right) - \rho c_v \frac{\delta T}{\delta t} = Q. \qquad (3)$$

The thermal resistance and capacity must also be known to determine temperature from generated heat for accurate calculation of voltage and current. Thermal resistance ($K°/W$) is defined as the temperature increase per watt according to the equation,

$$R_{th} = \frac{\Delta T}{P} = \frac{\Delta T}{I^2 R} \; . \qquad (4)$$

Although steady-state temperature can be calculated using the power dissipated in the device and its thermal resistance, the instantaneous temperature can not be calculated using these parameters. The finite time needed to transfer heat is captured by the thermal capacity, the combination of the ability of the material to store heat by raising its temperature and to conduct heat to its surrounding environment at a given rate. The thermal capacity can be expressed as

$$C_v = \left(\frac{\delta Q}{dT} \right)_v = T \left(\frac{\partial S}{\partial T} \right)_v \; . \qquad (5)$$

Here Q is heat and S is the entropy of the system.

The forcing function, in this case joule heating, can be substituted into the heat conduction equation in terms of corresponding electrical parameters to yield

$$\nabla \cdot \left(\frac{\nabla T}{R_{th}} \right) - \rho c_v \frac{\delta T}{\delta t} = J^2 \rho_0 \left(1 + \alpha T + \beta T^2 + ... \right). \qquad (6)$$

By inspection, it is evident this is a nonlinear system, even if only a linear dependence on temperature is considered. In practice, the first order coefficient of the TCR equation is several orders of magnitude larger than any higher order coefficient in most metals, leading to its dominance in the distortion spectrum of a resistive device.

Joule heating will result from any input signal to a RF system, including time harmonic signals, which directly drives the heat conduction equation. When a single carrier is applied to a resistive element, the electro-thermal processes responsible for modulating the device resistance will average out to provide a stable resistance because the thermal capacity cannot react quickly enough to the high frequency signal to significantly heat or cool the resistive material. The situation changes drastically when two or more signals are applied to the device.

In the simplest case, a two-tone input of equal amplitude can be applied to the system, resulting in a time varying signal envelope. The instantaneous power of this signal varies sinusoidally at the beat frequency of the two tone input to the device. If the beat frequency is within the bandwidth of the low pass filter created by the thermal resistance and capacity, periodic heating and cooling of the element will occur at baseband frequencies. The heating and cooling of the resistive element will periodically alter the resistance of the element, creating a passive mixer capable of facilitating intermodulation distortion generation by mixing the envelope frequencies at baseband back up to microwave frequencies.

The electro-thermal serves to demonstrate that fractional calculus is essential to understanding RF circuits operating at high power. Considering the heating of the resistor by pulsed RF will result in signals that will have rich spectra when the spectrum is evaluated over a realistic time window. Thus nonlinear intermodulation products that interfere with radios operating in adjacent channels will be generated

III. Filter Response to Pulsed RF

The electro-thermal situation above is an example how fractional calculus may be necessary to describe multiphysics phenomena. Similar phenomenology has also been seen with RF filters, more sophisticated electro-thermal effects, flicker noise, and passive intermodulation. It is becoming increasingly clear that fractional calculus and the solution of systems described by fractional calculus is key to the understanding of the response of systems excited by RF pulses as in nearly all modern communications systems.

The next stage in the investigation was investigating the response of RF filters to pulsed RF signals. An example of a time-frequency characteristic is shown in Figure 2. Figure 2(a) shows a switched RF tone applied to the input of an RF bandpass filter and Figure 2(c) shows the response with the same response simulated and measured. Analysis of the response, in Figure 2(d) shows apparent nonlinear distortion behavior from a system which is most definitely linear. What is happening can be described with the aid of Figure 2(b). The top of Figure 2(b) shows the transmission response of the filter and the bottom shows the group delay response. The frequency of the first part of the excitation signal is at 988 MHz, the frequency of the first dot. The second part of the excitation signal is at 988 MHz, the second dot. The initial concept was to exploit the different in group delay to combine the signals at the output of the filter. However the response does not correspond with this and we know that the group delay is a steady-state quantity and it cannot adequately describe what is happening with pulsed signals. The best way of thinking about this is to consider that the filter (in this case) is made up of three resonators and RF energy must pass from first the resonator at the input, to the intermediate resonator to the output resonator. Until steady-state is reached energy passes back forth among the resonators and takes a very long time to reach steady-state. This is another long-tail phenomenon resulting from an entirely different source from that of the lector-thermal effect described in the previous section.

While we were able to simulate this filter in a circuit simulator (using of course integer calculus) it took a very long time. A behavioral model (say for MATLAB) would require a fractional calculus solution to capture the response efficiently. Similar behavior can be seen on antennas, for example, indicating the co-site interference cannot be eliminated unless long-tail phenomena is taken into account. Throughout RF engineering similar effects can be observed.

The same diffusive phenomenology described above occurs with antennas (impacting co-site interference), transmission lines (carriers bouncing back-and-forth), and audio signals when illuminating a hollow object. It is apparent that these effects can be described by fractional calculus. Fractional calculus effects (in which the Laplacian variable, s, is not raised to an integer power) can be expected whenever diffusion occurs. Integer calculus has been central to electrical and audio engineering. We believe that the phenomenology requires fundamental re-thinking of electrical engineering at the extremes (high-power, co-site interference, and remote detection). It is not possible currently to simulate these effects adequately and experiments will guide the development of understanding, of mathematical models, and of exploitation of the phenomena in remote characterization.

The generation of intermodulation products is an artifact of windowed Fourier analysis of the time-windowed response but in any case the short term response is real and the generation of spectral regrowth is also real. The finite time for filters to respond should be accounted for in communication systems as it can result in inter-symbol interference in systems that use RF pulses as with most TDMA cellular systems and frequency hoping systems.

Another way of viewing this phenomenology is that the reduced order response of a linear system comprising coupled first-order differential equations is a fractional derivative.

IV. Co-Site Interference

Many radio operating in ad-hoc environments use frequency hoping techniques. This section presents preliminary results relating to interference of radios in a fast frequency hoping communication system. A frequency hoping system using the 20 MHz unregulated band at 900 MHz is being considered. The input RF filter has a bandwidth of 36 MHz. Figure 3 shows two pulsed RF sequences representing the signal captured by a receiver from two transmitters using two adjacent time slots but the same frequency channel. The signal received from the first transmitter at a higher level than that received by the second transmitter and exaggerates the effect of the long-tail response of the input RF filter.

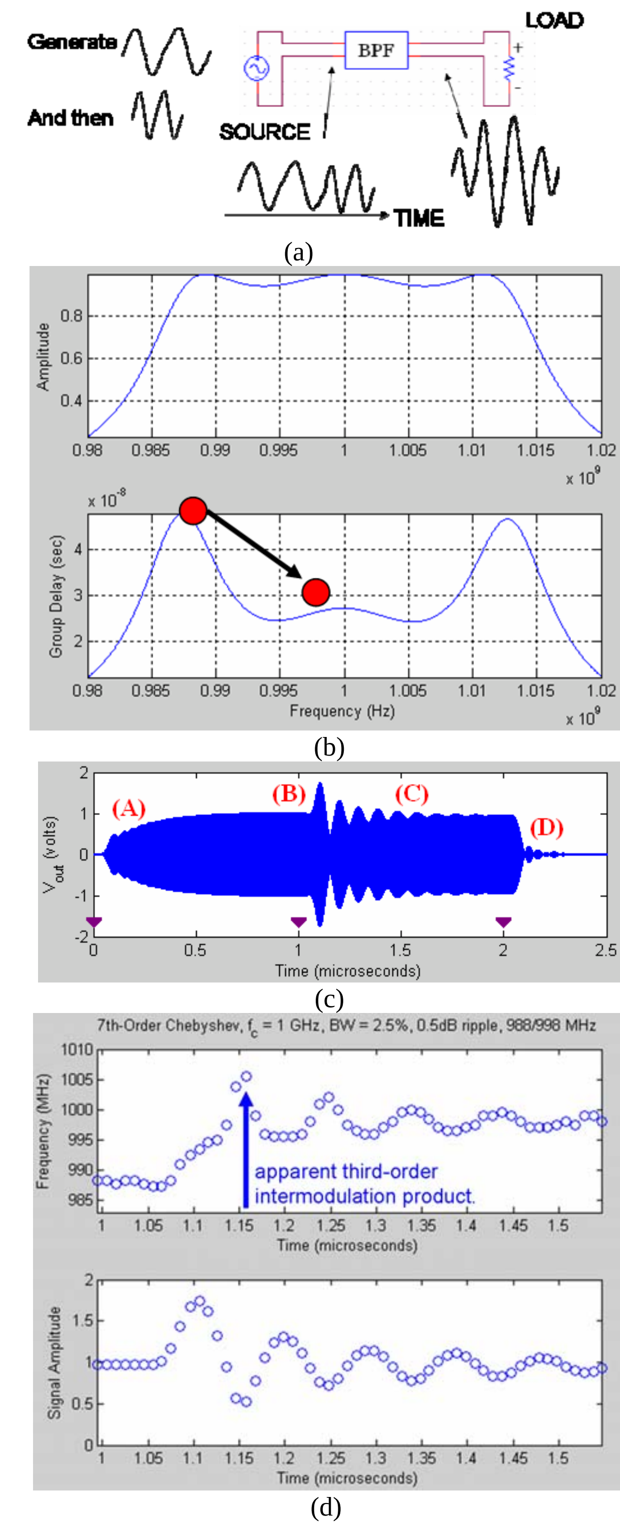

Figure 2. Response of a radio frequency filter: (a) bandpass filter with applied two-tone signal; (b) top is the transmission response and bottom is the group delay response; (c) the waveform measured at the output of the filter with 1,000 RF waveforms per microsecond so that only the envelope is shown; and (d) the time-varying spectra of (c) showing apparent nonlinear distortion components.

In Figure 3 the pulse of the first user starts at 0.1 μs and is turned off at 0.6 μs. The second user starts at 0.7 μs and is turned off at 1.2 μs. The guard band is 0.1 μs. The interference problem occurs as energy from the RF filter retains extended memory of the first user, the long-tail effect. The impact can be seen more closely by examining Figure 4 which is an expanded view of eth RF filter response of the first user showing considerable energy in the second time slot. This energy is comparable to the energy that is received from User 2 so that the SINAD (ratio of signal to interference, noise and distortion) for User 2 is 0 dB although User#1 is only 24 dB higher than User 1 and there is apparently ample guard-banding.

V. Conclusion

This paper reported the impact of long-tail effects on radio frequency communication hardware. The effect is pronounced and preliminary results indicate that it is a significant cause of co-site interference but clearly not the only cause. The work described here stresses the importance of using time-frequency characterizations in designing and experimentally characterizing microwave systems. The effects will not be seen using linear and nonlinear circuit analysis techniques that use steady-state methods such as the harmonic balance technique or shooting methods in Spice-like circuit simulations. One of the difficulties in using time-domain simulations is the difficulty of handling high Q, narrow bandwidth filters. Even with such capability, simulations of very long time duration and high dynamic range would be required to see the effect. Reduced order modeling of circuits having external long-tail responses requires simulation using fractional calculus. Currently there is not a viable technique for performing such simulations.

VI. Acknowledgement

This material is based upon work supported by the U.S. Army Research Office as a Multi-disciplinary University Research Initiative on Standoff Inverse Analysis and Manipulation of Electronic Systems under grant number W911NF-05-1-0337.

References

[1] J. R. Wilkerson, K. G. Gard and M. B. Steer, "Electro-thermal passive intermodulation distortion in microwave attenuators," *36 th European Microwave Conference*, Sept. 2006, pp. 157–160.

[2] K. B. Oldham and J. Spanier, *The Fractional Calculus*, Academic Press, New York, 1974.

[3] I. Podlubny, *Fractional Differential Equations*, Academic Press, New York, 1999.

[4] K. S. Miller and B. Ross, *Fractional Calculus and Fractional Differential Equations*, Wiley, New York, 1993.

[5] B.J. West, M. Bologna, and P. Grigolini, *Physics of Fractal Operators*, Springer, New York, 2003.

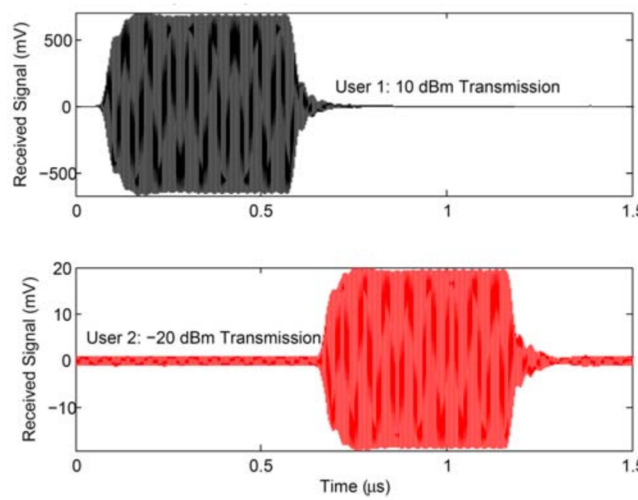

Figure 3. Received signal from two users operating in the same frequency hopping channels but adjacent time slices.

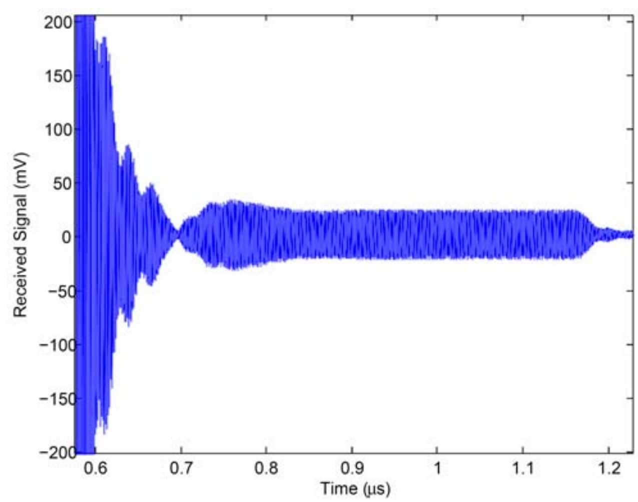

Figure 4. Close in detail of the residual long-tail response of the filter to the first user showing considerable power in time slot number 2.