Adaptive Modulation for MIMO Broadcast Channels

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***Abstract*— Adaptive modulation is a promising technique to increase the spectral efficiency of a wireless communication system. In this paper we investigate the effectiveness of adaptive modulation in maximizing the spectral efficiency of a MIMO multiuser downlink channel. The MIMO multiuser downlink transmission is carried out by the minimum-mean-squared-error feedback precoder. The overall spectral efficiency is maximized by adapting transmission power and/or transmission rate to the user channels. We show that there is a penalty associated with the use of MQAM and the penalty becomes more severe if only a finite set of discrete modulation sizes is provided to the transmitter.**

I. Introduction

The need for high-rate wireless communications has significantly increased over the past years as more and more subscribers demand more feature-rich contents to be delivered to their handsets. This has ignited the flare of research interests in the multiple-input multiple-output (MIMO) technologies. During its initial phase of development, the research in MIMO communications had been confined to the point-to-point link [1][2]. Later, the MIMO concept was extended to the multiple-access-channel (MAC) to provide additional diversity gain [3]. Until recently, in the information-theoretic work by Caire and Shamai [4], the set of achievable rates for the downlink MIMO broadcast channel (BC) has been characterized by applying the "dirty paper coding" technique. A practical realization of such a technique is the MMSE feedback precoder (MMSE-FBP) originally proposed in [5] and later optimized in [6]

The results from [5] and [6] also revealed one distinctive characteristic of multiuser transmission. It is statistically unlikely that all users within the coverage area of a base-station undergo the same level of channel disturbances in a given time period. The conventional way of combating the uncertainties in channel variations is to choose transmission parameters for the worst case scenario, thus leading to an uneconomical distribution of scarce resources. A more modern and efficient approach is to instruct the base-station to track the fluctuations in channel conditions and adjust transmission parameters accordingly. Adaptive modulation thus came into play and gradually became an emerging standard for future wireless communication systems. Adaptive modulation has been extensively studied for single-input single-output channel [7] and MIMO point-to-point channel [9]. It has been shown, in both cases, that a greater throughput can be attained by adapting transmission power and modulation scheme to channel variations. The contribution of this paper is the extension of adaptive modulation technique in the multiuser scenario, in particular the MIMO BC.

The remainder of this paper is organized as follows. Section II covers the system model and a brief summary of the MMSE-FBP. The formulation of adaptive modulation problem is presented at the end of Section II. The main contribution of this paper is presented in Section III, where we study adaptive modulation under different system constraints. The discrete version is included in Section IV while Section V concludes the paper.

II. System Model

A. MIMO Multiuser Downlink Model

We consider a MIMO multiuser downlink system with N_t transmit antennas at the access point serving N_r users, each with one receive antenna. The data received at the kth user is

$$y_k = \sum_{i=1}^{N_t} h_{k,i} x_i + n_k \tag{1}$$

where $h_{k,i}$ is a zero-mean, unit-variance complex Gaussian random fading coefficient associating the kth user's receiver and the ith transmit antenna at the access point. Data transmitted from the ith transmit antenna is denoted by x_i and n_k is the standard complex Gaussian noise observed at the kth user's receiver. If we group the received data at all users' receivers into a vector, we can describe the same system with the following matrix equation

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{2}$$

where $\mathbf{y} = [y_1, ..., y_{N_r}]^T$ is the received data vector, and $\mathbf{x} = [x_1, ..., x_{Nt}]^T$ is the vector of transmitted data. Furthermore, we use $\mathbf{x}_k$ to differentiate the per-user transmit data vector and the composite transmit data vector, $\mathbf{x}$. The noise vector is symbolically represented by $\mathbf{n} = [n_1, ..., n_{N_r}]^T$. The matrix channel $\mathbf{H}$ has $h_{k,i}$ as its constituents and its dimension is $N_r \times N_t$. For simplicity, we study the system where $N_r = N_t$ and consider only the slow block fading channel, i.e. the channel $\mathbf{H}$ stays constant over the duration of one transmission frame, which includes pilot and information symbols, but varies from one frame to the other. Also, the channel state information (CSI), $h_{k.i}$, are estimated precisely at the receivers and made available at the transmitter via delay-and-error free feedback links.

B. MIMO Multiuser Downlink (Broadcast Channel) Capacity

If we assume the total transmission power is subject to the following constraint

$$\sum_{k=1}^{N_r} P_k = P_{total} \tag{3}$$

where P_k is the power assigned to the kth user and P_{total} is the total available transmission power. The capacity of the kth user is given by

$$C_k = \log_2 \frac{\det\left[\mathbf{h}_k\left(\mathbf{\Sigma}_k + ... + \mathbf{\Sigma}_{N_r}\right)\mathbf{h}_k^{\dagger} + \sigma_n^2\mathbf{I}\right]}{\det\left[\mathbf{h}_k\left(\mathbf{\Sigma}_{k+1} + ... + \mathbf{\Sigma}_{N_r}\right)\mathbf{h}_k^{\dagger} + \sigma_n^2\mathbf{I}\right]} \tag{4}$$

where $\mathbf{\Sigma}_k = \mathbf{E}\left[\mathbf{x}_k\mathbf{x}_k^{\dagger}\right]$ is the transmit covariance matrix for user k and $P_k = \mathbf{Tr}\left(\mathbf{\Sigma}_k\right)$. The sum-capacity of the N_r-user MIMO BC is

$$C_{BC}^{sum} = \max\left\{\sum_{k=1}^{N_r} C_k\right\} \tag{5}$$

$$s.t. \left\{\mathbf{\Sigma}_k\right\}_{k=1}^{N_r} : \mathbf{\Sigma}_k > 0, \ \sum_{k=1}^{N_r} \mathbf{Tr}\left(\mathbf{\Sigma}_k\right) \leq P_{total}$$

There are plenty of algorithms available in the literature to solve the optimization problem described by (5), i.e. [10] and the references therein. We used the 2^{nd} algorithm proposed in [10] due to its simplicity and efficiency. The solutions provided by the algorithm can be conceptually understood as water-filling the matrix channel $\mathbf{H}$ across the spatial domain with joint power constraint. The water-filling characteristic of the solutions has also been identified in the scalar channel [7], and the MIMO point-to-point channel, i.e. eq. (6) and (7) in [9].

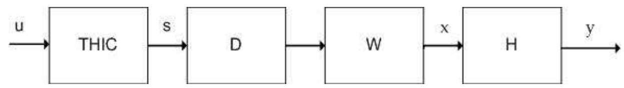

Fig. 1. Block Diagram of MMSE-FBP

C. MMSE-FBP

The MIMO multiuser downlink transmission is carried out using the minimum-mean-squared-error feedback precoder (MMSE-FBP). The precoder uses the knowledge of the channels to encode user information symbols so that the sum capacity of the MIMO BC can be achieved. Fig. 1 shows the block diagram of the MMSE-FBP.

We define a matrix $\mathbf{P}_b = \mathbf{H}\mathbf{H}^{\dagger} + K\sigma_n^2\mathbf{I}$ and using Cholesky decomposition to separate it into an upper triangular matrix $\mathbf{G}$, with ones being the main diagonal, and a diagonal matrix $\mathbf{S}$ having positive entries. The transmit filter $\mathbf{W}$ is set as

$$\mathbf{W} = \mathbf{H}^{\dagger}\mathbf{G}^{-1}\mathbf{S}^{-1} \tag{6}$$

The matrix $\mathbf{D}$ is the power allocation matrix and its elements are given by

$$\mathbf{E}\left[d_{kk}\right] = \sqrt{\frac{P_k}{P_{s_k}}} \tag{7}$$

where P_{s_k} is the average power of the symbols at the output of the Tomlinson-Harashima interference canceller (THIC), s_k, and its value is

$$P_{s_k} = \frac{\tau^2}{6} \tag{8}$$

where τ is the modulation-dependent modulo parameter. The value of P_k in (7) depends on the type of power control policy implemented. If the power control policy is required to achieve the sum capacity of the MIMO BC at all SNR, P_k will be the solutions of (5), i.e. $P_k = \mathbf{Tr}\left(\Sigma_k\right)$. On the other hand, if equal power policy is implemented, P_k will assume a value of $1/N_r$ for all k.

If we further define an effective channel between the received signal vector, $\mathbf{y}$, and the output of THIC, $\mathbf{s}$, as $\tilde{\mathbf{H}} = \mathbf{HWD}$, the THIC encodes the user information symbols obeying the dirty-paper-coding rule

$$s_k = f_\tau\left(u_k - \sum_{j=1}^{k-1}\frac{\tilde{h}_{k,j}s_j}{\tilde{h}_{k,k}}\right) \quad for\ k = 1, ..., N_r \tag{9}$$

where u_k denotes the information symbol for user k and the modulo function f_τ is

$$f_\tau\left(y\right) = \left\{\Re\left(y\right) - \lfloor\frac{\Re\left(y\right) + \tau/2}{\tau}\rfloor \times \tau\right\} + \left\{\Im\left(y\right) - \lfloor\frac{\Im\left(y\right) + \tau/2}{\tau}\rfloor \times \tau\right\} \tag{10}$$

with $\lfloor \rfloor$ denoting the largest integer less than or equal to its argument.

Finally, in order to define the per-user SINR in Section III in a more informative manner, let us introduce here the normalized transmit filter $\mathbf{w}_k$ as follows:

$$(\mathbf{w}_k)^{norm} \triangleq \frac{\mathbf{w}_k}{||\mathbf{w}_k||} \tag{11}$$

where $\mathbf{w}_k$ is the k^{th} column of the transmit filter matrix $\mathbf{W}$. We only summarized the important aspects of the MMSE-FBP here. Full derivations of components and discussions about choices of parameters can be found in [5] and [6].

D. Problem Formulation

The following optimization problem characterizes the adaptive modulation (AM) system we consider and it is the multiuser version of the system discussed in [9]:

$$\max \text{ASE} = \mathbf{E_H}\left[\sum_{i=1}^{N_r} k_i\right] \tag{12}$$

subject to

$$\mathbf{E_H}\left[\sum_{i=1}^{N_r} P_i\right] = P_{total} \tag{13}$$

$$\mathbf{E_H}\left[\frac{\sum_{i=1}^{N_r} k_i \text{BER}_i}{\sum_{i=1}^{N_r} k_i}\right] \leq \text{BER} \tag{14}$$

$$for\ k_i \geq 0;\ P_i \geq 0,\ i = 1, \cdots, N_r$$

If we use M_i to symbolize the levels of modulation used for the i^{th} user, then $\log_2(\text{M}_i) = k_i$ represents the spectral efficiency (SE). The corresponding transmit power is represented by P_i and the average sum of transmit powers of all users need to comply with the power constraint (13). The objective here is to maximize the average spectral efficiency (ASE) of a MIMO BC. From the above characterization we can see that k_i, P_i, and BER_i must be adapted in both spatial domain, i.e. across user space, and temporal domain, i.e. over a series of channel realizations. This results in a space-time optimization problem and is prohibitively complex to analyze. One technique to get around this hurdle is to freeze the channel in time and design the same AM system for that instantaneous instance of channel. In other words, instead of directly solving (12) to (14), we solve the following:

$$\max \text{ASE} = \sum_{i=1}^{N_r} k_i \tag{15}$$

subject to

$$\sum_{i=1}^{N_r} P_i = P_{total} \tag{16}$$

$$\frac{\sum_{i=1}^{N_r} k_i \text{BER}_i}{\sum_{i=1}^{N_r} k_i} \leq \text{BER} \tag{17}$$

$$for\ k_i \geq 0;\ P_i \geq 0,\ i = 1, \cdots, N_r$$

The significance of this simplication is that although a closed form solution might not exist, for example due to the highly non-linear structure of (17), we can at least resort to the exhaustive numerical search method [9].

III. Continuous MQAM Adaptive Modulation

An AM system can emcompass a range of modulation techniques, however, we confine our study to the quadrature amplitude modulation (QAM) due to the availability of a closed form expression for its uncoded BER performance. The uncoded BER for an M level QAM is upper bounded by [7]:

$$\text{BER}_i = 0.2 \exp\left[\frac{-1.6 P_i \lambda_i}{\sigma_n^2 \left(2^{k_i} - 1\right)}\right] \quad for\ i = 1, ..., N_r \tag{18}$$

where λ_i denotes the channel gain for the i^{th} user and $P_i \lambda_i / \sigma_n^2$ is the instantaneous received SNR. This upper bound is tight for $\text{M} \geq 4$ and $0 \leq \text{SNR} \leq 30\text{dB}$. Obviously, the value of $k_i = \log_2(\text{M}_k)$ can theoretically be taken from a continuous range, however, non-integer values are not practically favorable as they result in sophisticated modulators. Nonetheless, the treatment bears some important implications. We will show in the coming subsections that continuous AM (CAM) studied herein yields closed form solutions for some of the scenarios that we consider. Furthermore, it provides an upper bound on the ASE that can be achieved by integer values of k_i.

In a multiuser context, such as the MIMO BC, the SNR is generally not an appropriate parameter to characterize system throughput. A more indicative parameter should include the multiuser inteferences, such as the SINR. Referring to Figure 1, we can express (18) in a more precise form:

$$\text{BER}_i = 0.2 \exp\left[\frac{-1.6}{\left(2^{k_i} - 1\right)} \cdot \frac{P_i \left|\left|\mathbf{h}_i \mathbf{w}_i\right|\right|^2}{\sum_{j \geq i}^{N_r} \left|\left|\mathbf{h}_i \mathbf{w}_j\right|\right|^2 P_j + \sigma_n^2}\right] \tag{19}$$

where $\mathbf{w}_i, (i = 1, \cdots, N_r)$ is defined previously in Section II and P_i is the average power of the symbols in one transmission frame at the output of the power allocation block $\mathbf{D}$. We need to point out while base-station is able to compute (19) precisely provided our assumption regarding perfect channel feedback holds true, the users on the other end do not have such a luxury; in particular, the i^{th} user has no knowledge of the transmit filters that correspond to users $j \neq i, j = 1, \cdots, N_r$, hence it decodes the received signal using only the noise statistics. Mathematically this means the base-station needs to use

$$\text{BER}_i = 0.2 \exp\left[\frac{-1.6}{(2^{k_i}-1)} \cdot \frac{P_i \left|\left|\mathbf{h}_i \mathbf{w}_i\right|\right|^2}{\sigma_n^2}\right] \qquad (20)$$

to ensure coherence between transmitter and receivers. Fortunately, simulation results show that the difference of (19) and (20) is indistinguishable. Thus, eq. (20) is used for the following analysis and simulations.

A. Equal BER Constraint

An even simpler set of characterizing optimization expressions can be arrived at by providing equal BER to all users. More specifically we set

$$\text{BER}_i \leq \text{BER}_{\text{target}} \quad for\ i = 1, ..., N_r \qquad (21)$$

where $\text{BER}_{\text{target}}$ is a design parameter. Substituting (20) into (21) and rearranging, we obtain

$$\begin{aligned} k_i &\leq \log_2\left[1 - \frac{1.6}{\ln\left(\text{BER}_{\text{target}}/0.2\right)} \cdot \frac{P_i \left|\left|\mathbf{h}_i \mathbf{w}_i\right|\right|^2}{\sigma_n^2}\right] \\ &= \log_2\left[1 + K \cdot \frac{P_i \left|\left|\mathbf{h}_i \mathbf{w}_i\right|\right|^2}{\sigma_n^2}\right] \qquad (22) \end{aligned}$$

with $K := -1.6/\ln\left(\text{BER}_{\text{target}}/0.2\right)$

where $0 < K < 1$ is known as the penalty factor due to MQAM [7] [9]. The objective function (15) is updated as

$$\max \text{ASE}^{equal\_BER} = \sum_{i=1}^{N_r} \log_2\left[1 + K \cdot \frac{P_i \left|\left|\mathbf{h}_i \mathbf{w}_i\right|\right|^2}{\sigma_n^2}\right] \qquad (23)$$

subject to

$$\sum_{i=1}^{N_r} P_i = P_{total};\ P_i \geq 0 \qquad (24)$$

Recall a similar optimization problem exists for the MIMO point-to-point channel, i.e. eq. (15) in [9]:

$$\max \text{ASE}^{point-to-point} = \sum_{i=1}^{N_r} \log_2\left(1 + K \cdot \frac{\lambda_i P_i}{\sigma_n^2}\right) \qquad (25)$$

subject to

$$\sum_{i=1}^{N_r} P_i = P_{total};\ P_i \geq 0 \qquad (26)$$

where N_r is the number of receive antennas and λ_i is the channel gain of the i^{th} branch of the parallel non-interferring AWGN channels, which is obtained by singular value decomposing (SVD) the MIMO point-to-point channel. The closed form solutions for the branch power and branch SE of the point-to-point system are:

$$P_i = \left[\mu - \sigma_n^2/K\lambda_i\right]^+ \qquad (27)$$

$$k_i = \left[\log_2\left(\mu K \lambda_i/\sigma_n^2\right)\right]^+ \qquad (28)$$

where μ is a constant determined by the total power constraint. Let us now examine (27) in detail. Assuming appropriate scaling is applied, we can set $\lambda_i = 1$. If σ_n^2 is large, i.e. when the channel quality is poor, the resulting P_i will be small while a much larger P_i prevails when the channel disturbance σ_n^2 is small, indicating a much favorable transmission condition. This type of power allocation has been called the *water-filling* strategy in information theory literature [11].

Comparing (23) and (24) with (25) and (26), we immediately see that they share a common structure. This correspondence motivates us to conjecture that water-filling the MIMO BC with joint power constraint can also maximize (23). Thus, the solutions to the equal BER constraint AM can be readily obtained as follows:

$$P_i = \mathbf{Tr}\left(\mathbf{\Sigma}_i^{optimal}\right) \qquad (29)$$

where $\mathbf{\Sigma}_i^{optimal}$ is the water-filling transmit covariance matrix for the i^{th} user. The i^{th} user's SE is subsequently given by

$$\text{SE}_i = k_i = \log_2\left(1 + K \cdot \frac{P_i \left|\left|\mathbf{h}_i \mathbf{w}_i\right|\right|^2}{\sigma_n^2}\right) \qquad (30)$$

B. Equal-BER and Equal-Power Constraint

We now impose a further constraint on the optimization problem by equally allocating the transmission power among users. In this equal-BER and equal-power scenario, the only transmission degrees of freedom (DoF) is the number of *active* users denoted by r'. Once a subset of users S is selected with $|S| = r'$, the power is distributed equally as P_{total}/r' and the SE is obtained by (22) with $P_i = P_{total}/r'$. The characterizing optimization problem in this case is

$$\max \text{ASE}^{equal\_power} = \sum_{i \in S} \log_2\left(1 + K \cdot \frac{P_{total} \left|\left|\mathbf{h}_i \mathbf{w}_i\right|\right|^2}{r' \sigma_n^2}\right)$$
$$S \subset \{1, \cdots, N_r\}; \quad |S| = r' \qquad (31)$$

We need to numerically evaluate (31) for all possible $|S| = r' = 1, \cdots, N_r$ and select the one achieving the maximum ASE. We want to emphasize at this point that the number of user subsets that the numerical search needs to traverse is dependent upon whether (19) or (20) is opted; clearly, order of users needs to be taken into account when interference is included. As an example, consider a system with $N_t = N_r = 4$. There are 64 subsets if (19) is used and only 15 subsets are required to be visited for (20). Generally, the number of subsets that needs to be considered for the (19) case is

$$\sum_{i=1}^{N_r}\left(\prod_{j=1}^{i-1}(N_r-j)\right) \tag{32}$$

and

$$\sum_{i=1}^{N_r}\left(\binom{N_r}{i}\right) \tag{33}$$

if (20) is used.

Let us now study the asymptotic behavior of the equal-BER and equal-power AM. At high channel SNR, the base-station has enough power to support all users, therefore, $S=\{1,\cdots,N_r\}$ and $|S|=r'=N_r$. Eq. (31) becomes

$$\begin{aligned}&\text{ASE}^{equal\_power}\\&=\sum_{i=1}^{N_r}\log_2\left(1+\frac{P_{total}}{N_r}\cdot\frac{K\,||\mathbf{h}_i\mathbf{w}_i||^2}{\sigma_n^2}\right)\\&\approx\sum_{i=1}^{N_r}\log_2\left(\frac{P_{total}}{N_r}\cdot\frac{K\,||\mathbf{h}_i\mathbf{w}_i||^2}{\sigma_n^2}\right)\\&=\sum_{i=1}^{N_r}\log_2\left(\frac{P_{total}}{N_r}\right)+\sum_{i=1}^{N_r}\log_2\left(\frac{K\,||\mathbf{h}_i\mathbf{w}_i||^2}{\sigma_n^2}\right)\\&=N_r\log_2\left(\frac{P_{total}}{N_r}\right)+\sum_{i=1}^{N_r}\log_2\left(\frac{K\,||\mathbf{h}_i\mathbf{w}_i||^2}{\sigma_n^2}\right)\end{aligned} \tag{34}$$

Since the water-filling power allocation in equal BER scenario converges to the equal-power allocation scheme at high channel SNR, i.e. (23) approaches

$$\begin{aligned}&\text{ASE}^{equal\_BER}\\&=\sum_{i=1}^{N_r}\log_2\left[1+K\cdot\frac{P_i\,||\mathbf{h}_i\mathbf{w}_i||^2}{\sigma_n^2}\right]\\&\approx\sum_{i=1}^{N_r}\log_2(P_i)+\sum_{i=1}^{N_r}\log_2\left(\frac{K\,||\mathbf{h}_i\mathbf{w}_i||^2}{\sigma_n^2}\right)\\&\approx N_r\log_2\left(\frac{P_{total}}{N_r}\right)+\sum_{i=1}^{N_r}\log_2\left(\frac{K\,||\mathbf{h}_i\mathbf{w}_i||^2}{\sigma_n^2}\right)\end{aligned} \tag{35}$$

therefore, the additional power constraint brings insignificant impact to the ASE of the system at high channel SNR region. On the other end of the spectrum, we can expect a similar convergence result since at the lower end of the channel SNR, the base-station can only dedicate all its transmit power to the strongest user.

C. Equal-BER and Equal-SE Constraint

So far we have studied AM systems that always adjust the SEs of the users as a function of the channels. We now shift our attention to the case where the SE is kept at a constant level of k_0. We are again left with a single transmission DoF, which is the number of *active* users r'. Once the base-station decides how many users to receive transmission, the same SE k_0 is assigned to every user. The i^{th} user's transmission power is related to k_0 as follows:

$$P_i=\frac{\left(2^{k_0}-1\right)\sigma_n^2}{K\,||\mathbf{h}_i\mathbf{w}_i||^2};\quad for\ i=1,\cdots,r' \tag{36}$$

and the overall transmission power is subject to the constraint

$$\sum_{i=1}^{r'}\frac{\left(2^{k_0}-1\right)\sigma_n^2}{K\,||\mathbf{h}_i\mathbf{w}_i||^2}=P_{total} \tag{37}$$

Rearranging (37) we can obtain the constant SE k_0:

$$k_0=\log_2\left(1+\frac{KP_{total}}{\sigma_n^2\sum_{i=1}^{r'}||\mathbf{h}_i\mathbf{w}_i||^{-2}}\right) \tag{38}$$

The optimization problem characterizing the equal-BER and equal-SE constraint case is thus the following

$$\begin{aligned}\max\text{ASE}^{equal\_SE}&=r'k_0=r'\log_2\left(1+\frac{KP_{total}}{\sigma_n^2\sum_{i=1}^{r'}||\mathbf{h}_i\mathbf{w}_i||^{-2}}\right)\\&for\ r'=1,\cdots,N_r\end{aligned} \tag{39}$$

We evaluate (39) for all possible r' and select the one that achieves the maximum ASE.

Similar to the equal-BER and equal-power case, at low channel SNR, the base-station can only concentrate its transmission power to the user with the strongest channel gain. Thus, we can expect the equal-BER and equal-SE system to perform comparatively against the equal BER system. At high channel SNR, transmission to all users is possible and $r'=N_r$. Following the same derivation as [9], we can approximate (39) as follows

$$\begin{aligned}\max\text{ASE}^{equal\_SE}&=N_rk_0\approx N_r\log_2\left(\frac{KP_{total}}{\sigma_n^2\sum_{i=1}^{N_r}||\mathbf{h}_i\mathbf{w}_i||^{-2}}\right)\\&\le N_r\log_2\left(\frac{KP_{total}}{N_r\sigma_n^2}\left(\prod_{i=1}^{N_r}||\mathbf{h}_i\mathbf{w}_i||^2\right)^{\frac{1}{N_r}}\right)\\&=\sum_{i=1}^{N_r}\log_2\left(\frac{KP_{total}\,||\mathbf{h}_i\mathbf{w}_i||^2}{N_r\sigma_n^2}\right)\\&=N_r\log_2\left(\frac{P_{total}}{N_r}\right)+\sum_{i=1}^{N_r}\log_2\left(\frac{K\,||\mathbf{h}_i\mathbf{w}_i||^2}{\sigma_n^2}\right)\end{aligned} \tag{40}$$

with the last line of (40) being the maximum ASE achieved by the equal BER AM.

The gap between ASE^{equal_SE} and ASE^{equal_BER} is

$$\text{ASE}^{equal\_BER}-\text{ASE}^{equal\_SE}=\sum_{i=1}^{N_r}\log_2\left(\frac{||\mathbf{h}_i\mathbf{w}_i||^2N_r}{\sum_{j=1}^{N_r}||\mathbf{h}_j\mathbf{w}_j||^2}\right) \tag{41}$$

k	0	2	4	6	8	10
SNR(dB) BER $= 1 \times 10^{-3}$	N/A	9.97	16.96	23.19	29.27	35.30
SNR(dB) BER $= 1 \times 10^{-6}$	N/A	13.59	20.58	26.81	32.88	38.92

TABLE I

SWITCHING THRESHOLDS FOR UNCODED MQAM

which is obviously independent of the channel SNR and the target BER. In other words, a constant SE gap exists between the equal-SE and the equal-BER AM at high channel SNR region as well as at any target BER level.

IV. DISCRETE ADAPTIVE MODULATION

In the previous section we have investigated the effects of power and rate adaptations on the average spectral efficiency of a MIMO BC. We did not, throughout the treatment, impose any restrictions on the levels of modulation that could be used apart from demanding the modulation technique to be QAM. In this section, we introduce a practical constraint that limits the levels of modulation to a finite set of numbers that are integer multiples of two. More precisely, we assume the available set of SEs $\mathbf{B} = [0, 2, 4, 6, 8, 10]$ is supplied to the base-station and it can only use $\left[2^0, 2^2, 2^4, 2^6, 2^8, 2^{10}\right]$-QAM to transmit user information. Due to the discrete natural of the system, analytical expressions for the power and/or rate adaptations are unavailable and numerical search needs to be unertaken to obtain the optimal transmission parameters.

A. *Equal Power and/or Equal BER Constraint*

Similar to the continuous adaptive modulation, we start by considering the equal BER scenario. For continuous adaptive modulation we performed a numerical search over all possible user subsets for maximum ASE. The same procedure must be undertaken for the discrete counterpart with an additional modulation level down coversion. More precisely, searching over the user subsets returns a set of continuous SEs for a given r' and S, we need to down convert those SEs to the nearest member of the list $\mathbf{B}$. The down conversion requires us to first identify the *switching thresholds* for the modulation levels. These switching thresholds can be obtained directly from (18), simulation results or other more accurate analytical expressions such as [12]. Table I lists the calculated thresholds from (18)

For a given r' and S, the base-station, after receiving the channel parameters from users, computes the SNRs using

$$\gamma_i = \frac{P_i \left|\left|\mathbf{h}_i \mathbf{w}_i\right|\right|^2}{\sigma_n^2} \tag{42}$$

where $P_i = P_{total}/r'$ for equal-BER and equal-power case and $P_i = \mathbf{Tr}\left(\mathbf{\Sigma}_i^{optimal}\right)$, with $\Sigma_i^{optimal}$ being

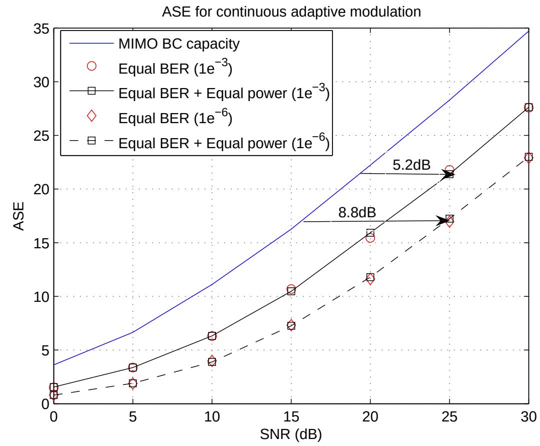

Fig. 2. ASE for continuous adaptive modulation (Equal BER and Equal BER and Equal Power cases)

the i^{th} user's water-filling transmit covariance matrix, for equal BER scenario. These SNRs are compared against the tabulated switching thresholds in Table I and appropriate values of k are selected. Mathematically

$$\begin{aligned} &if\ \gamma_\alpha \leq \gamma < \gamma_{\alpha+2};\ \alpha \subset \mathbf{B} = [0, 2, 4, 6, 8, 10] \\ &k_i = \alpha \end{aligned} \tag{43}$$

V. SIMULATION RESULTS

Figure 2 shows the ASE for continuous adaptive modulation under different system constraints. It clearly shows the addition equal power constraint brings an insignificant degradation in the ASE for the range of channel SNRs considered. This result agrees with our previous observations from Eq. (34) and (35). Furthermore, there is a constant gap of 5.2dB between the equal BER case and the MIMO BC sum capacity for a target BER of 1×10^{-3} and 8.8dB if target BER is lowered to 1×10^{-6}. This gap is independent of the channel SNR and is quantified by the penalty factor K defined in (22). Because channel coding was not considered in the simulation, therefore, K can also be regarded as the maximum possible coding gain for the system [7].

Figure 3 shows that imposing an additional equal SE causes a slight reduction in the ASE. The ASE reduction is more visible at higher channel SNRs, a result that coincides with our observation in the previous section. In additional, the simulation result also supports our previous statement regarding the ASE gap between equal BER and equal BER with equal SE cases, i.e. it is irrelevant of the target BER.

We next present the simulation results for the discrete adaptive modulation. Figure 4 compares the continuous and discrete adaptive modulation under equal BER constraint. Limiting the SEs to a set of finite levels incurs an ASE penalty and the severity of the ASE descent is

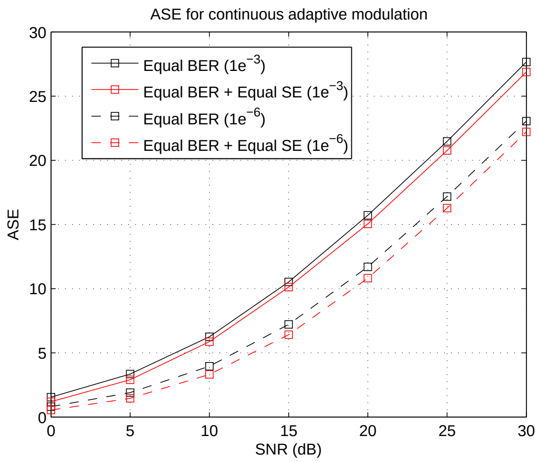

Fig. 3. ASE for continuous adaptive modulation (Equal BER and Equal BER with Equal SE cases)

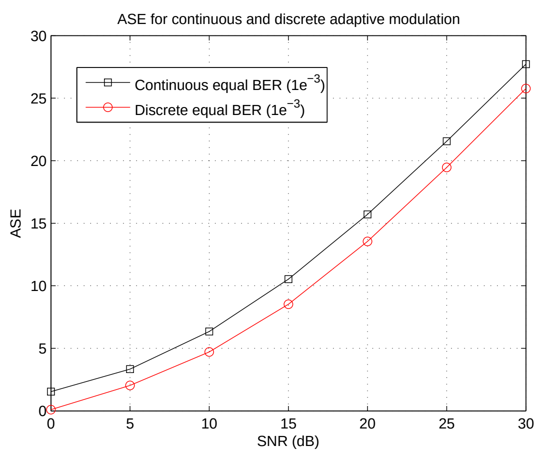

Fig. 4. ASE for continuous and discrete adaptive modulation (Equal BER case)

independent of the channel SNRs. Eq. (22) reveals the reason for the loss in ASE; the equality always holds if the SEs are continuous while for discrete SEs, the inequality is always true. Lastly we use Figure 5 to justify our choice of (20) over (19); there is no noticeable difference between the two curves which suggests the results obtained by adopting (20) is equally indicative for our purpose.

VI. Conclusion

In this paper we investigated AM techniques that can maximize the ASE while maintaining an acceptable BER for the MIMO BC system. We found that there was an ASE penalty if we confined ourselves to MQAM. The ASE penalty is a function of the target BER and is constant for all SNR if the modulation sizes are continuous. Our simulation results further showed that

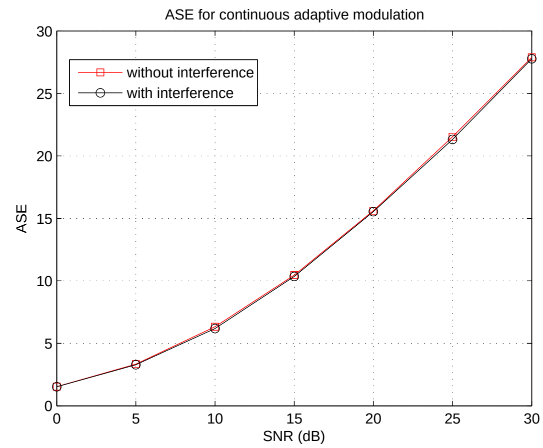

Fig. 5. ASE for continuous adaptive modulation (Equal BER and Equal Power case)

if only a finite set of discrete modulation sizes was supplied to the transmitter, the ASE gap expanded and it maintained the same distance from the continuous AM case. In addition, we have identified that rate adaptation is more important than power adaptation in maximizing the ASE of the system.

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