A Novel Low Complexity Clipping Method for OFDM Signals

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***Abstract*—In this paper, we propose a low complexity clipping method to reduce the peak power of OFDM signals. The proposed clipping method exploits the conventional square clipping, which clips signal according to the amplitude in I/Q-channels separately, to avoid obtaining a real amplitude of each sample which costs relatively high computational complexities. By combining the square clipping and phase rotations of signals, the proposed octagon clipping achieves close clipping performances to the amplitude based normal clipping with a small amount of computations. The proposed method requires 4 comparison operations, one addition, and one subtraction to clip a sample of signal.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) systems have many attractive features suitable for wide band mobile communications [1]. Because of the benefits, OFDM systems are widely adopted to wireless standards such as wireless LAN, wireless MAN, and digital radio and television broadcasting. On the other hand, one of crucial problems of OFDM systems is high Peak-to-Average Power Ratio (PAPR) due to the use of several hundred or thousand of subcarriers.

There are plenty of previous studies on peak reduction methods for OFDM systems, such as Partial Transmit Sequence (PTS) [2], SeLectied Mapping (SLM) [3], companding techniques [4], and Clipping And Filtering (CAF) [5]. In this study, we consider simple PAPR reduction methods that receivers do not require additional mechanism corresponding to the operation performed at the transmitter.

Several clipping algorithms such as Lucent algorithm, vector subtraction, and sector clipping are introduced and proposed [6]. From the view point of complexity for clipping operations, sector clipping may be the most attractive method, because sector clipping needs 2, 3, or 9 comparison operations per signal sample for the cases of 2 sectors (equivalent to square clipping), 3 sectors, or 5 sectors, respectively.

In this paper, we propose octagon clipping as a variant of polygon clipping method which clips unallowable high amplitude with light computation. The octagon clipping utilizes square clipping and phase rotation which is achieved by a addition and a subtraction. This computational complexity is almost equivalent to sector clipping with 3 sectors. Moreover, the Error Vector Magnitude (EVM) of the proposed clipping is similar to circle clipping, which is an ideal clipping, and less than above mentioned sector clippings.

II. System Model

We evaluate the peak performance of OFDM signals to satisfy both requirements for the Complementary Cumulative Distribution Function (CCDF) of instantaneous power and EVM.

The transmitter model is shown in Fig. 1. A serial stream of M-PSK/QAM modulated data symbols is converted to N_u parallel streams. Then the symbols are transformed to a time domain signal by Inverse Fast Fourier Transform (IFFT). The discrete time OFDM signal without Guard Interval (GI) can be shown as

$$s(n) = \sum_{k=-N_u/2}^{N_u/2} d_k e^{j2\pi kn/N} \quad n = 0, 1, \ldots, N-1. \tag{1}$$

where d_k represents the data symbols for k-th subcarrier and N denotes the FFT point. Then Cyclic-Prefix (CP) is added and the signal is converted to a serial stream. CAF is applied at the following block to reduce the extra power components and to suppress the out-of-band emission. Finally the signal is amplified and transmitted from the antenna.

For PAPR evaluation, we use the CCDF of instantaneous power normalized by the average power. Let ξ be the ratio of an instantaneous power to the average power mathematically defined as follows.

$$\xi(n) = \frac{|s(n)|^2}{\frac{1}{N}\sum_{n=0}^{N-1} |s(n)|^2} \tag{2}$$

The CCDF can be denoted as the probability $\Pr(\xi(n) > \xi_0)$ that the instantaneous power of a sample is above a normalized instantaneous power ξ_0.

As a performance comparison purpose, we use EVM defined by the following equation in this paper.

$$\mathrm{EVM}(\%) = \sqrt{\frac{\sum_i |d_i - \hat{d}_i|^2}{\sum_i |d_i|^2}} \cdot 100 \tag{3}$$

where d_i represents a transmitted data symbol, $\hat{d}_i$ is a received data symbol, and the received data symbols are assumed be normalized as $\sum_i |\hat{d}_i|^2 = \sum_i |d_i|^2$. EVM can represent the distortion caused by CAF.

In addtion,We perform the evaluation of BER.BER is the number of erroneous bits received divided by the total number of bits transmitted.

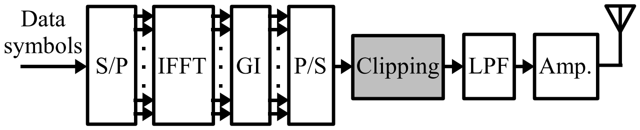

Fig. 1. OFDM transmitter with clipping and filtering.

To evaluate the out-of-band emission, we introduce ACLR (Adjacent Channel Leakage Ratio). ACLR is defined as

$$\mathrm{ACLR} = \frac{P_d}{P_s} \tag{4}$$

where P_d denotes the signal power in the adjacent channel and P_s is the signal power within the assigned channel. We do not consider the effect of thermal noise for the evaluation. Thus, if we don't use any clipping operation, the ACLR becomes 0.

III. Clipping Methods

In this section, we introduce some conventional clipping methods and propose a novel clipping method. Although there are some previous studies on clipping algorithms [6], we focus on the computational complexity relevant to clipping operations in this paper.

As the conventional clipping methods, we show three kinds of clipping method, that is, circle clipping, square clipping, and sector clipping [6]. Then the proposed polygon clipping method is given.

A. Circle clipping

The normal clipping method clips the signal based on the comparison between a clipping threshold and an amplitude of the signal component. We call this clipping method circle clipping because the constellation of (time domain) clipped signal becomes a circle whose radius is a clipping threshold.

Let $s(n) = s_I(n) + s_Q(n)$ be the n-th component of the OFDM signal. Besides, its polar expression is assumed to be $s(n) = |s(n)|\, e^{j\phi(n)}$. Then, the clipping function can be shown as follows.

$$y(n) = \begin{cases} s(n) & (|s(n)| \le A_{th}) \\ A_{th} e^{j\phi(n)} & (|s(n)| > A_{th}) \end{cases} \tag{5}$$

where $y(n)$ is the output signal component and A_{th} is a clipping threshold. In order to calculate the amplitude of the signal component, calculating $|s(n)| = \sqrt{s_I^2(n) + s_Q^2(n)}$ is required. However, direct calculation of the square root is complex in the sense of hardware implementation.

Practically, CORDIC (COordinate Rotation DIgital Computer) algorithm is used to obtain the amplitude [7]. Because CORDIC algorithm requires light operations such as bit shift, additions, subtractions, and read from lookup table, this is suitable for hardware implementation. However, CORDIC algorithm utilizes iterative process. Thus, the amount of calculations to obtain the amplitude and to clip the signal is much more than that by the following clipping methods.

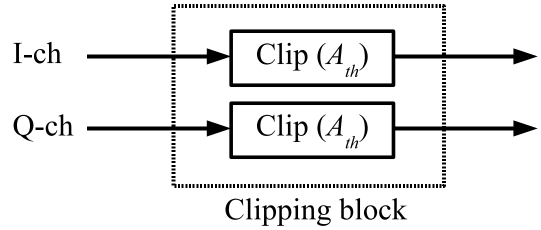

Fig. 2. Block diagram of square clipping method

B. Square clipping

If the I-ch and Q-ch components of the signal are clipped separately, the computational costs for clipping can be reduced significantly. After checking the sign of the components, it requires a comparison operation between the absolute value of the component and the clipping threshold. Since the constellation after clipping becomes a square with sides $2A_{th}$ long, we call this method square clipping. If the absolute value of an input component is x which can be $|s_I(n)|$ or $|s_Q(n)|$ and the corresponding output is y, the clipping function is given as follows.

$$y = \begin{cases} x & (0 \le x < A_{th}) \\ A_{th} & (x \ge A_{th}) \end{cases} \tag{6}$$

This clipping block can be simply shown as Fig. 2. Here A_{th} in the box denotes the clipping threshold.

Note that square clipping method is equivalent to the sector clipping method described in the following subsection when the number of sectors is 2.

Although the computational complexity of square clipping is very small, there is 3 dB gap between the deepest clipped signal and the shallowest in the instantaneous power. For example, the power ratio between the clipped signal at $A_{th} + j \cdot 0$ and $A_{th} + jA_{th}$ is 2. The former may cause larger peak regrowth, undesired components to the other samples, or out-of-band components. The noise due to the above reasons makes the signal quality degrade. To avoid the gap, it is essential that the clipping function generates the constellation of the signal after clipping being close to a circle.

C. Sector clipping

Sector clipping is proposed in the literature [6]. In the sector clipping, the absolute values of I-ch and Q-ch components are compared to multiple thresholds. As the results of the comparisons, the signal components are divided into a corresponding sector. The increase of the number of clipping thresholds makes the signal constellation be close to a circle. Of course, that increases the number of comparisons as well.

The least number of sectors is 2 in sector clipping. In this case, the number of threshold is one for each channel. Since the signal constellation after the clipping becomes square, this is the same function as square clipping.

D. Polygon clipping (octagon clipping)

We propose a new clipping method which combines the square clipping and phase rotation. From the resultant constellation shape, it can be called as polygon clipping.

First the input component is clipped by the square clipping. Let the clipped component be $s'(n) = s'_I(n) + js'_Q(n)$. Then the complex component is multiplied by the rotation matrix R_n which rotates the signal by $\pi/(2n)$ radian, where $n = 2, 3, \ldots$. The matrix is given as follows.

$$R_n = \begin{pmatrix} \cos\left(\frac{\pi}{2n}\right) & -\sin\left(\frac{\pi}{2n}\right) \\ \sin\left(\frac{\pi}{2n}\right) & \cos\left(\frac{\pi}{2n}\right) \end{pmatrix} \tag{7}$$

A pair of output components $s''(n)$ is shown as

$$\begin{aligned} \begin{pmatrix} s''_I(n) \\ s''_Q(n) \end{pmatrix} &= R_n \begin{pmatrix} s'_I(n) \\ s'_Q(n) \end{pmatrix} \\ &= \begin{pmatrix} s'_I(n)\cos\left(\frac{\pi}{2n}\right) - s'_Q(n)\sin\left(\frac{\pi}{2n}\right) \\ s'_I(n)\sin\left(\frac{\pi}{2n}\right) + s'_Q(n)\cos\left(\frac{\pi}{2n}\right) \end{pmatrix}. \end{aligned} \tag{8}$$

Then the output is clipped by the square clipping with the clipping threshold A_{th}. This square clipping reduces the components whose instantaneous power exceeds the clipping threshold. The pair of square clipping and rotation continues $n-1$ times. By the polygon clipping, the signal constellation becomes the polygon with $4n$ sides. Clearly, the increase of n make the constellation be close to the circle, however, the computational complexity also increases. Since the computation includes multiplications, the complexity is more than sector clipping.

However, if $n = 2$, i.e., the case of octagon clipping, the matrix (7) can be changed to the following form.

$$\begin{aligned} R_2 &= \begin{pmatrix} \cos\left(\frac{\pi}{4}\right) & -\sin\left(\frac{\pi}{4}\right) \\ \sin\left(\frac{\pi}{4}\right) & \cos\left(\frac{\pi}{4}\right) \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \end{aligned} \tag{9}$$

If we ignore the coefficient, the multiplication of the matrix becomes an addition and a subtraction of the components. To compensate the coefficient, the clipping threshold should be $\sqrt{2}A_{th}$ at the following square clipping. The block diagram of the octagon clipping method is shown in Fig. 3.

After the second square clipping, the signal is transfered to the low pass filter, then the output is transmitted to the amplifier. Although both the amplitude and the phase are changed by the simple matrix, the amplitude is adjustable at the following block and the phase rotation may be absorbed at the channel. Since any OFDM system needs channel compensation mechanism, neither the transmitter nor the receiver requires additional cost to re-rotate the phase.

From the practical point of view, polygon clipping costs too much for $n \geq 3$. Note that the performances by octagon clipping are sufficiently close to the those by circle clipping as shown in the following section. Therefore, we consider $n = 2$ is sufficient for both performances and complexities.

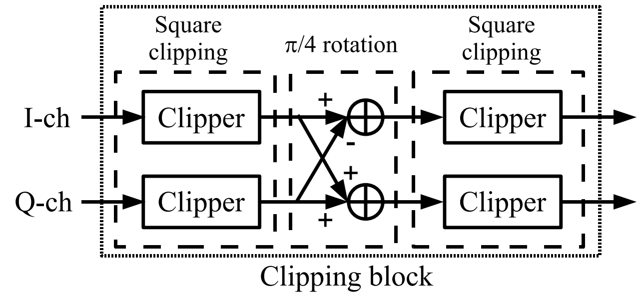

Fig. 3. Block diagram of octagon clipping method

TABLE I
SIMULATION PARAMETERS

Modulation scheme	QPSK
Number of subcarriers N_u	1703
Oversampling factor	4
FFT point N	2048
Filter type	Sinc function with Kaiser window
Number of filter taps	161
Normalized cut-off frequency	0.93
Side-lobe attenuation factor	45 dB

IV. NUMERICAL RESULTS

In this section, we compare the performances of clipping methods described in the previous section. The parameters used in the following simulations are shown in Table I. We employ sinc function with Kaiser window for low pass filtering.

A. Signal constellation of clipped signal

The signal constellations of one OFDM symbol signal after clipping are shown in Fig. 4. The vertical axis shows I-channel component, and the horizontal axis is Q-channel component. Each blue point means a signal sample. The plots are for circle clipping, square clipping, octagon clipping, and sector clipping with 3 sectors and 5 sectors. The clipping threshold $A_{th} = 1$ is used for all the plots. A red circle is drawn as the clipping threshold of circle clipping for reference. For the octagon clipping, the scale of amplitudes is shrunk by $1/\sqrt{2}$ to be the same clipping threshold for comparison purpose.

From the figure, we can observe the gap between the clipping threshold and clipped signal component. The relatively large gap by square clipping is cut efficiently by the second square clipping of the octagon clipping. We can see also that increase of the number of sectors decreases the gap in sector clipping.

B. Computational complexity for clipping

Then, we compare computational complexities per signal sample for clipping methods, i.e., CORDIC based circle clipping, square clipping, sector clipping (3 sectors and 5 sectors), and the proposed octagon clipping. Here the number of iterations for CORDIC algorithm is set at 12, which is sufficient to converge. Because of the iterative operations, CORDIC algorithm requires the most computations. Square

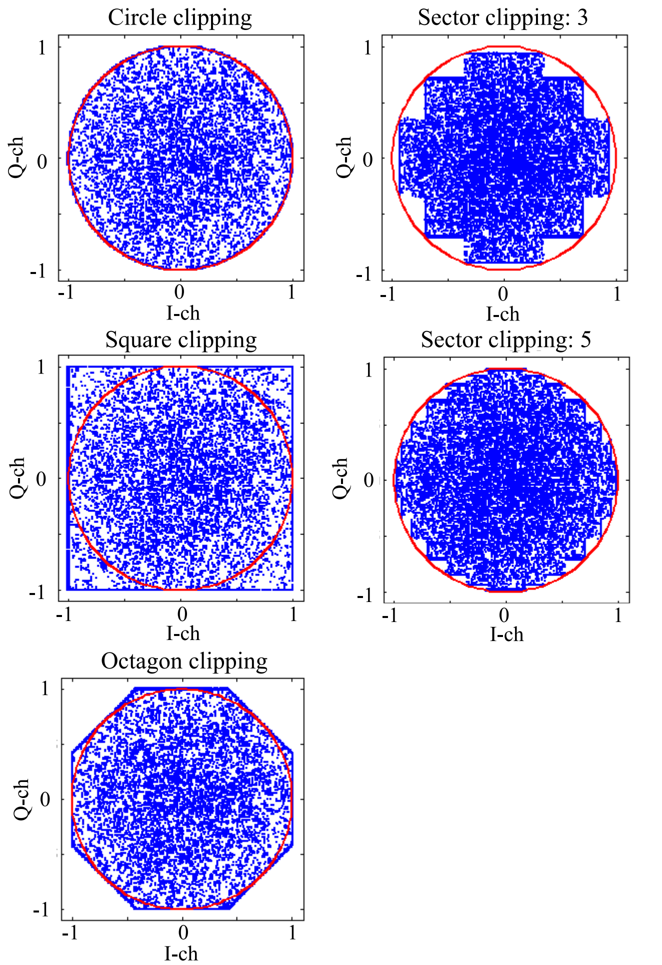

Fig. 4. Signal constellations after clipping for different clipping methods.

TABLE II
COMPUTATIONAL COMPLEXITIES PER SAMPLE OF SIGNALS.

	Comparison	Addition	Subtraction	Bit Shift
CORDIC	14	36	0	24
Square	2	0	0	0
3 Sectors	5	0	0	0
5 Sectors	9	0	0	0
Octagon	4	1	1	0

clipping only needs twice of comparisons, that is the least among the clipping methods. Octagon clipping is comparable to sector clipping with 3 sectors. Since comparison operation uses subtraction, the number of computations can be regarded as 6 and 5, respectively. Computations of sector clipping with 5 sectors are 1.5 times as many as those of octagon clipping.

C. PAPR performances

The CCDF performance of instantaneous power is shown in Fig 5. The dotted lines shows the distribution of instantaneous power after clipping before filtering. The solid lines denotes the distribution after filtering, that is, peak regrowth is included in the performance. For all clipping methods, the CCDF value

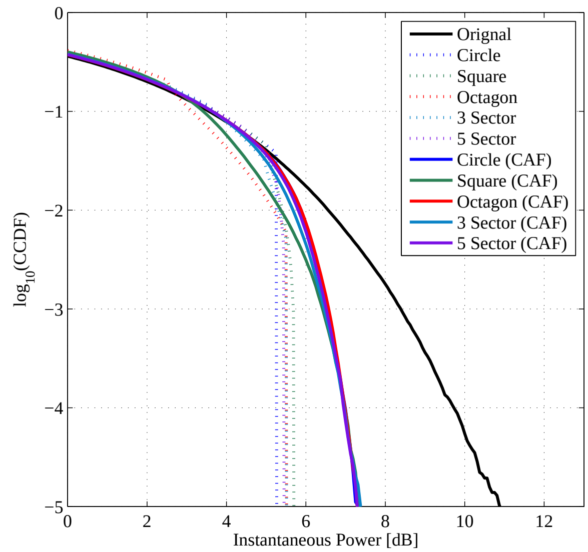

Fig. 5. PAPR performances of circle, square, octagon, and 3 sector and 5 sector clipping methods. PAPR performances after clipping are shown by dotted lines, and those after filtering are illustrated by solid lines.

is 10^{-4} when the instantaneous power is 7 dB. The clipping thresholds are adjusted to satisfy the above requirements.

The CCDFs of more than -1.5 in log scale for circle clipping, sector clipping with 5 sectors, and octagon clipping perform almost the same. This means the clipping threshold of these methods are similar value and the amount of peak regrowth is also similar to each other. On the other hand, to achieve the requirement, square clipping needs relatively smaller clipping threshold which is about 2.5 dB.

D. EVM performances

The EVM performance comparison is shown in Fig. 6. The horizontal axis shows the instantaneous power at the point where CCDF is 10^{-4}. Since EVM represents the error due to CAF, the value should be as small as possible.

We can see that the performances of circle clipping and octagon clipping are almost equivalent to each other within the region of measured instantaneous power. The octagon clipping outperforms sector clipping methods with 3 sectors and 5 sectors, despite the computational complexity is comparable or less. When the required EVM is 10%, the gain by the proposed method is 0.3 and 0.5 dB against 5 sector and 3 sector clipping method, respectively.

On the other hand, because of the difference between deepest and shallowest clipped components, square clipping requires PAPR penalty of 1 dB compared with the proposed clipping.

E. ACLR performances

Finally we evaluate the ACLR performances of the proposed clipping method to confirm the ability to suppress the out-of-band emission. The performances are shown in Fig. 7.

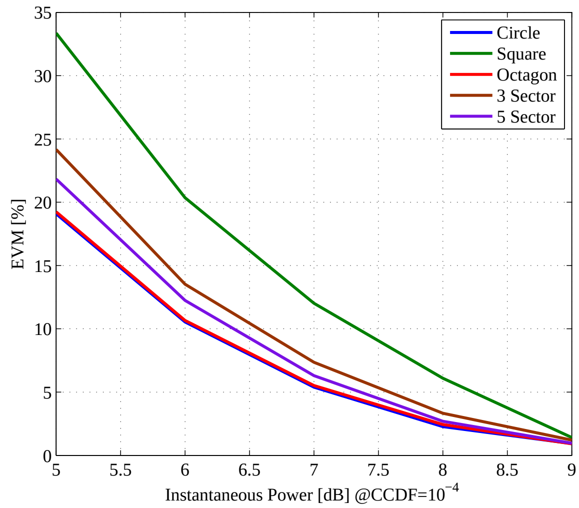

Fig. 6. EVM performances of circle, square, 3 sector, 5 sector, and octagon clipping methods.

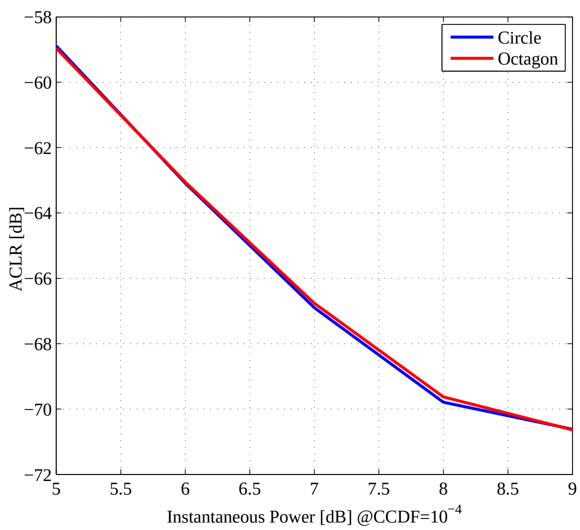

Fig. 7. ACLR performances of circle and octagon clipping methods.

As the reference, ACLR of circle clipping is also shown in the figure. The horizontal axis is the same as that of Fig. 6. We can see that the proposed method achieves almost the same suppression performance with the ideal clipping (circle clipping method). The increase of allowable peak power converges the ACLR value.

V. Conclusion

In this paper, we proposed a novel low complexity clipping method, which is octagon clipping, for OFDM systems. The proposed clipping is a combination of square clipping and phase rotation which makes the shape of the clipped signal constellation be a polygon. This operation requires only light weight computations such as comparisons, additions, and subtractions when the resultant constellation is a octagon. Although the shape approaches to the ideal (circle) by increasing the number of operations, the gain by the additional computational complexities is little.

We numerically compared the EVM and ACLR performances among the conventional circle clipping, square clipping, sector clipping, and the proposed clipping under the same PAPR requirement From the numerical results of EVM, we can obtain almost the same performance to the circle clipping by using the octagon clipping. Compared with conventional sector clipping with 3 sectors and 5 sectors, 0.5 dB and 0.3 dB gains can be achieved by octagon clipping, respectively. The ability to suppress the out-of-band emission by the proposed method is also confirmed to be equivalent to circle clipping.

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