Near Field Source Localization and Tracking Using a Passive Sensor Array

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ABSTRACT

In this paper, we consider the problem of passive localization and tracking of multiple sources in near field. For high resolution source localization, the multiple signal classification (MUSIC) algorithm can be extended to its 2-D version by accounting for spherical curvature and spreading factors in the array manifold. For tracking the sources, simultaneous estimation of *x* and *y* coordinates of sources using the 2-D MUSIC spectrum requires exhaustive two dimensional search in each snapshot. To reduce the computational complexity, an alternative Newton type algorithm is proposed for target tracking. Simulation results show good performance of the proposed algorithms.

***Index Terms*—** Source localization, near field, MUSIC algorithm, tracking algorithm.

1. INTRODUCTION

In the last three decades, considerable efforts have been made to estimate the parameters of the signals arriving at the sensors of an array. Estimation of the direction of arrival angles (DOAs) from the received signals has attracted considerable interest in wireless communication, radar, sonar, and mobile systems. The classical subspace methods such as MUSIC [1]-[2] and ESPRIT [3]-[4] are two of the most popular algorithms for DOA estimation.

Localization of radiating sources by a passive sensor array is an important problem in various applications. The aforementioned methods and several other algorithms that have been proposed for DOA estimation assume that the sources are located in the far field so that the propagating waves emanated from them have planar wavefronts when reach the array. When sources are close to the array, these wavefronts can not be assumed to be planar anymore. Generally, the wavefront curvature is spherical in the near field region. So, the conventional DOA Estimation algorithms cannot be used in this case.

For near field sources, Huang and Barkat [5] proposed a version of MUSIC algorithm which needs search in bearing-range domain. Weiss and Friedlander [6] proposed an algorithm which involves search in the range domain combined with polynomial rooting. Starer and Nehorai [7] developed an algorithm based on path-following which can be used only with uniform linear arrays.

In situations that the sensors or the sensor arrays are distributed in a relatively vast region, it is better to use two-dimensional Cartesian coordinates instead of polar coordinates. In this paper, we use a two-dimensional MUSIC algorithm for estimating the Cartesian coordinates of the sources. In addition, we develop a Newton type tracking algorithm for updating these estimates in each snapshot. Therefore, the two-dimensional search in each snapshot is avoided. Simulation results are presented which show the good performance of the proposed algorithm.

2. MODEL OF OBSERVATIONS

We assume *r* near field omnidirectional sources at unknown locations. The sources are emitting spherical waves impinging on an array with *n* omnidirectional sensors ($n>r$) where the locations of sensors are arbitrary. Assume that the vector $\mathbf{z}\in C^n$ contains sensor outputs at each time instant. This vector satisfies the following model at time instant *m*:

$$\mathbf{z}(m)=\mathbf{A}(x,y)\mathbf{s}(m)+\mathbf{n}(m) \qquad (1)$$

where $\mathbf{s}\in C^r$ is the vector of complex signal amplitudes, $\mathbf{n}\in C^n$ is an additive noise vector, $\mathbf{A}(x,y)=[\mathbf{v}(x_1,y_1),\mathbf{v}(x_2,y_2),\ldots,\mathbf{v}(x_r,y_r)] \in C^{n\times r}$ is the matrix of the steering vectors, and x_j and y_j , $j=1,2,\ldots,r$ are the coordinates of sources in Cartesian system. We assume that the elements of $\mathbf{s}(m)$ are stationary random processes, and the elements of $\mathbf{n}(m)$ are zero-mean stationary random processes which are uncorrelated with the elements of $\mathbf{s}(m)$. The $\mathbf{v}(x_k,y_k)$ elements in the steering vector, depend on the geometrical structure of the array and the sources. The complex response of the

ith sensor to the kth impinging signal can be expressed as

$$v_i(x_k, y_k) = \frac{1}{u_{ik}} \exp(-j\frac{2\pi}{\lambda} u_{ik}) \quad (2)$$

where j is the unit imaginary number, λ is the wavelength of the impinging waves, and u_{ik} is the distance between the kth source and the ith sensor defined as

$$u_{ik} = [(x_k - \tilde{x}_i)^2 + (y_k - \tilde{y}_i)^2]^{\frac{1}{2}} \quad (3)$$

where $\tilde{x}_i$ and $\tilde{y}_i$ (i=1,2,…,n) are the coordinates of the sensors in Cartesian system. u_{ik} also represents the spherical spreading factor. This factor is used for describing the fact that the amplitude of a spherical wave decays as the wave propagates away from the source and is inversely proportional to the propagating distance. We assume that both sensors and sources are omnidirectional. d_0 defines the minimum distance between sources and sensors. Therefore, the SNR of each of sensors for each source in (dB) can be obtained as

$$(SNR)_{ik} = 10\log E\{\mathbf{s}_k^2\} - 10\log E\{\mathbf{n}_i^2\} - 10\log(u_{ik}^2) \quad (4)$$

where $(SNR)_{ik}$ is the SNR of the signal of the kth source at ith sensor, $E\{\mathbf{s}_k^2\}$ is the power of kth source and $E\{\mathbf{n}_i^2\}$ is the power of the noise of the ith sensor. We assume that the noises of sensors are uncorrelated and have equal powers.

3. TWO DIMENSIONAL MUSIC ALGORITHM

The one-dimensional MUSIC algorithm can be easily modified to estimate the location of sources in two-dimensional Cartesian coordinate system. Let $\hat{\mathbf{R}}$ be the sample correlation matrix given by

$$\hat{\mathbf{R}} = \frac{1}{M}\sum_{i=1}^{M} \mathbf{z}(i)\mathbf{z}^H(i) \quad (5)$$

where the transcript H denotes the conjugate transposition and M determines the number of snapshots. Let λ_i and $\mathbf{u}_i$ (i=1,2,…,n) be the eigenvalues and the corresponding orthonormal eigenvectors of $\hat{\mathbf{R}}$ respectively. If the number of signal sources r is less than n, then the eigenvalues of $\hat{\mathbf{R}}$ which are sorted in descending order are given by:

$$\lambda_1 > \lambda_2 > ... > \lambda_r > \lambda_{r+1} = ... = \lambda_n \quad (6)$$

The dominant eigenpairs ($\lambda_i, \mathbf{u}_i$) for i=1,..,r are termed the signal eigenvalues and signal eigenvectors while ($\lambda_i, \mathbf{u}_i$) for i=r+1,..,n are referred to as the noise eigenvalues and noise eigenvectors. The column spans of

$$\mathbf{U}_S = [\mathbf{u}_1, ..., \mathbf{u}_r] \quad \text{and} \quad \mathbf{U}_N = [\mathbf{u}_{r+1}, ..., \mathbf{u}_n] \quad (7)$$

are called as the signal and noise subspaces, respectively.

The steering vector $\mathbf{v}(x,y)$ is defined as

$$\mathbf{v}(x, y) = [\frac{1}{u_1}\exp(-j\frac{2\pi u_1}{\lambda}), ..., \frac{1}{u_n}\exp(-j\frac{2\pi u_n}{\lambda})]^T \quad (8)$$

where the transcript T denotes the transpose of a vector and u_i is defined by

$$u_i = [(x - \tilde{x}_i)^2 + (y - \tilde{y}_i)^2]^{\frac{1}{2}}, \; i=1,2,\ldots,n \quad (9)$$

The MUSIC estimates of the source locations in Cartesian coordinate are calculated as the parameters which minimize the following expression

$$P(x, y) = \mathbf{v}^H(x, y)\mathbf{U}_N\mathbf{U}_N^H\mathbf{v}(x, y) \quad (10)$$

The estimates of the source locations can be obtained also by the following normalized cost function [8]

$$Q(x, y) = \frac{\mathbf{v}^H(x, y)\mathbf{U}_N\mathbf{U}_N^H\mathbf{v}(x, y)}{\mathbf{v}^H(x, y)\mathbf{v}(x, y)} \quad (11)$$

Using Q has the advantage that the level of background in MUSIC spectrum is equal for near and far sources. At the same time, using Q causes a far source to have a lower peak in the spectrum than a near source with equal power. As far sources usually produce high sidelobes in the spectrum, using P may cause these sidelobes to be greater than real peaks that are near to the sensors. The cost function Q reduces the risk of encountering such situation by attenuating peaks that are far from sensors. Thus, we use the cost function Q for source localization.

4. A TRACKING ALGORITHM FOR NEAR FIELD SOURCES

For localization of sources, one can search in two dimensional (Cartesian coordinate system) once and find the nulls of the normalized spectrum. But for tracking the sources, simultaneous estimation of x – coordinate and y – coordinate of source using the normalized 2-D MUSIC spectrum, requires exhaustive two dimensional search in each snapshot. To reduce the computational complexity, an alternative Newton type algorithm is proposed.

Suppose that the kth source exists in ith snapshot and we want to track the source in (i+1)th snapshot. Thus, according to the Newton algorithm, the estimation of source location in (i+1)th snapshot can be achieved by

$$\mathbf{w}_k(i+1) = \hat{\mathbf{w}}_k(i) - ((\nabla^2 P(\mathbf{w}))^{-1}\nabla P(\mathbf{w}))\Big|_{\mathbf{w}=\hat{\mathbf{w}}_k(i)} \quad (12)$$

where

$$\hat{\mathbf{w}}_k(i) = [\hat{x}_k(i), \hat{y}_k(i)]^T \quad (13)$$

$$\nabla P = [\frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}]^T \quad (14)$$

$$\nabla^2 P = \begin{bmatrix} \frac{\partial^2 P}{\partial x^2} & \frac{\partial^2 P}{\partial x \partial y} \\ \frac{\partial^2 P}{\partial x \partial y} & \frac{\partial^2 P}{\partial y^2} \end{bmatrix} \quad (15)$$

where $\hat{x}_k(i)$ and $\hat{y}_k(i)$ are the estimated coordinates of the *k*th source in *i*th snapshot, ∇P and $\nabla^2 P$ are the gradient vector and Hessian matrix of *P* respect to *x* and *y*. After some manipulations, we have

$$\frac{\partial P}{\partial x} = 2\,\mathrm{Re}\{\mathbf{v}_x^H \mathbf{U}_N \mathbf{U}_N^H \mathbf{v}\} \quad (16)$$

$$\frac{\partial P}{\partial y} = 2\,\mathrm{Re}\{\mathbf{v}_y^H \mathbf{U}_N \mathbf{U}_N^H \mathbf{v}\} \quad (17)$$

$$\frac{\partial^2 P}{\partial x^2} = 2\,\mathrm{Re}\{\mathbf{v}_{xx}^H \mathbf{U}_N \mathbf{U}_N^H \mathbf{v}\} + 2\mathbf{v}_x^H \mathbf{U}_N \mathbf{U}_N^H \mathbf{v}_x \quad (18)$$

$$\frac{\partial^2 P}{\partial y^2} = 2\,\mathrm{Re}\{\mathbf{v}_{yy}^H \mathbf{U}_N \mathbf{U}_N^H \mathbf{v}\} + 2\mathbf{v}_y^H \mathbf{U}_N \mathbf{U}_N^H \mathbf{v}_y \quad (19)$$

$$\frac{\partial^2 P}{\partial x \partial y} = 2\,\mathrm{Re}\{\mathbf{v}_{xy}^H \mathbf{U}_N \mathbf{U}_N^H \mathbf{v} + 2\mathbf{v}_x^H \mathbf{U}_N \mathbf{U}_N^H \mathbf{v}_y\} \quad (20)$$

where $\mathbf{v}_x = \frac{\partial \mathbf{v}}{\partial x}$, $\mathbf{v}_y = \frac{\partial \mathbf{v}}{\partial y}$, $\mathbf{v}_{xx} = \frac{\partial^2 \mathbf{v}}{\partial x^2}$, $\mathbf{v}_{yy} = \frac{\partial^2 \mathbf{v}}{\partial y^2}$, $\mathbf{v}_{xy} = \frac{\partial^2 \mathbf{v}}{\partial x \partial y}$. Equations (12)-(20) describe the recursive adaptive algorithm for tracking the *k*th source. For initialization, one can use the proposed two dimensional MUSIC algorithm for estimation of the primary locations of the sources.

5. SIMULATION RESULTS

We consider two uniform linear arrays where the number of sensors in each array is *n*=20 and the distance between adjacent sensors in each array is equal to half wavelength. We assume that the wavelength is equal to 2. The first array is located between the points (40,0) and (59,0) and the second array is located between the points (0,40) and (0,59) in Cartesian coordinate system. The noise processes of sensors are uncorrelated white Gaussian processes with equal powers. In all scenarios in this section, the number of simulation runs used for obtaining each point is equal to 100 and the step size used in searching for peaks is 0.3.

In the first scenario, we investigate the performance of the proposed two dimensional MUSIC algorithm in estimating the source locations. To do so, we consider two sources with equal powers that are located at (50,50) and (45,20) in Cartesian coordinate system. The SNR of the first and second sources at the origin is equal to -6.98 dB and -3.84 dB respectively. Figure 1 shows the normalized MUSIC spectrum (inverse of *Q*) for this scenario. It can be seen that two sharp peaks appear in the spectrum that their locations determine the estimated locations of the sources. To evaluate the performance of the algorithm in each SNR, the first scenario is repeated for various SNRs

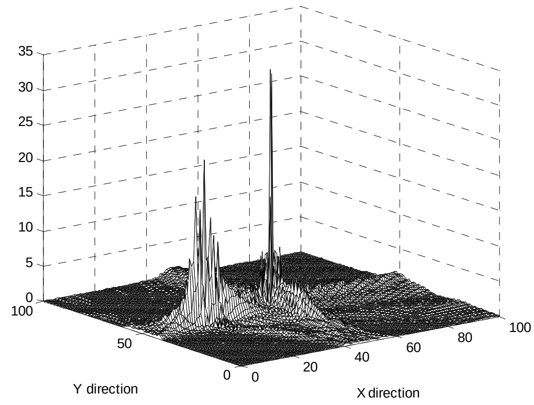

Figure 1. Two dimensional MUSIC spectrum

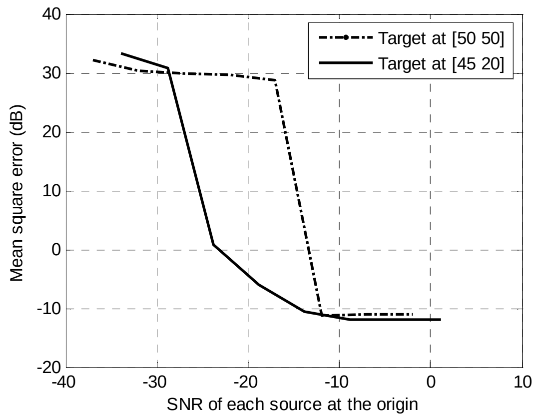

Figure 2. Mean square error of each source vs. SNR of that source at the origin

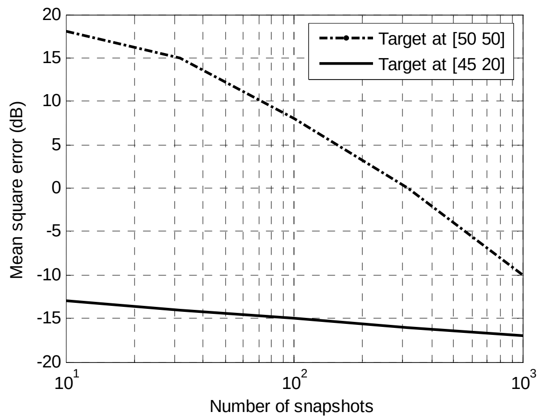

Figure 3. Mean square error vs. number of snapshots for each source

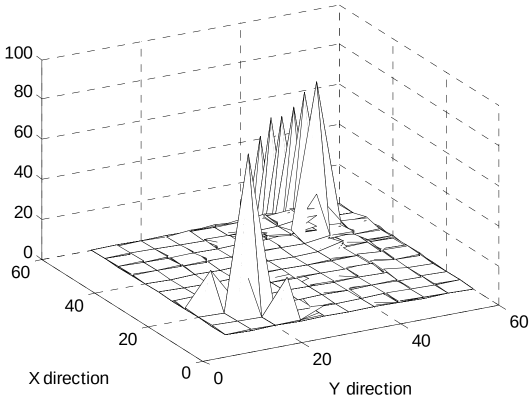

Figure 4. Two dimensional MUSIC spectrum

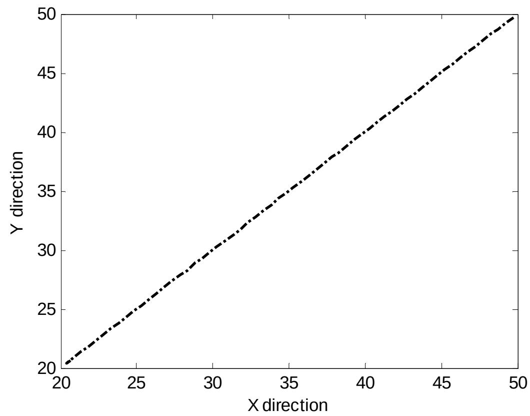

Figure 5. Estimated trajectory of the source

and the mean square error in each SNR is evaluated. Figure 2 shows mean square error of the localization algorithm for each source versus the SNR of that source at the origin. The number of snapshots used in figures 1 and 2 is equal to 1000.

To investigate the effect of number of snapshots on the performance of the MUSIC algorithm, the mean square localization error for each source is shown in figure 3 versus number of snapshots. In this figure, the SNRs of the two sources at the origin are equal to -6.98 dB and -3.84 dB, respectively.

In the second scenario, the performance of the proposed tracking algorithm has been investigated. To do so, we consider a constant velocity source that changes its location from (20,20) to (50,50) in Cartesian coordinate system. In this test, the source has constant power but the SNR in origin changes from 10 dB to 3 dB during the motion. Number of snapshots used in each run of the MUSIC algorithm was equal to 100. All 2-D MUSIC spectra obtained during this location change are shown in figure 4. In figure 5, the estimated trajectory of the source derived by the proposed tracking algorithm is depicted. This figure shows the high performance of the proposed algorithm in tracking the source in this scenario.

Note that in the first scenario the MUSIC estimates are obtained using cost function Q but, in the second scenario cost function P is used for tracking ,however, figure 4 which is depicted using Q.

6. CONCLUSION

In this paper, we used a two-dimensional MUSIC algorithm for source localization in Cartesian system. This 2-D MUSIC algorithm is suitable for cases where sources are in near field and sensors are distributed in a relatively vast region. For tracking the sources, finding the peaks of the two-dimensional MUSIC spectrum is time consuming. To reduce the computational complexity, a Newton type algorithm was proposed that recursively estimated the positions of the sources in Cartesian coordinate system. Two scenarios were used to evaluate the performance of the proposed estimation and tracking algorithms. Simulation results showed that the algorithms are successful in locating and tracking the sources.

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