PAPR Reduction of OFDM Signals Using Deliberate Clipping and Pre-scrambling Technique

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***Abstract*—Orthogonal Frequency Division Multiplexing (OFDM) is considered to be a promising technique against the multipath fading channel for wireless communications. One of the disadvantages of OFDM is peak to average power ratio (PAPR) problem. In this paper, we analyze a PAPR reduction system which combines a selected mapping technique (SLM) and a deliberate clipping technique. The numerical analysis and computer simulation show that the system has effective PAPR reduction capability, moderate system complexity and reasonable bit error rate (BER) performance.**

***Index Terms*—Selected Mapping, Clipping, PAPR, OFDM**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is considered to be a promising technique against the multipath fading channel for wireless communications. However, OFDM has a serious peak to average power ratio (PAPR) problem. The simplest and most effective method to reduce PAPR might be the clipping and filtering [1], but the error performance of clipped OFDM signal is degraded due to the distortion of the original signals. In non-distortion techniques, several symbol selection schemes, such as partial transmit sequence (PTS) [2] and selected mapping (SLM) [3], are widely used. Symbol selection schemes can obtain a moderate PAPR reduction ability but increase the complexity of OFDM system. To obtain an effective PAPR reduction ability and a moderate system complexity, [4] proposed an OFDM system which combines the deliberate clipping and the symbol selection schemes.

In the paper, base on [4], we analyze the OFDM system which combines deliberate clipping technique and SLM technique. Through the analysis in PAPR reduction capability, system complexity and error performance, we demonstrate that the system has proper performance in real applications.

II. OFDM SIGNALS AND PAPR REDUCTION

OFDM symbols can be given as the sum of a numbers of independent symbols which are modulated onto subchannels of equal bandwidth. Let $X_k\left(k=0,1,\ldots,N-1\right)$ denote the input data symbol whose period is T. Then the complex representation of an OFDM symbol is given as

$$x(t)=\sum_{k=0}^{N-1} X_k \cdot e^{j2\pi k \triangle f t}, 0 \le t < NT \qquad (1)$$

where N is the number of subcarriers, and $\triangle f = 1/NT$ is the subcarrier spacing. The samples are denoted by $x_n\left(n=0,1,\ldots,LN-1\right)$ for the OFDM symbols with the sampling rate L. In the following, we consider the sampling rate to be the Nyquist rate which corresponds to the case of $L=1$. The amplitude of the n th sample of an OFDM symbol is given as $r_n \triangleq |x_n|$. As N is a sufficiently large number, r_n is considered to be approximately equal to a Rayleigh random variable with probability density function
(pdf) given as [4]

$$f_R(r_n)=\frac{2r_n}{P_{in}} e^{-r_n^2/P_{in}}, \quad r_n \ge 0 \qquad (2)$$

where $P_{in}=2\sigma^2$ is the input power of the OFDM signal. The PAPR of the OFDM symbol is defined as the ratio of the peak power and the average power as

$$PAPR \triangleq \frac{\max\left[r_n^2\right]}{E\left[r_n^2\right]}=\frac{\max\left[r_n^2\right]}{P_{in}}, n\in\left[0,N-1\right] \qquad (3)$$

where $E[\cdot]$ denotes the statistical expectation function and $\max[\cdot]$ gives the highest value among the samples. For one OFDM symbol, the probability of the peak amplitude being smaller than a given threshold W can be obtained by [5]

$$\begin{aligned} F_l(W) &= \Pr\left(\max_{0\le n<N} r_n < W\right) \\ &= \Pr(r_0 < W)\cdot\Pr(r_1 < W)\cdots\Pr(r_{N-1} < W) \\ &= \left(1-e^{-W^2/P_{in}}\right)^N \end{aligned} \quad (4)$$

Thus, the cumulative distribution and the complementary cumulative distribution of the peak amplitude can be respectively given as

$$F_l(l) = \left(1-e^{-l^2/P_{in}}\right)^N \quad (5)$$

and

$$F_l^c(l) \triangleq 1-F_l(l) = 1-\left(1-e^{-l^2/P_{in}}\right)^N . \quad (6)$$

To solve the PAPR problem, an OFDM system combining deliberate clipping and SLM can be used. The system model is shown in Fig. 1.

A Deliberate Clipping

Deliberate clipping might be the simplest method to reduce PAPR. This method limits the samples' amplitudes of the input OFDM signal to a predetermined value. The amplitude of the nth output sample of the clipped OFDM signal is given as

$$\tilde{r}_n \triangleq \begin{cases} r_n, & for\ r_n \le W \\ W, & for\ r_n \ge W \end{cases}, \quad (7)$$

and the power of the clipped OFDM signal therefore becomes

$$P_{out} = E\left[\tilde{r}_n^2\right] = \int_0^\infty \tilde{r}_n^2 f_R(r_n) dr_n . \quad (8)$$

Equation (7) shows that deliberate clipping is a memoryless nonlinear transformation. The output of the memoryless nonlinear transformation of OFDM signal x_n can be expressed as

$$\tilde{x}_n = \alpha x_n + d_n \quad (9)$$

where d_n is the distortion term uncorrelated with x_n, and α is an attenuation factor that can be calculated as [4]

$$\begin{aligned} \alpha &= \frac{E_{r_n,\phi_n}\left[r_n \cos\phi_n \tilde{r}_n \cos\phi_n\right]}{\sigma^2} \\ &= \frac{\int_0^\infty r_n \tilde{r}_n f_R(r_n) dr_n \cdot \int_0^{2\pi} \cos^2\phi_n \cdot \frac{1}{2\pi} d\phi_n}{\sigma^2} \end{aligned} \quad (10)$$

where ϕ_n is a random variable which has uniformly distribution on $[0, 2\pi)$. As derived in [4], the SNDR of the clipped OFDM signal $\tilde{x}_n$ can be presented as

$$SNDR = \frac{\alpha^2 P_{in}}{P_{out} - \alpha^2 P_{in} + N_0} \quad (11)$$

where N_0 is the total variance of the AWGN. Therefore, the BER of QPSK OFDM signal after deliberate clipping can be calculated by

$$P_b = Q\left(\sqrt{SNDR}\right) \quad (12)$$

where $Q(x) = (1/2) erfc\left(x/\sqrt{2}\right) = \left(1/\sqrt{2\pi}\right)\int_x^\infty e^{-t^2/2} dt$.

A. B Selected Mapping Technique

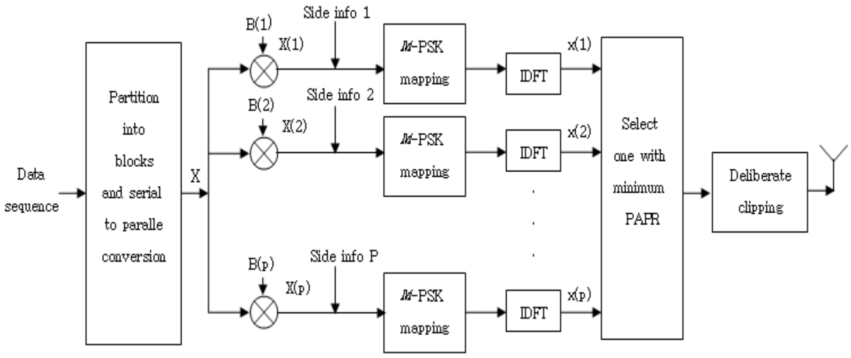
The SLM for PAPR reduction is a non-distortion technique. In this approach, the transmitter generates a set of sufficiently different candidate data symbols, all representing the same information as the original data symbol. Among these symbols, the symbol which has the smallest PAPR value is selected and the information of the selected data symbol is transmitted as the side information. Assume P candidate symbols are generated in the transmitter, then the transmitter needs P IFFT operations and the bits number of required side information is larger than $\log_2 P$ for each symbol. As given in [5], the probability of the peak amplitude of the selected OFDM symbol exceeding the given threshold W can be given as

$$\begin{aligned} F_l^c(W) &= \Pr\left(\min_{1\le p\le P} l_p > W\right) \\ &= \Pr(l_1 > W)\Pr(l_2 > W)\cdots\Pr(l_P > W) \\ &= \left(1-F_{l_p}(W)\right)^P \end{aligned} \quad (13)$$

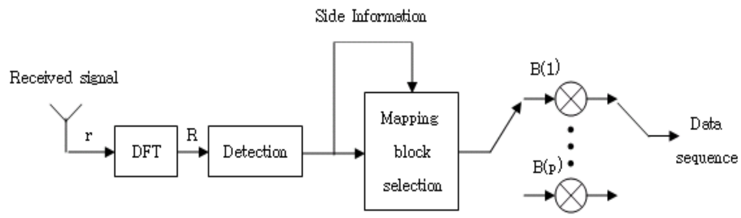
where l_p is the peak amplitude of the pth OFDM symbol and $F_{l_p}(l)$ is the complementary cumulative distribution of l_p . Therefore, the cumulative distribution of the peak amplitude of the selected OFDM symbol can be given as

$$F_l(l) = 1-F_l^c(l) = 1-\left(1-F_{l_p}(l)\right)^P . \quad (14)$$

For the selected OFDM symbol without over-sampling, there are N statistically independent samples for one OFDM symbol period. Just as in (4), (14) can be expressed by



(a)



(b)

Fig. 1. The proposed OFDM system combining clipping and SLM: (a) transmitter and (b) receiver

multiplying N cumulative density functions. Since these cumulative density functions are considered to be same for all samples of one OFDM symbol, the cumulative density function of the amplitude of the n th sample can be calculated from (5) and (14) as

$$\Pr\left(r_n < l\right) = \left(F_l(l)\right)^{1/N} = \left(1-\left(1-\left(1-e^{-l^2/P_{in}}\right)^N\right)^P\right)^{1/N}. \quad (15)$$

By calculating the derivative of the function (15) and substituting r_n for l, we obtain the pdf of r_n by

$$\overline{f}_R(r_n) = P\cdot\left(1-\left(1-\left(1-e^{-r_n^2/P_{in}}\right)^N\right)^P\right)^{1/N-1} \cdot\left(1-\left(1-e^{-r_n^2/P_{in}}\right)^N\right)^{P-1}\cdot\left(1-e^{-r_n^2/P_{in}}\right)^{N-1}\cdot\frac{2r_n}{P_{in}}e^{-r_n^2/P_{in}}. \quad (16)$$

Fig. 2 shows the pdf of the samples' amplitudes of the original OFDM signal and the samples' amplitudes of the selected OFDM signal. We can see that the probability distribution for the non-selected OFDM signal can be approximated as the Rayleigh distribution, and the probability distribution for the adaptively selected OFDM signal shows little difference from the Rayleigh distribution.

C Combination of Deliberate Clipping and SLM

The implementation of deliberate clipping is quite simple and effective in PAPR reduction, but larger clipping ratio results in the severe BER performance degradation. On the other hand, the SLM technique does not cause distortion on the error performance if there are no errors in the sideinformation; however, in order to obtain effective PAPR reduction ability like clipping method, the system complexity becomes challenging as the number of the candidate symbols increases. Thus, the use of only deliberate clipping or only SLM technique cannot obtain satisfactory error performance and moderate system complexity simultaneously. However, if these two approaches are combined, the effective PAPR reduction can be achieved with reasonable BER performance and suitable system complexity. Applying $\hat{f}_R(r_n)$ instead of $f_R(r_n)$ in the BER analysis by substituting (17) into (8), (10)-(12), the BER performance for clipped selected OFDM signals can be calculated.

III. SIMULATION RESULTS

In the simulation, QPSK systems with 256 subcarriers were used. OFDM symbols were transmitted over an AWGN channel. Fig. 3 shows the error performance of the proposed system with different numbers of selective mapping symbols and clipping ratio (CR). CR is given as the ratio of the maximum permissible amplitude and root mean square power of the OFDM signal. Because the selected OFDM signal has smaller PAPR than non-selected OFDM signal, the distortion caused by clipping on the selected OFDM signals is more serious than that on the non-selected OFDM signals. From the figure, we can see that the BER performance of the clipped-selected OFDM signal is improved as the number of selective mapping symbols increases. Fig. 4 shows the PAPR reduction ability of the system combining SLM and deliberate clipping. It is clear that the clipping method is much more effective in PAPR reduction than SLM with smaller value P. Therefore, performing clipping after taking SLM can significantly reduce PAPR and system complexity.

IV. CONCLUSION

In this paper, an OFDM system which combines SLM technique and deliberate clipping technique was discussed. The effect of symbol selection scheme on the deliberate clipping was analyzed by deriving the pdf of the samples' amplitude of the adaptively-selected OFDM symbol. From the analysis and the computer simulation, we can see that there is the tradeoff between the PAPR reduction ability, system complexity and the BER performance.

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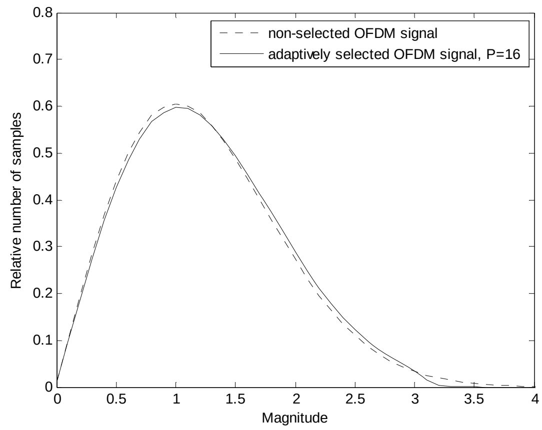

Fig. 2. Probability distribution of samples' amplitudes for non-selected OFDM symbols and selected OFDM symbols (QPSK, 256 subcarriers).

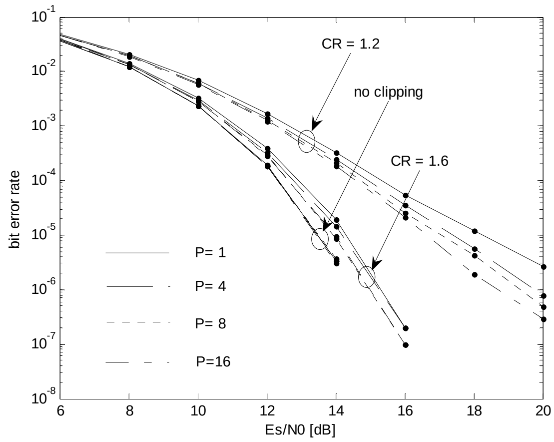

Fig. 3. BER perfromance of clipped-selected OFDM signal, $N = 256$

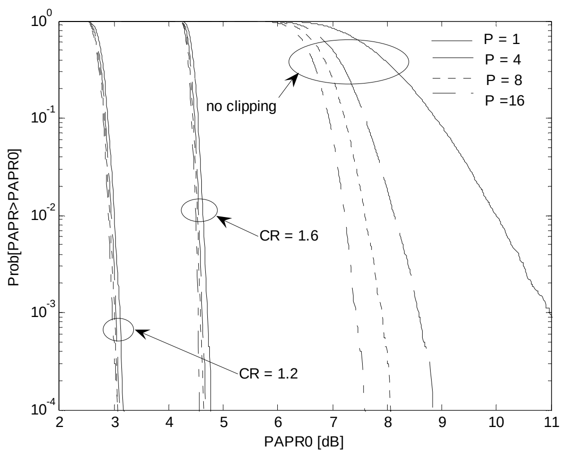

Fig. 4. Complementary cumulative distribution functions of PAPR of an OFDM signal with 256 subcarriers for QPSK modulation.