A Fast Signal Subspace Tracking Algorithm Based on a Subspace Information Criterion

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ABSTRACT

In this paper, we present a new algorithm for tracking the signal subspace recursively. It is based on a new interpretation of the signal subspace. We introduce a novel information criterion for signal subspace estimation. We show that the solution of the proposed constrained optimization problem results the signal subspace. In addition, we introduce three adaptive algorithms which can be used for real time implementation of the signal subspace tracking. The computational complexity of the proposed signal subspace tracking algorithms are $O(nr^2)$ which is much less than the direct computation of singular value decomposition or even some algorithms. To reduce the computational complexity, a fast alternative algorithm is proposed. The complexity of the latter algorithm is $O(nr)$ which makes it feasible in real time applications. Simulation results in the direction of arrival (DOA) tracking context depict excellent performance of the proposed algorithms.

Index Terms— subspace tracking, constrained optimization, adaptive algorithm.

I. INTRODUCTION

Subspace-based signal analysis methods play a major role in contemporary signal processing, with applications including direction of arrival estimation in array processing and frequency estimation of sinusoidal signals in spectral analysis. As their distinguishing feature, these methods seek to extract the desired information about the signal and noise by first estimating either a part or all of the eigenvalue decomposition (EVD) of the data covariance matrix. For example, knowledge of the eigenvalues can be used in connection with a criterion such as AIC or MDL to estimate the number of dominant signal sources present in the observed data [1]. Additional knowledge of the eigenvectors can be used in a high resolution procedure such as MUSIC to estimate unknown parameters of these dominant sources [2].

In recent years, several computationally efficient methods in the form of recursive algorithms have been proposed for sequential estimation and tracking of some or all of the EVD components of a time-varying data covariance matrix. A commonly used approach for the derivation of subspace trackers is to formulate the determination of the desired EVD components as the optimization of the specific cost function involving the unknown data covariance matrix. To arrive at a recursive algorithm, the optimization is accomplished adaptively via an appropriate stochastic search algorithm. Algorithms of this type have been derived based on the constrained gradient search [3], the Gauss-Newton search [4] and the recursive least squares [5]. Another type of approach consists of using classical algorithms from numerical analysis to compute exactly, at regular intervals, the EVD of a time-varying sample covariance matrix or, equivalently, the singular value decomposition (SVD) of a corresponding data matrix. Such a technique based on orthogonal iterations is proposed in [6]. Another approach consists of interlacing the recursive update of a sample covariance or data matrix with only a few steps of certain standard iterations for EVD or SVD computation. Subspace tracker based on the inverse power method [7] is an example algorithm of this approach.

From the computational point of view, we may distinguish between methods having $O(n^2r)$, $O(nr^2)$, or $O(n^3)$ operation counts where n is the number of sensors in the array and r is the dimension of signal subspace. Real time implementation of subspace tracking is needed in some applications and regarding that the number of sensors is usually much more than the number of sources ($n >> r$), algorithms with $O(n^3)$ or even $O(n^2r)$ are not preferred in these cases. It is noteworthy that in this paper operation counts are expressed in terms of multiply/accumulate (MAC) operations.

In this paper, we propose recursive algorithms for tracking the signal subspace spanned by the eigenvectors corresponding to the r largest eigenvalues. These algorithms rely on a new interpretation of the signal subspace as the constrained optimization problem. Therefore, we call our approach as subspace information criterion (SIC). We show that the solution of the optimization problem results signal subspace. Then, three adaptive algorithms for implementation of the optimization problem are proposed. In order to reduce the computational complexity, fast SIC (FSIC) is proposed. Simulation results are given to evaluate the performance of the SIC in the context of adaptive DOA estimation.

This paper is organized as follows. Section II introduces the signal model. In section III, our approach as a constrained optimization problem presented and derivation of the solution is described. Recursive implementation of the proposed solution is derived in section IV. Section V introduces low complexity version of the SIC algorithm. In section VI, simulations are used to evaluate the performance of the proposed algorithms. Finally, the main conclusions of this paper are summarized in section VII.

II. SIGNAL MODEL

Consider the samples $\mathbf{x}(t)$, recorded during the observation time on the n sensor outputs of an array, satisfying the following model

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t) \qquad (1)$$

where $\mathbf{x} \in C^n$ is the vector of sensor outputs, $\mathbf{s} \in C^r$ is the vector of complex signal amplitudes, $\mathbf{n} \in C^n$ is an additive noise vector, $\mathbf{A}(\theta)=[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\ldots,\mathbf{a}(\theta_r)] \in \mathrm{C}^{n\times r}$ is the matrix of the steering vectors $\mathbf{a}(\theta_j)$, and θ_j ; j=1,2,…,r is the parameter of the jth source, for example its DOA. It is assumed that $\mathbf{a}(\theta_j)$ is a smooth function of θ_j and that its form is known (i.e. the array is calibrated). We assume that the elements of $\mathbf{s}(t)$ are stationary random processes, and the elements of $\mathbf{n}(t)$ are zero-mean stationary random processes which are uncorrelated with the elements of $\mathbf{s}(t)$. The covariance matrix of the sensors' outputs can be written in the following form

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{ASA}^H + \mathbf{R}_n \qquad (2)$$

where $\mathbf{S} = E\{\mathbf{s}(t)\mathbf{s}^H(t)\}$ is the signal covariance matrix assumed to be nonsingular (''H'' denotes Hermitian transposition), and $\mathbf{R}_n$ is the noise covariance matrix. A large number of methods such as SVD or EVD use covariance matrix of data to estimate the signal subspace.

III. A NEW SIGNAL SUBSPACE INTERPRETATION

Let $\mathbf{x} \in \mathrm{C}^n$ be a complex valued random vector process with the autocorrelation matrix $\mathbf{C} = E\{\mathbf{xx}^H\}$ which is assumed to be positive definite. The normalized orthonormal eigenvectors and the positive eigenvalues of $\mathbf{C}$ are denoted by $\mathbf{q}_i$ and λ_i (i=1,2,…,n) respectively. If the number of signal sources r is less than n, then the eigenvalues of $\mathbf{R}$ are given by

$$\lambda_1 > \lambda_2 > \ldots > \lambda_r > \lambda_{r+1} = \ldots = \lambda_n \qquad (3)$$

The signal subspace is defined as the column span of

$$\mathbf{Q}_s = [\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_r] \qquad (4)$$

and the noise subspace is defined as the column span of

$$\mathbf{Q}_n = [\mathbf{q}_{r+1}, \mathbf{q}_{r+2}, \ldots, \mathbf{q}_n] \qquad (5)$$

We consider the following constrained minimization problem

$$\underset{\mathbf{W}}{\text{minimize}} \;\; J(\mathbf{W}) = E\left\{\left\|\mathbf{xx}^H - \mathbf{WW}^H\right\|_F^2\right\}$$
$$\textit{subject to} \;\; \mathbf{W}^H(t)\mathbf{W}(t) = \mathbf{I}_r \qquad (6)$$

where $\|.\|_F$ denotes the Frobenius norm and $\mathbf{W} = [\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_r]$ is an $n \times r$ matrix $\mathbf{W} \in C^{n\times r}\,(r<n)$ which is assumed to be full rank. This is no restriction on $\mathbf{W}$, if the rank of $\mathbf{W}$ is $\hat{r} < r$, $\mathbf{W}$ in (6) can be replaced by a full rank $n \times \hat{r}$ matrix $\hat{\mathbf{W}}$ satisfying $\hat{\mathbf{W}}\hat{\mathbf{W}}^H = \mathbf{WW}^H$. The orthonormality constraint can be accomplished by the Gram-Schmidt orthonormalization (GS-orth) procedure.

Now, we want to consider the following questions about the aforementioned minimization problem:

- Is there a global minimum of $J(\mathbf{W})$?
- Are there any local minima of $J(\mathbf{W})$?
- What is the relation between the minimum of $J(\mathbf{W})$ and the signal subspace of $\mathbf{C}$?

We will answer the first and the second questions by the following theorem.

Theorem 1: $J(\mathbf{W})$ has one and only one global minimum and there are not any other minima or maxima in $J(\mathbf{W})$.

Proof. We can write the minimization problem as follows

$$J(\mathbf{W}) = \alpha - 2tr(\mathbf{CWW}^H) + tr((\mathbf{WW}^H)^2) \qquad (7)$$

where

$$\alpha = tr(E\{(\mathbf{xx}^H)^2\}) \qquad (8)$$

We define $\mathbf{Z} = \mathbf{WW}^H$, so (7) can be changed to the following form

$$J(\mathbf{Z}) = \alpha - 2tr(\mathbf{CZ}) + tr(\mathbf{Z}^2) \qquad (9)$$

For discussing about the minima and maxima of $J(\mathbf{W})$, we expand (9) as below

$$J = \sum_{i=1}^{n} \; [(z_{1i}^2 + z_{2i}^2 + \ldots + z_{ni}^2) - 2(c_{1i}z_{i1} + c_{2i}z_{2i} + \ldots + c_{ni}z_{ni})] + \alpha \qquad (10)$$

where z_{ij} and c_{ij} ($i,j = 1,2,\ldots,n$) are the corresponding elements of the ith row and jth column of the matrices $\mathbf{Z}$ and $\mathbf{C}$ respectively.

Since (10) shows a quadratic equation, it is clear that it has convex shape and has one and only one minimum and it has not any other minima or maxima.

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To answer the third question about the relation of the minimum point and the signal subspace, the following theorem is presented.

Theorem 2: J reaches its minimum when $\mathbf{W}$ spans the signal subspace. In this case, $\mathbf{W}$ is an arbitrary basis for the signal subspace.

Proof. We can write (7) as follows

$$J(\mathbf{W}) = \alpha - 2tr(\mathbf{W}^H\mathbf{C}\mathbf{W}) + tr(\mathbf{W}^H\mathbf{W}\mathbf{W}^H\mathbf{W}) \quad (11)$$

Respect to the constraint $\mathbf{W}^H(t)\mathbf{W}(t) = \mathbf{I}_r$, (6) can be replaced with the following problem

$$\underset{\mathbf{W}}{\text{maximize}} \quad \hat{J}(\mathbf{W}) = tr(\mathbf{W}^H\mathbf{C}\mathbf{W})$$
$$\textit{subject to} \quad \mathbf{W}^H(t)\mathbf{W}(t) = \mathbf{I}_r \quad (12)$$

It is well known that the $\mathbf{W}$ which maximizes the aforementioned constrained optimization problem is equal to $\mathbf{Q}_s$. Thus, $\mathbf{W}$ is the signal subspace of the received data.

■

It is noteworthy that at the global minimum of $J(\mathbf{W})$, $\mathbf{W}$ does not contain the signal eigenvectors necessarily. Instead, we attain an arbitrary orthonormal basis of the signal subspace. This is because $J(\mathbf{W})$ is invariant with respect to rotation of the parameter space, indeed $J(\mathbf{W})= J(\mathbf{WU})$ when $\mathbf{U}\mathbf{U}^H=\mathbf{I}_n$. In other words, $\mathbf{W}$ is not determined uniquely when we minimize $J(\mathbf{W})$.

IV. ADAPTIVE SIGNAL SUBSPACE TRACKING

Subspace tracking methods have applications in numerous domains, including the fields of adaptive filtering, source localization, and parameter estimation. In many of these applications we have a continuous stream of data. Thus, developing adaptive algorithms is very useful for these applications. In this section, we propose three adaptive algorithms for signal subspace tracking.

The constrained minimization of the cost function (6) can be accomplished by a constrained gradient search procedure. If the step size be fixed, the weight matrix is updated as

$$\mathbf{W}(k) = \mathbf{W}(k-1) - \mu\nabla J \quad (13)$$

with

$$\mathbf{W}(k) := \text{orthonormalization of the columns of } \mathbf{W}(k) \quad (14)$$

where μ is the step size and ∇J is the gradient of J respect to $\mathbf{W}$. The gradient of $J(\mathbf{W})$ is given by

$$\nabla J = -2\mathbf{C}\mathbf{W} + 2\mathbf{W}\mathbf{W}^H\mathbf{W} \quad (15)$$

Thus, the signal subspace update can be written as

$$\mathbf{W}(t) = \mathbf{W}(t-1) - \mu[-\hat{\mathbf{C}}(t)\mathbf{W}(t-1) + \mathbf{W}(t-1)\mathbf{W}^H(t-1)\mathbf{W}(t-1)] \quad (16)$$

where $\hat{\mathbf{C}}(t)$ is an estimate of the correlation matrix $\mathbf{C}$ at the instant t. We may use an exponentially weighted or a sliding window estimate for $\hat{\mathbf{C}}(t)$. The simplest choice is the instantaneous estimate $\hat{\mathbf{C}}(t) = \mathbf{x}(t)\mathbf{x}^H(t)$ as used in the least mean square (LMS) algorithm for adaptive filtering. The obtained subspace update is expressed by

$$\mathbf{y}(t) = \mathbf{W}^H(t-1)\mathbf{x}(t) \quad (17)$$

$$\mathbf{W}(t) = \mathbf{W}(t-1) + \mu[\mathbf{x}(t)\mathbf{y}^H(t) - \mathbf{W}(t-1)\mathbf{W}^H(t-1)\mathbf{W}(t-1)] \quad (18)$$

We note that a further simplification of the above algorithm can be obtained by replacing $\mathbf{W}^H(t-1)\mathbf{W}(t-1)$ in (18) with identity matrix achieved by the constraint in the previous step. So, we have

$$\mathbf{W}(t) = \mathbf{W}(t-1) + \mu[\mathbf{x}(t)\mathbf{y}^H(t) - \mathbf{W}(t-1)] \quad (19)$$

Table 1 summarizes the so called SIC1 algorithm. Another estimation of $\hat{\mathbf{C}}(t)$ can be achieved by estimate it with exponentially weighted window. Thus, (16) can be changed as follows

$$\hat{\mathbf{C}}(t) = \sum_{i=1}^{t} \beta^{t-i}\mathbf{x}(i)\mathbf{x}^H(i) = \beta\hat{\mathbf{C}}(t-1) + \mathbf{x}(t)\mathbf{x}^H(t) \quad (20)$$

$$\mathbf{W}(t) = \mathbf{W}(t-1) + \mu[\hat{\mathbf{C}}(t)\mathbf{W}(t-1) - \mathbf{W}(t-1)] \quad (21)$$

where β is the forgetting factor used to ensure that data in the past are downweighted in order to afford the tracking capability when the system operates in a nonstationary environment. Table 2 summarizes the so called SIC2 algorithm.

A simplification is obtained by approximating the second $\mathbf{W}(t\text{-}1)$ term in (21) by $\mathbf{W}(i\text{-}1)$. For stationary or slowly varying signals, the difference between $\mathbf{x}(i)\mathbf{W}^H(t\text{-}1)$ and $\mathbf{x}(i)\mathbf{W}^H(i\text{-}1)$ is small, in particular when i is close to t. So, in this case, (16) can be expressed in the following form

$$\mathbf{C}_{xy}(t) = \sum_{i=1}^{t} \beta^{t-i}\mathbf{x}(i)\mathbf{y}^H(i) = \beta\mathbf{C}_{xy}(t-1) + \mathbf{x}(t)\mathbf{y}^H(t) \quad (22)$$

$$\mathbf{W}(t) = \mathbf{W}(t-1) + \mu[\mathbf{C}_{xy}(t) - \mathbf{W}(t-1)] \quad (23)$$

In table 3 summary of the so called SIC3 algorithm has been shown. It should be noted that all proposed adaptive algorithms require orthonormalization procedure, such as Gram-Schmidt (GS) orthonormalization, at the end of each iteration. It can be easily shown that the SIC1, SIC2, and SIC3 have $O(nr)$, $O(n^2r)$, and $O(nr)$ respectively. Moreover, the orthonormalization procedure which needs at the end of each iteration requires additional $O(nr^2)$ operations in the algorithms.

TABLE 1. The SIC1 Algorithm

FOR $t = 1,2,...$ DO
$\mathbf{y}(t) = \mathbf{W}^H(t-1)\mathbf{x}(t)$
$\mathbf{W}(t) = \mathbf{W}(t-1) + \mu[\mathbf{x}(t)\mathbf{y}^H(t) - \mathbf{W}(t-1)]$
$\mathbf{W}(t) :=$ GS-orth. of the columns of $\mathbf{W}(t)$

TABLE 2. The SIC2 Algorithm

FOR $t = 1,2,...$ DO
$\hat{\mathbf{C}}(t) = \sum_{i=1}^{t} \beta^{t-i}\mathbf{x}(i)\mathbf{x}^H(i) = \beta\hat{\mathbf{C}}(t-1) + \mathbf{x}(t)\mathbf{x}^H(t)$
$\mathbf{W}(t) = \mathbf{W}(t-1) + \mu[\hat{\mathbf{C}}(t)\mathbf{W}(t-1) - \mathbf{W}(t-1)]$
$\mathbf{W}(t) :=$ GS-orth. of the columns of $\mathbf{W}(t)$

TABLE 3. The SIC3 Algorithm

FOR $t = 1,2,...$ DO
$\mathbf{y}(t) = \mathbf{W}^H(t-1)\mathbf{x}(t)$
$\mathbf{C}_{\mathbf{xy}}(t) = \beta\mathbf{C}_{\mathbf{xy}}(t-1) + \mathbf{x}(t)\mathbf{y}^H(t)$
$\mathbf{W}(t) = \mathbf{W}(t-1) + \mu[\mathbf{C}_{\mathbf{xy}}(t) - \mathbf{W}(t-1)]$
$\mathbf{W}(t) :=$ GS-orth. of the columns of $\mathbf{W}(t)$

SIC1 requires the minimum operations and the SIC2 needs the maximum operations. The SIC3 requires a little more operations than the SIC1.

V. FAST SIC ALGORITHM

In this section, a fast implementation of the SIC algorithm is proposed. SIC3 has $O(nr)$ complexity and an additional $O(nr^2)$ operations for orthonormalization. The main computational complexity corresponds to orthonormalization step. To reduce the computational complexity, Gram-Schmidt orthonormalization procedure is replaced with an alternative approach.

To this end, using (22) and (23) leads to the following expression

$$\mathbf{W}(t) = (1-\mu)\mathbf{W}(t-1) + \mu[\beta\mathbf{C}_{xy}(t-1) + \mathbf{x}(t)\mathbf{y}^H(t)] \tag{24}$$

It can be inferred from (23) that

$$\mathbf{C}_{xy}(t-1) = \frac{1}{\mu}[\mathbf{W}(t-1) - (1-\mu)\mathbf{W}(t-2)] \tag{25}$$

Substituting (25) into (24) results

$$\mathbf{W}(t) = (1-\mu+\beta)\mathbf{W}(t-1) - \beta(1-\mu)\mathbf{W}(t-2) + \mu\mathbf{x}(t)\mathbf{y}(t) \tag{26}$$

The classical projection approximation [5] is equivalent to $\mathbf{W}(t-1) \approx \mathbf{W}(t-2)$ at each time step. Thus, (26) can be written in the following form

$$\mathbf{W}(t) = \gamma\mathbf{W}(t-1) + \mu\mathbf{x}(t)\mathbf{y}(t) \tag{27}$$

where

$$\gamma = 1 - \mu + \beta\mu \tag{28}$$

The fast orthonormal SIC algorithm consists of the expression (27) plus an orthonormalization step of the weight matrix at each iteration

$$\mathbf{W}(t) := \mathbf{W}(t)[\mathbf{W}^H(t)\mathbf{W}(t)]^{\frac{-1}{2}} \tag{29}$$

where $[\mathbf{W}^H(t)\mathbf{W}(t)]^{\frac{-1}{2}}$ denotes an inverse square root of $[\mathbf{W}^H(t)\mathbf{W}(t)]$. To compute the (29), we use the updating equation of $\mathbf{W}(t)$. Keeping in mind that $\mathbf{W}(t\text{-}1)$ is now an orthonormal matrix.

By invoking (27) we have

$$\mathbf{W}^H(t)\mathbf{W}(t) = \gamma^2\mathbf{I}_r + \gamma\mu\mathbf{W}^H(t-1)\mathbf{x}(t)\mathbf{y}^H(t) + \gamma\mu\mathbf{y}(t)\mathbf{x}^H(t)\mathbf{W}(t-1) + \mu^2\mathbf{y}(t)\mathbf{x}^H(t)\mathbf{x}(t)\mathbf{y}^H(t) \tag{30}$$

Using projection approximation (17) leads to

$$\mathbf{W}^H(t)\mathbf{W}(t) = \gamma^2\mathbf{I}_r + \gamma\mu\mathbf{y}(t)\mathbf{y}^H(t) + \gamma\mu\mathbf{y}(t)\mathbf{y}^H(t) + \mu^2\mathbf{y}(t)(\mathbf{x}^H(t)\mathbf{x}(t))\mathbf{y}^H(t) \tag{31}$$

For reduce the complexity further, we apply the matrix inversion lemma to (31). The matrix inversion lemma (MIL) can be written as follows

$$(\mathbf{A}+\mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}(\mathbf{DA}^{-1}\mathbf{B} + \mathbf{C}^{-1})^{-1}\mathbf{DA}^{-1} \tag{32}$$

By employing (32), the following expression can be obtained

$$[\mathbf{W}^H(t)\mathbf{W}(t)]^{-1} = \frac{1}{\gamma^2}[\mathbf{I}_r - \frac{\mathbf{y}(t)\mathbf{y}^H(t)}{\mathbf{y}^H(t)\mathbf{y}(t) + \delta(t)}] \tag{33}$$

where

$$\delta(t) = \frac{\gamma^2}{2\gamma\mu + \mu^2\mathbf{x}^H(t)\mathbf{x}(t)} \tag{34}$$

It can be shown that $[\mathbf{W}^H(t)\mathbf{W}(t)]^{\frac{-1}{2}}$ can be obtained in the following form

$$[\mathbf{W}^H(t)\mathbf{W}(t)]^{\frac{-1}{2}} = \gamma^{-1}[\mathbf{I}_r + \tau(t)\mathbf{y}(t)\mathbf{y}^H(t)] \tag{35}$$

where

$$\tau(t) = \frac{1}{(\mathbf{y}^H(t)\mathbf{y}(t))}\left(\frac{1}{\sqrt{1+\delta^{-1}(t)\mathbf{y}^H(t)\mathbf{y}(t)}} - 1\right) \tag{36}$$

Substituting (27) and (35) into (29) results that

$$\mathbf{W}(t) = [\gamma\mathbf{W}(t-1) + \mu\mathbf{x}(t)\mathbf{y}(t)]\gamma^{-1}[\mathbf{I}_r + \tau(t)\mathbf{y}(t)\mathbf{y}^H(t)] \tag{37}$$

Finally, the following recursive expression is used for updating the signal subspace

Table 4. The FSIC algorithm

FOR $t = 1,2,...$ DO

$$\mathbf{y}(t) = \mathbf{W}^H(t-1)\mathbf{x}(t)$$

$$\delta(t) = \frac{\gamma^2}{2\gamma\mu + \mu^2\mathbf{x}^H(t)\mathbf{x}(t)}$$

$$\tau(t) = \frac{1}{(\mathbf{y}^H(t)\mathbf{y}(t))}\left(\frac{1}{\sqrt{1+\delta^{-1}(t)\mathbf{y}^H(t)\mathbf{y}(t)}} - 1\right)$$

$$\mathbf{W}(t) = \mathbf{W}(t-1) + \tau(t)(\mathbf{W}(t-1)\mathbf{y}(t))\mathbf{y}^H(t) + \left(\frac{\mu + \mu\tau(t)\mathbf{y}^H(t)\mathbf{y}(t)}{\gamma}\right)\mathbf{x}(t)\mathbf{y}^H(t)$$

Table 5. Computational complexity of the algorithms

Algorithm	Cost (MAC)
SIC1	$O(nr^2)+3nr$
SIC2	$n^2r+2n^2+O(nr^2)+2nr$
SIC3	$O(nr^2)+5nr$
FSIC	$4nr+n+r^2+O(r)$

$$\mathbf{W}(t) = \mathbf{W}(t-1) + \tau(t)(\mathbf{W}(t-1)\mathbf{y}(t))\mathbf{y}^H(t) + \left(\frac{\mu + \mu\tau(t)\mathbf{y}^H(t)\mathbf{y}(t)}{\gamma}\right)\mathbf{x}(t)\mathbf{y}^H(t) \quad (38)$$

This fast SIC algorithm, which is referred as to FSIC, requires only $O(nr)$ operations which makes it feasible in real time applications. Table 4 summarizes the FSIC algorithm.

Table 5 is given to compare the computational complexity of the proposed algorithms. It can be inferred from this table that the FSIC and SIC2 have the lowest and highest computational complexity, respectively. Since the complexity of SIC2 is $O(n^2r)$, this algorithm is not preferred in on-line applications. On the other hand, the FSIC algorithm can be used in on-line applications.

VI. SIMULATION RESULTS

In this section, we use simulations to demonstrate the applicability and performance of the SIC algorithms. To do so, we consider the proposed algorithm in DOA estimation context. We use MUSIC algorithm for finding the DOAs of signal sources impinging on an array of sensors. We consider a uniform linear array where the number of sensors is n=21 and the distance between adjacent sensors is equal to half wavelength. In the first scenario of this section the number of simulation runs used for obtaining each point is equal to 100.

In the first scenario, we assume that two signal sources are located constantly at $(-40^{o},40^{o})$ and their

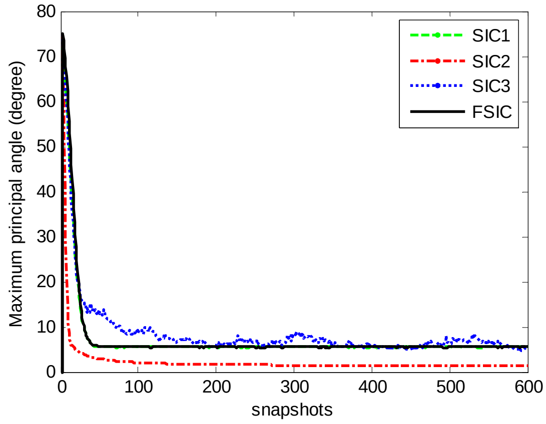

Figure 1. Maximum principal angle vs. snapshots

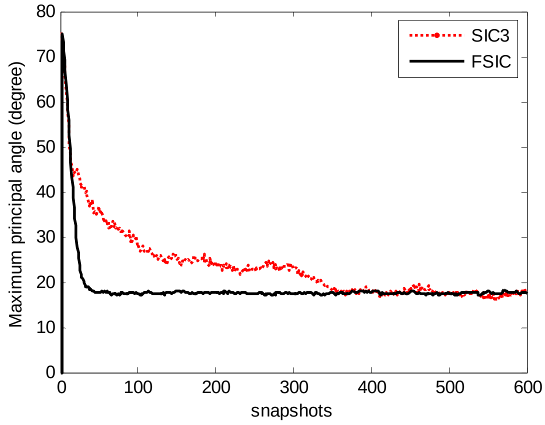

Figure 2. Maximum principal angle vs. snapshots

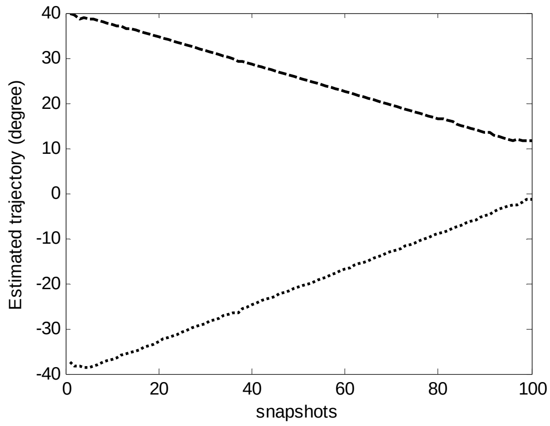

Figure 3. Estimated trajectory vs. snapshots

SNR is equal to 10 dB. In addition, we let the step size μ equal to 0.001. To evaluate the performance of the proposed SIC algorithms in tracking of the signal subspace, the maximum principal angles of the adaptive algorithms are measured. Principal angle is a measure of the difference between the estimated subspace and the real subspace. The principal angles are zero if the compared subspaces are identical. In figure 1, we have depicted the maximum principal angle for this scenario. From this figure, it can be seen that the performance of the SIC1 and FSIC are approximately equal. Figure 1 shows that performance of SIC2 outperforms the other algorithms. The SIC1, SIC3, and FSIC have the same performance.

To demonstrate the performance of the proposed algorithms in low SNR, two sources with 0 dB SNR are considered. In addition, we let the step size μ equal to 0.01. Figure 2 shows the maximum principal angles for SIC3 and FSIC algorithms. It depicts the appropriate performance of the SIC3 and FSIC algorithms.

For investigation the performance of the proposed algorithms in nonstationary environments, we consider the first scenario. To this end, we assume that two signal sources change their locations from $(-40^o, 40^o)$ to $(0^o, 10^o)$ and SNR of each source is equal to 10 dB as the previous scenario. Figure 3 shows the estimated trajectory of the sources achieved by the MUSIC and FSIC algorithms. This figure depicts that the SIC's algorithms can be used for subspace tracking in DOA tracking context.

VII. CONCLUDING REMARKS

In this paper, we introduced a new interpretation of the signal subspace which is based on a novel constrained optimization problem. We proved that the solution of the proposed constrained minimization results the signal subspace. Then, we derived three adaptive algorithms for signal subspace tracking. To reduce the complexity, FSIC proposed. The total computational complexity of the SIC1, SIC2, SIC3, and FSIC are $O(nr^2)$, $O(n^2r)$, $O(nr^2)$, and $O(nr)$ respectively. Simulation results in DOA tracking context showed the perfect performance of the proposed algorithms. Respect to the computational complexity and the performance of the algorithms shown in the simulation results section, the FSIC algorithm is superior to the other algorithms and has perfect performance.

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