A COOPERATIVE METHOD FOR TX/RX MATRIX ESTIMATION IN A MULTI-ANTENNA COMMUNICATION SYSTEM

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Abstract

In this paper we assume multi-antenna communication systems which are able to transmit and receive signal in a same or very near frequency bounds. Our goal is to improve the system performance by using weighting matrices at transmitter and receiver. We propose and study a cooperative algorithm in order to find the proper transmit beamforming and receive combining matrices for increasing the system performance without direct channel measurement and additional computations for beamforming.

1. INTRODUCTION

Using multiple antennas at both transmitter and receiver is an attractive method to combat the destructive effect of channel fading and significantly increase the spectral efficiency in wireless systems [1][2]. Multiple-input multiple-output (MIMO) systems can provide a diversity gain in proportion to the product of the number of transmit and receive antennas. One method for exploiting the significant diversity gain and simplifying the detection in a MIMO communication system is to use some proper set of transmit beamforming vectors and some proper set of receive combining vectors (through this manuscript they are called the transmit beamforming matrix and receive combining matrix, respectively) [2]. Such a transmit/receive scheme can result in considerable improvement in signal to noise ratio [3].

Gaining all of the advantages related to beamforming, requires knowledge about the channel matrix or knowledge of proper beamforming matrices at both transmitter and receiver.

Channel training is an important way to extract the channel information at the receiver side[4]. It is also possible that receiver inform the transmitter about channel for proper beamforming with some sort of feedback through a low bandwidth feedback channel [5][6].

We assume multi-antenna communication systems which are able to both transmit and receive data. Our goal is to improve the system performance by using weighting matrices at both sides. Here, a cooperative scheme is proposed to directly compute the beamforming matrix at transmit and receive stations. Thus, it is possible to compute a proper beamforming matrices at transmitter and receiver without direct channel knowledge.

Throughout this paper $E\{.\}$, $\|.\|$, $(.)^T$, $(.)^*$, $(.)^H$ denote the expected value, Frobenius norm, transpose, conjugate and hermitian of matrix, respectively. Notation $[\mathbf{A}]_{ij}$ shows the element which lays in the ith row and jth column of matrix $\mathbf{A}$. $\mathbf{I}_k$ shows k-dimensional identity matrix. $\mathbb{C}^{m\times n}$ is used to show the set of $m \times n$ dimensional complex matrices and $\mathbb{C}^m$ shows the set of m dimensional complex vectors.

2. SYSTEM MODEL

We consider two transmit/receive systems where one is equipped with M_A antennas and the other has M_B antennas. Here, these two systems are called node A and node B, respectively. Node A has the ability to transmit signal at carrier frequency f_A for B via channel matrix $\mathbf{H}_{AB}$ and to receive the transmit signal from B at carrier frequency f_B via channel matrix $\mathbf{H}_{BA}$ and vice versa.

In a general case, $\mathbf{H}_{AB}$ and $\mathbf{H}_{BA}$ may differ from each other, but in the case that f_A and f_B are the same (like in a time division duplex system) or when these two frequencies are close to each other, because of the *reciprocity principle*, it is logical to assume that,

$$\mathbf{H}_{AB} = \mathbf{H}_{BA}^T \stackrel{\text{def}}{=} \mathbf{H}$$

We also assume that the channel is stationary during a sufficiently long period of time. Additionally, the elements of $M_B \times M_A$ channel matrix $\mathbf{H}$ are considered to be identically independent with complex Gaussian distribution and unity variance. Such a distribution for the elements of channel is a proper model in full scattering environments.

In such a scenario, when the baseband vector $\mathbf{a}_t = [a_{t,1}, \ldots, a_{t,M_A}]^T$ is transmitted from node A, the baseband signal vector $\mathbf{b}_r = [b_{r,1}, \ldots, b_{r,M_B}]^T$ received by node B can be expressed as,

$$\mathbf{b}_r = \mathbf{H}\mathbf{a}_t + \mathbf{n}_B \tag{1}$$

Here, $\mathbf{n}_B \in \mathbb{C}^{M_B}$ is a zero mean, circularly symmetric complex Gaussian noise vector with covariance matrix $E\{\mathbf{n}_B\mathbf{n}_B^H\} = \sigma_B^2 \mathbf{I}_{M_B}$.

In a similar way, if $\mathbf{b}_t = [b_{t,1}, \ldots, b_{t,M_B}]^T$ is the baseband transmitted signal from node B the baseband received signal vector $\mathbf{a}_r = [a_{r,1}, \ldots, a_{r,M_A}]^T$ by A can be written as,

$$\mathbf{a}_r = \mathbf{H}^T \mathbf{b}_t + \mathbf{n}_A \tag{2}$$

where $\mathbf{n}_A \in \mathbb{C}^{M_A}$ is the additive Gaussian noisev with covariance matrix $E\{\mathbf{n}_A\mathbf{n}_A^H\} = \sigma_A^2 \mathbf{I}_{M_A}$.

3. BEAMFORMING AND COMBINING FOR MAXIMUM CHANNEL CAPACITY

As shown in Fig. 1a, we would like to use some $M_A \times m_A$ mapping matrix $\mathbf{W}_A$ at node A, in order to transmit the mapped signal $\mathbf{a}_t = \mathbf{W}_A \mathbf{s}_A$ where $\mathbf{s}_A = [s_{A,1}, \ldots, s_{A,m_A}]^T$ is the desired transmit vector. Also, at node B, we use an $M_B \times m_B$ combining matrix $\mathbf{Z}_B$ as shown in Fig. 1b before further processing for transmit signal detection (here, we also call $\mathbf{Z}_B$ as the receive beamforming matrix).

In this way, the received signal $\mathbf{x}_B$ at the output of combiner can be expressed as,

$$\mathbf{x}_B = \mathbf{Z}_B^H \mathbf{H} \mathbf{W}_A \mathbf{s}_A + \mathbf{Z}_B^H \mathbf{n}_B \tag{3}$$

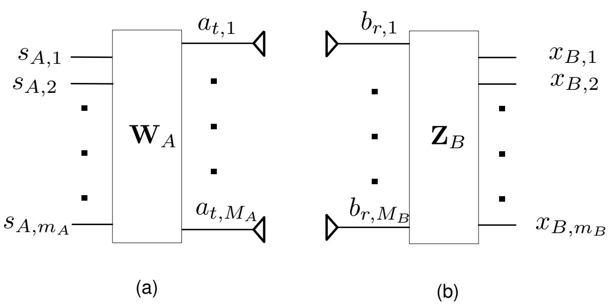

Figure 1. (a) Multi-antenna transmit system with beamforming matrix $\mathbf{W}_A$, (b) multi-antenna receive system with combining matrix $\mathbf{Z}_B$.

For the case that $m_A \leq M_A$, to maximize the achievable data rate when the total transmit power is bounded to value P, it can be shown that the transmit beamforming matrix $\mathbf{W}_A$ has to be computed as [1],

$$\mathbf{W}_A = \mathbf{D}^{+} \mathbf{U}_{m_A} \tag{4}$$

where $\mathbf{D}^{+}$ is obtained from water-filling (also known as water pouring) as,

$$\mathbf{D}^{+} = \mathrm{diag}\big(\sqrt{D_1}, \ldots, \sqrt{D_{M_A}}\big) \tag{5}$$

Here, $D_i = \max\{\mu - \sigma_B^2/\lambda_i, 0\}$ where λ_i is the ith eigenvalue of $\mathbf{H}^H\mathbf{H}$ (we assume that $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots$) and μ is a constant adjusted to satisfy the power constraint $\|\mathbf{W}_A \mathbf{s}_A\|^2 = P$. m_A is equal to the nonzero diagonal elements of matrix $\mathbf{D}^{+}$, and the columns of $\mathbf{U}_{m_A} \in \mathbb{C}^{M_A \times m_A}$ are the first m_A principle eigenvectors of $\mathbf{H}^H\mathbf{H}$.

To simplify the detection of transmitted symbols, it can be readily shown that the receive combining matrix has to be computed from,

$$\mathbf{Z}_B = \mathbf{V}_{m_B} \tag{6}$$

where the columns of $\mathbf{V}_{m_B}$ contains the first m_B principle eigenvectors of $\mathbf{H}\mathbf{H}^H$. Interestingly, m_B is not required to be more than m_A. That is because for $i > (m_B - m_A)$, $[\mathbf{x}_B]_i$ is just a signal free noisy term which bear no information about the transmitted data. Additionally, it can be shown that m_B has not to be less than m_A.

Therefore, for optimal beamforming and proper combining the transmitter has to know the eigenvectors and eigenvalues of $\mathbf{H}^H\mathbf{H}$ and the receiver has to have the eigenvectors of $\mathbf{H}\mathbf{H}^H$.

4. COOPERATIVE METHOD FOR TRANSMIT/RECEIVE BEAMFORMING

4.1. Finding the desired eigenvectors

Let us assume that A sends an arbitrary normalized vector $\mathbf{a}_t^1$. Due to this transmission, node B receives $\mathbf{b}_r^1$ and transmits back $\mathbf{b}_t^1 = \mathcal{N}(\mathbf{b}_r^1)$ to A, where

$$\mathcal{N}(\mathbf{x}) = \frac{\mathbf{x}^*}{\|\mathbf{x}\|}$$

For transmit signal $\mathbf{b}_t^1$, A receives $\mathbf{a}_r^1$ and transmits $\mathbf{a}_t^2 = \mathcal{N}(\mathbf{a}_r^1)$ in turn. This procedure is repeated and can be stopped at kth loop of iteration whenever $\|\mathbf{a}_t^k - \mathbf{a}_t^{k-1}\|$ is less than ε, where ε is some sufficiently small positive value.

Using the above procedure, at kth iteration the transmit signal $\mathbf{a}_t^k$ at node A and the received signal $\mathbf{b}_r^k$ at node B, can be expressed in terms of $\mathbf{a}_t^1$ as,

$$\mathbf{a}_t^k = \frac{(\mathbf{H}^H\mathbf{H})^{(k-1)}}{\|(\mathbf{H}^H\mathbf{H})^{(k-1)}\mathbf{a}_t^1\|}\mathbf{a}_t^1 \tag{7}$$

$$\mathbf{b}_r^k = \frac{(\mathbf{H}\mathbf{H}^H)^{(k-1)}}{\|(\mathbf{H}^H\mathbf{H})^{(k-1)}\mathbf{H}\mathbf{a}_t^1\|}\mathbf{H}\mathbf{a}_t^1 \tag{8}$$

It is interesting to note that the above relations are similar to the so called *power method* that is widely used for obtaining the principal eigenvector of a square matrix[7, 8]. Using the same principle, it is straight to show that $\mathbf{a}_t^k$ and $\mathbf{b}_r^k$ converge to the principal eigenvectors of $\mathbf{H}^H\mathbf{H}$ and $\mathbf{H}\mathbf{H}^H$, respectively.

Now, let us assume that the eigenvectors $\mathbf{q}_1, \ldots, \mathbf{q}_i$ for $\mathbf{H}^H\mathbf{H}$ are known at node A. Thus, node A is able to compute the projection matrix $\mathbf{P}_i$ as,

$$\mathbf{P}_i = \prod_{k=1}^{i} \Big(\mathbf{I}_{M_A} - \frac{\mathbf{q}_k\mathbf{q}_k^H}{\|\mathbf{q}_k\|^2}\Big) \tag{9}$$

Then, A sends $\mathcal{N}(\mathbf{P}_i \mathbf{a}_t^1)$ where $\mathbf{a}_t^1$ is an arbitrary vector. Node B receives $\mathbf{b}_r^1$ and transmits back $\mathbf{b}_t^1 = \mathcal{N}(\mathbf{b}_r^1)$ to A. For transmit signal $\mathbf{b}_t^1$, A receives $\mathbf{a}_r^1$ and transmits $\mathbf{a}_t^2 = \mathcal{N}(\mathbf{P}_i \mathbf{a}_r^1)$. This procedure is repeated and can be stopped whenever $\|\mathbf{a}_t^k - \mathbf{a}_t^{k-1}\| < \varepsilon$ for some sufficiently small positive value ε.

Using this procedure, in a similar way as before, it can be shown that $\mathbf{a}_t^k$ converges to $\mathbf{q}_{i+1}$ and $\mathbf{b}_r^k$ converges to the $(i+1)$th eigenvector of $\mathbf{H}\mathbf{H}^H$.

As a result, knowing the principal eigenvectors of $\mathbf{H}^H\mathbf{H}$ and $\mathbf{H}\mathbf{H}^H$, with the above mentioned cooperative method, the transmitter and receiver are able to find the second principal eigenvectors required for their transmit and receive beamforming matrices. The same procedure can be applied to compute all columns of matrices $\mathbf{U}_{m_A}$ and $\mathbf{Z}_B$.

4.2. Finding the desired eigenvalues

The eigenvalues of matrix $\mathbf{H}^H\mathbf{H}$ has to be known at the transmitter in order to compute $\mathbf{D}^+$ and calculate $\mathbf{W}_A$ from (4).

Consider that we want to estimate the principal eigenvalue λ_n. At the kth iteration of the proposed method for finding the nth column of $\mathbf{U}_{m_A}$ and $\mathbf{Z}_B$, A sends $\mathbf{a}_t^k = \mathcal{N}(\mathbf{P}_{n-1}\mathbf{a}_r^{k-1})$ and B receives $\mathbf{b}_r^k$. Then B transmits the vector $\mathbf{b}_t^k = \mathcal{N}(\mathbf{b}_r^k)$ and A receives $\mathbf{a}_r^k$ for it.

With such a procedure, it is easy to show the following relation between $\mathbf{a}_r^k$ and $\mathbf{a}_t^k$.

$$\mathbf{a}_r^k = \frac{\left(\mathbf{H}^H\mathbf{H}\mathbf{a}_t^k\right)^*}{\|\mathbf{H}\mathbf{a}_t^k\|} \tag{10}$$

Whenever $\mathbf{a}_t^k$ is converged to $\mathbf{q}_n$, (10) becomes,

$$\mathbf{a}_r^k = \frac{\left(\mathbf{H}^H\mathbf{H}\mathbf{q}_n\right)^*}{\|\mathbf{H}\mathbf{q}_n\|} \tag{11}$$

From singular value decomposition, we know that,

$$\mathbf{H} = \sum_{i=1}^{r} \sqrt{\lambda_i}\ \frac{\mathbf{H}\mathbf{q}_i\mathbf{q}_i^H}{\|\mathbf{H}\mathbf{q}_i\|} \tag{12}$$

where $r = Rank(\mathbf{H})$. Thus we have,

$$\mathbf{H}\mathbf{q}_n = \sqrt{\lambda_n}\ \frac{\mathbf{H}\mathbf{q}_n}{\|\mathbf{H}\mathbf{q}_n\|} \quad \rightarrow \quad \|\mathbf{H}\mathbf{q}_n\| = \sqrt{\lambda_n} \tag{13}$$

Putting (13) back into (11) and using the fact that $\mathbf{H}^H\mathbf{H}\mathbf{q}_n = \lambda_n\mathbf{q}_n$ we have,

$$\mathbf{a}_r^k = \sqrt{\lambda_n}\mathbf{q}_n^* \tag{14}$$

As a result, the nth eigenvalue of $\mathbf{H}^H\mathbf{H}$ can be approximated from the norm of received signal vector at node A, i.e. $\widehat{\lambda_n} = \|\mathbf{a}_r^k\|^2$.

Thus, in conjunction with the algorithm of previous subsection, the nth eigenvector and eigenvalue can be found at node A, simultaneously.

It is interesting to note that the advantage of the above method for finding the required eigenvectors and their corresponding eigenvalues, is that it can be used to avoid extra transmission for finding the eigenpairs that are not required for transmit/receive beamforming.

Our proposed procedure is briefly described in following steps,

Initialization: $i = 1,\ \mathbf{D}^+ = \mathbf{0}_{M_A}$.

Step 1: Find the ith eigenvector and eigenvalue of $\mathbf{H}^H\mathbf{H}$ at node A and the ith eigenvector of $\mathbf{H}\mathbf{H}^H$ at node B with the proposed cooperative method.

Step 2: Using (5) to compute the D_j for $1 \le j \le i$

Step 3: If $D_i \le 0$: go to step 4. If $D_i > 0$: $[\mathbf{D}^+]_{jj} = D_j$, jth column of $\mathbf{U}_{m_A} = j$th eigenvector of $\mathbf{H}^H\mathbf{H}$ and jth column of $\mathbf{V}_{m_B} = j$th eigenvector of $\mathbf{H}\mathbf{H}^H$ for $1 \le j \le i$, $i = i+1$; go to Step 1.

Step 4: $m_A = i - 1$, $\mathbf{W}_A = \mathbf{D}^+\mathbf{U}_{m_A}$ and $\mathbf{Z}_B = \mathbf{V}_{m_B}$

Using this procedure, m_A, $\mathbf{W}_A$ and $\mathbf{Z}_B$ are computed at nodes A and B, respectively.

5. COMPUTER SIMULATIONS

For the first simulation, we have assumed that $M_A = 6$, $M_B = 6$ and $\varepsilon = 10^{-4}$ for checking the algorithm convergence. Also, the initial vector $\mathbf{a}_t^1$ for estimation of each eigenvector is selected as $[1, 1, ..., 1]^T/\sqrt{M_A}$.

The Ferobenus norm $\|\mathbf{q}_i - \hat{\mathbf{q}}_i\|$, for $i = 1, 2, 3$, is plotted in Fig. 2 as a function of the number of iterations in a noise free scenario. Here, $\mathbf{q}_i$ is the exact ith eigenvector of $\mathbf{H}^H\mathbf{H}$ and $\hat{\mathbf{q}}_i$ is its estimated value using our proposed method. As this figure shows the error decreasing with the number of iterations.

Relative error in the eigenvalues estimation is plotted in Fig. 3. Here, the additive noise is considered in our simulations assuming that the noise power is the same at both nodes and the training SNR is the system SNR during our proposed cooperative method for eigenpairs and eigenvalues estimation. In this simulation, the eigenvectors are the output of our algorithm at 5th iteration. The relative error in estimation of λ_i increases with index i as shown in this simulation.

For the next simulation we have assumed that $M_A = 2$ and $M_B = 2$. Fig. 4 compares the Shannon capacity of a system which uses proper beamforming matrices computed from

$$C = \log_2 |\mathbf{I}_{m_B} + \tfrac{1}{\sigma^2}\mathbf{Z}_B^H\mathbf{H}\mathbf{W}_A\mathbf{W}_A^H\mathbf{H}^H\mathbf{Z}_B| \tag{15}$$

with the capacity of the other one which uses estimated weight matrices with our method. These estimation results are plotted for training SNRs$= 5, 10, 20$dB. From this figure it is easily seen that the capacity of our system with estimated beamforming matrices has a trivial difference with the capacity of the system when the exact weight matrices are used for transmit and receive beamforming.

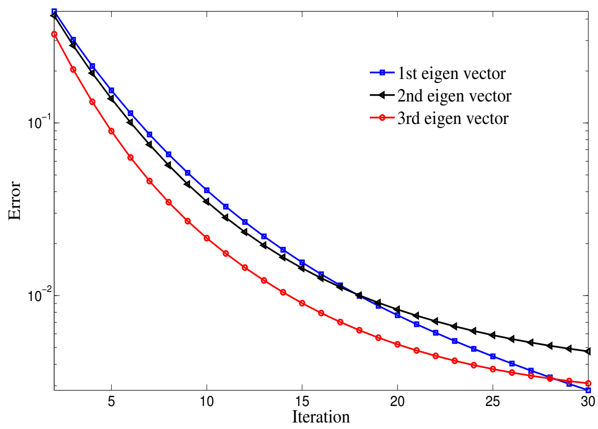

Figure 2. Error $= E\{\| \mathbf{q}_i - \mathbf{a}_t^k \|\}$ versus number of iteration(k) for $M_A = 6$ and $M_B = 6$.

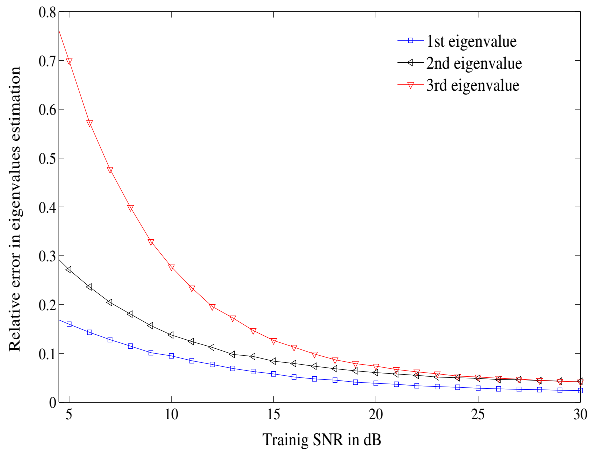

Figure 3. Relative error in eigenvalue estimation as a function of training SNR for $M_A = 6$ and $M_B = 6$.

6. CONCLUSION

In this manuscript we considered MIMO communication systems which are able to transmit and receive data at both sides in a same or very near frequency band. An iterative cooperative method was proposed to obtain the proper transmit matrix at transmitter and receive beamforming matrix at receiver without direct measurement of the channel. The advantage of the above method for finding the required eigenvectors and their corresponding eigenvalues, is that it can be used to avoid extra transmission for finding the eigenvalues and eigenvectors that are not required for beamforming and combining regarding the transmit power budget. Simulation results show that the system performance is near to the case that precise optimal weight matrices are known the transmitter and receiver.

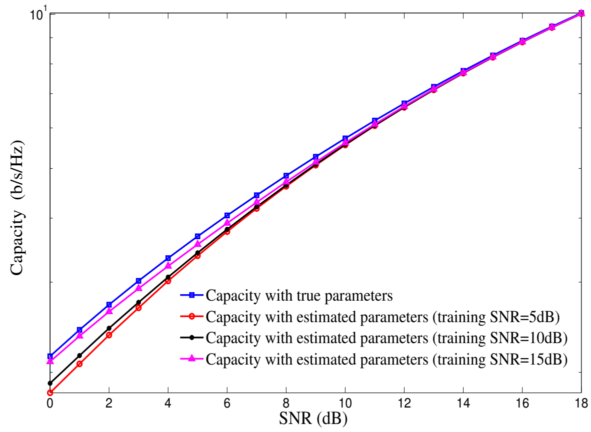

Figure 4. System capacity with perfect beamforming matrices and the capacity of system when our estimated beamforming matrices are used for a system with $M_A = M_B = 2$ antennas (the SNR during the proposed scheme are 5,10, and 20dB.

7. REFERENCES

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