The Modificatin of AIC using Denoising by Wavelet

H. keivani[1] , N. Tayyarzadeh[2], M. Bakhshi[2] , A. Kazerooni[1]
1-Islamic Azad university, branch of kazeroon
2-Islamic Azad university, branch of gachsaran

Abstract- Akaike's Information Criterion (AIC) is a popular method for estimation the number of sources impinging on an array of sensors, which is a problem of great interest in several applications. The performance of AIC degrades under low Signal-to-Noise Ratio (SNR) conditions due to errors in estimating the data covariance matrix from finite data. This paper explores the possibility of employing the wavelet denoising technique to arrest the degradation in the finite-data performance of AIC under low SNR. We propose the application of wavelet denoising to the noisy signal at each sensor to boost the SNR before performing estimation of the number of the sources by AIC. A comparative study of the finite data performance of AIC is presented for the undenoised and denoised data, and it is shown that denoising leads to the enhancement of the AIC method.

Keywords: Wavelet Denoising, AIC

I. INTRODUCTION

The problem of detecting the number of sources impinging on an array of sensors has received wide interest in many research problems. Termed as a model order selection problem, Akaike's Information Criterion (AIC) is the most widely known information theoretic methods to solve this problem in a noisy environment, using an array of L sensors [1]. The method involves eigen decomposition of the spectral covariance matrix R of the L dimensional data vector. The matrix R is estimated from a finite number of samples of the data vector. For a given data size N, reduction of the signal to- noise ratio (SNR) at the sensor array output causes an increase in the covariance matrix estimation error and a corresponding increase in the AIC estimation error. The estimation errors may be reduced by increasing N, but requirements of temporal coherence and speed impose an upper limit on the permissible value of N. Inevitably, the performance of The AIC estimator suffers a progressive degradation as the SNR is reduced.

In this paper, we explore the possibility of using a wavelet denoising technique to improve the performance of AIC in a low SNR environment. The wavelet denoising algorithm of Donoho and Johnstone [2] is used to enhance the SNR at the output of each sensor. It is shown that denoising leads to a significant improvement in the performance of the AIC estimator.

II. AIC PRINCIPLE

Consider an array of *L* sensors that receives signals transmitted from *M* narrowband far field sources with direction parameters $\theta_1, \ \ldots\ldots, \ \theta_M$. The complex signals received at the *L* sensor elements at time *t* can then be expressed as the *L*-dimensional vector:

$$x(t) = \sum_{m=1}^{M} s_m(t) a(\theta_m) + n(t) \tag{1}$$

Where, $s_m(t)$ is the scalar complex waveform referred to as the *m*-th signal, $a(\theta_m)$ is the steering vector corresponding to direction θ_m, and $n(t)$ is the *L*-dimensional complex vector containing noise.

We assume that the *M* (*M* < *L*) signals $s_1(t), \ \ldots\ldots., \ s_M(t)$, are complex (analytic), stationary, and ergodic Gaussian random processes, with zero mean and positive definite covariance matrix. The noise vector *n(t)* is assumed to be complex, stationary, and ergodic Gaussian vector process, independent of the signal , with zero mean and covariance matrix given by $\sigma^2 I$, where σ^2 is an unknown scalar constant and *I* is the identity matrix.

A crucial problem associated with the model described in (1) is that of determining the number of signals M from a finite set of observations $x(t_1) \ \ldots \ x(t_N)$.

A promising approach to this problem is based on the structure of the covariance matrix of the observation vector *x(t)*. To introduce this approach, we first rewrite (1) as:

$$x(t) = As(t) + n(t) \tag{2}$$

Where $A = [a(\theta_1), \ \ldots\ldots\ldots., \ a(\theta_M)]$ and $s^T(t) = [s_1(t), \ \ldots\ldots, \ s_M(t)]$.

Because the noise is zero mean and independent of the signals, it follows that covariance matrix of *x(t)* is given by:

$$R = \psi + \sigma^2 I \tag{3}$$

Where $\psi = ASA^{\dagger}$. With $\dagger$ denoting the conjugate transpose, and *S* denoting the covariance matrix of the signals, i.e, $S = E[s(t)s(t)^{\dagger}]$.
Assuming that the matrix A is of full column rank, i.e., the vectors $a(\theta_m)(m = 1, \ldots\ldots, M)$ are linearly independent and that the covariance matrix of the signals *S* is nonsingular, it follows that the rank of ψ is *M*, or equivalently, the *L-M* smallest eigenvales of θ are equal to zero. Denoting the eigenvalues of *R* by $\lambda_1 \geq \lambda_2 \ldots\ldots \geq \lambda_L$ it follows, therefore, that the smallest *L-M* eigenvalues of *R* are equal to σ^2, i.e.,

$$\lambda_{M+1} = \lambda_{M+2} = \ldots\ldots = \lambda_L = \sigma^2 \tag{4}$$

As stated earlier that in practice only sample estimates which is denoted by a hat "^" are available. An estimation of R is presented in (5).

$$\hat{R} = \frac{1}{N}\sum_{i=1}^{N} X(t)X(t)^H \tag{5}$$

The number of signals *M* can hence be determined from the multiplicity of the smallest eigenvalues of *R*. The problem is that the covariance matrix *R* is unknown in practice. When estimated from a finite sample size, the resulting eigenvalues are all different with probability one, thus making it difficult to determine the number of signals merely by "observing" the eigenvalues. Amore sophisticated approach to the problem developed by Rartlett and Lawley [3], is based on a sequence of hypothesis tests. The problems associated with this approach are the subjective judgment needed in the selection of the threshold levels for the different tests.
The Akaike's Information Criterion (AIC) method [4] used for estimation the number of sources. The method required computation of the likelihood ratio for the (*L-M*) lowest eigenvalues of $\hat{R}$, and this has been shown to be:

$$\delta_P(U) = \frac{\frac{1}{L-P}\sum_{i=P+1}^{L} U_i}{\left[\prod_{i=P+1}^{L} U_i\right]^{\frac{1}{L-P}}} \tag{6}$$

Where U_i denote the eigenvalues of $\hat{R}$ which is the ratio of the arithmetic and geometric mean of the eigenvalues [5].
Aaike's information criterion (AIC) can be written as:

$$AIC(P) = 2N(L-P)\ln\delta_P(U) + 2v(P,L) \tag{7}$$

Where $v(P,L)$ denotes the number of independent parameters that are to be estimated for a given P. This is found to be [6]:

$$v(P,L) = P(2L-P) + 1. \tag{8}$$

The optimal solution is the value of P that minimizes (7).

III. WAVELET DENOISING

The wavelet transform is a time-scale representation technique, which describes a signal by using the correlation with translation and dilation of a function called mother wavelet. The translation operation allows signal features to be isolated in time, while the dilation operation allows features existing at different scales to be identified. In this way, the wavelet transform represents a signal as a sum of wavelets with different locations and scales [7,8,9]. The definition of a discrete wavelet transform is given by:

$$C_{j,k} = \sum_{t\in z} x(t) g_{j,k}(t), t = 1,\ldots,Q \tag{9}$$

Where $C_{j,k}$ are wavelet coefficients and $g_{j,k}(t) = 2^{-j/2} g(2^{-j}t - k)$ is the scaling function. We consider the following model of a discrete noisy signal.

$$x(t) = s(t) + \sigma e(t), t = 1,\ldots,Q \tag{10}$$

Where x(t) represents the noisy signal, s(t) is the deterministic useful signal s(t), the white Gaussian signal, e(t) modeled as N(0,1), will be distributed across scales with a white spectrum and its energy will be preserved, here we assume that

$C_{j,k} >> C_{j,k}$. We use soft thresholding method to eliminate noise from the wavelet coefficients by replacing the coefficients that are in the range of $[-\delta, \delta]$ with zero [2]. Therefore the wavelets coefficient $C_{j,k}$, between $-\delta$ and δ is set to zero, while the others are shrunk in absolute value. Figure (1) shows the Donoho's "soft thresholding" or "shrink" function.

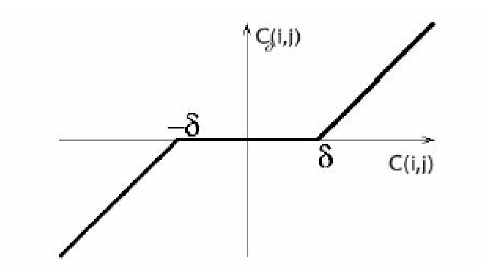


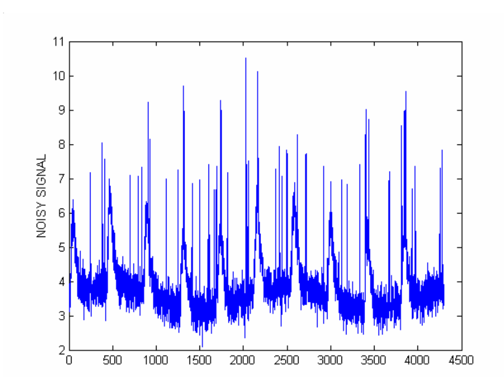
Figure 1: Soft-threshold function

The threshold δ proposed by Donoho is $\delta = \sqrt{2\log(M)\bar{\sigma}^2}$. Here $\bar{\sigma}^2$ is estimation of the noise variance σ^2 given by [2] $\bar{\sigma}^2 = median(|C_{j,k}|)/0.6745$.

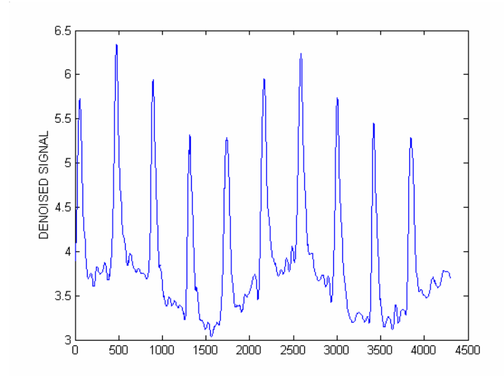
The algorithm used for denoising of the signal by wavelet thresholding is outlined below:

1. Perform a suitable wavelet transform of the noisy data *x(t)* [10].
2. Calculate the threshold *d* depending upon the noise variance.
3. Apply thresholding to the resulting wavelet coefficients
4. Perform inverse wavelet transform to obtain the denoised signal.

Figure (2) shows input and output of the algorithm above.



(a)



(b)

Figure 2: (a) Noisy input signal and (b) Denoised output signal.

IV. THE PROPOSED MODEL

Figure (3) shows the block diagram of the proposed model for AIC method. The main procedure of the system is described as follows. First, the received signals by antennas. Next, computation of the covariance matrix. Then, wavelet denoising by one type of wavelet family for the covariance matrix. Finally, AIC method is calculated to estimate the number of signals.

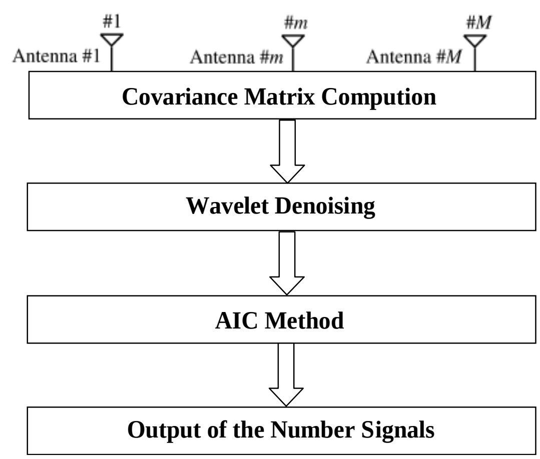


Figure 3: Block Diagram of the Proposed Model for AIC Method

V. SIMULATION RESULTS

The response of the AIC method is investigated according to the proposed model by using type of wavelet. A ULA of 10 sensors is considered with a half-wave length inter-element spacing, used to separate two uncorrelated emitters based on a batch of N=128 data samples with SNR=10 dB. The first source is at 80^{o} while the second source is at 82^{o}. The type of wavelet used in proposed model includes "*db*".

The first case is without applied proposed model for AIC method. It is noticed the eigenvalues of the sample – covariance matrix are 6.6883, 0.0348, 0.0085, 0.0097, 0.0122, 0.0160, 0.0139 and 0.0148, and the response of AIC method shown in table (1), the minimum value of AIC is obtained incorrectly for M = 2.

Table 1: The Response of the AIC Method without Proposed Model.

P	AIC	P	AIC
0	2731.6	4	99.57
1	2522.7	5	2111.23
2	556.8	6	120.4
3	87.03	7	126

The second case is with applied proposed model for AIC method by using db4 wavelet. It is noticed these eigenvalues of the sample – covariance matrix are 6.7416, 0.0211, 0.0013, 0.0016, 0.0026, 0.0019, 0.0022 and 0.0023, and response of AIC method shown in table (2), the minimum value of AIC is obtained correctly for M = 2.

Table 2: The Response of the AIC Method with Proposed Model.

P	AIC	P	AIC
0	2841.1	4	273.6
1	86.1	5	561.1
2	22.2	6	1113.7
3	91.3	7	2728.4

If the response of AIC method without the proposed model is compared with the response of the proposed model by using db4 wavelet, it is seen that the first does not yield the correct number of the sources but the proposed model gives the correct number of the sources as seen in Table (1) and Table (2).

VI. CONCLUSION

We have introduced a new technique to determine the number of sources in a noisy environment by applying (AIC) to the output of wavelet denoising, based on the ideal that wavelet denoising improves the SNR of a noisy signal. We proceeded to perform wavelet denoising of the signal from each sensor of the array independently. Wavelet denoising helps to reduce the error of the covariance matrix estimation. The wavelet of db4 yield higher response for AIC method.

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