

# Iterative Decision-Feedback Equalizer with Cyclic Detection for DFT-S OFDM System

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**Abstract**—This paper proposes a simple and efficient decision feedback equalizer (DFE) for Discrete Fourier Transform-Spread Orthogonal Frequency Division Multiplex (DFT-S OFDM) system in Evolved-Universal Telecommunication Radio Access (E-UTRA) [1] uplink transmission. Through cyclic detection and iteration, the new algorithm obtains a noticeable gain than conventional DFE, and is almost as effective as the modified UW based methods but with much higher bandwidth efficiency. Furthermore, it does not need to change the frame structure of existing protocol.

**Keywords**—iterative DFE; cyclic detection; DFT-S OFDM;

## I. INTRODUCTION

One of the most challenging problems in high data rate wireless transmission is to reduce inter-symbol interference (ISI) resulted from the time dispersion caused by multi-path propagation. To solve this problem, equalization is introduced. Blockwise equalization can be implemented both in time domain and frequency domain, but the later [2] is more attractive for its reasonable signal processing complexity. Generally, DFE holds better performance than linear equalizer for the cancellation of ISI without noise enhancement. However, its inherent error propagation has some limitation for further improvement. In fact, as shown later in simulation results, conventional DFE almost cannot be used in DFT-S OFDM systems. Unique word (UW) is proposed in the literatures [3-5], as a known symbol sequence, can make error propagation beyond one FFT-block impossible since the last symbols of every block are always decided correctly. But the UW based scheme is not appropriate for DFT-S OFDM which will be detailed later. In this paper, a novel method, named iterative DFE with cyclic detection, is proposed, which not only holds the same performance as the UW scheme, but also overcomes its shortcomings.

## II. CONVENTIONAL EQUALIZATION SCHEMES IN DFT-S OFDM SYSTEM

For E-UTRAN, the uplink transmission scheme is based on single-carrier FDMA, more specifically DFT S-OFDM.

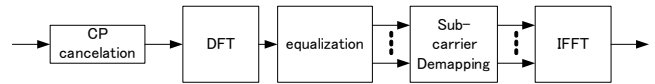
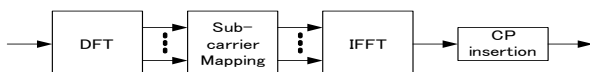


Fig.1. Transmitter and Receiver scheme of DFT-S OFDM

To obtain better performance, a hybrid time-frequency domain Decision Feedback Equalization is involved. This approach would be to use frequency domain filtering only for the forward filter part of the DFE, and conventional transversal feedback filter for the feedback part.

$W$  and  $f$  present the tap-coefficients of Feed Forward Filter (FFF) and Feed Back Filter (FBF), respectively. FBF includes  $B$  taps with complex coefficients  $\{f_k\}$ ,  $k \in F_B$ , where  $F_B$  is a set of non-zero indices corresponding to the delays (in symbol periods) of the  $B$  feedback coefficients. That is, the time delay of first path is  $k_1$ , the second is  $k_2$ , and so on. Through calculating the feedback coefficients  $f_{k_1}, f_{k_2}, \dots$  corresponding to each path, the interference of these paths will be induced effectively. Therefore, the symbols that have been equalized are

$$z_m = \sum_{l=1}^M W_l R_l \exp\left(j \frac{2\pi}{M} lm\right) - \sum_{k \in F_B} f_k^* a_{m-k} \quad (1)$$

Where  $\{R_l\}$  is the discrete Fourier transformation (DFT) of the received information symbols in a LB (long block), and  $\{a_m\}$  is the detected symbols. The superscript ‘\*’ stands for the complex conjugate.

A common problem with DFE is the error propagation due to incorrectly decided symbol values for feedback. The Long Block (LB) structure which includes 512 sub-carriers in Fig.2 (a) is for DFT S-OFDM systems. When the data is transmitted in a channel, the delayed cyclic prefix (CP) will corrupt the head data bits. Since the CP is the replica of the tail data bits, we can take the detection of the tail data bits as the detection of the CP. Thus, the detection of the tail data bits will influence the detection of the head data bits. On the other hand, the detection of the tail data bits depends on the head ones for feedback. Thus, the errors are propagated cyclically.

UW structure is introduced to improve the performance of DFE as in Fig.2 (b) [3-4]. It replaces some data bits and also plays a similar role as the CP. Since the decision of UW is

always correct, the error propagation can be reduced to a large extent. However, this structure cannot be used in DFT-S OFDM since it does not abide the LB structure in protocol, thus we modify it to what is showed as Fig.2 (c). But as the UW carries no information, the two-UW structure obtains the performance gain at the cost of lower bandwidth efficiency.

Based on the joint optimization of both the performance and bandwidth efficiency, we proposed the following iterative DFE with cyclic detection for DFT-S OFDM system.

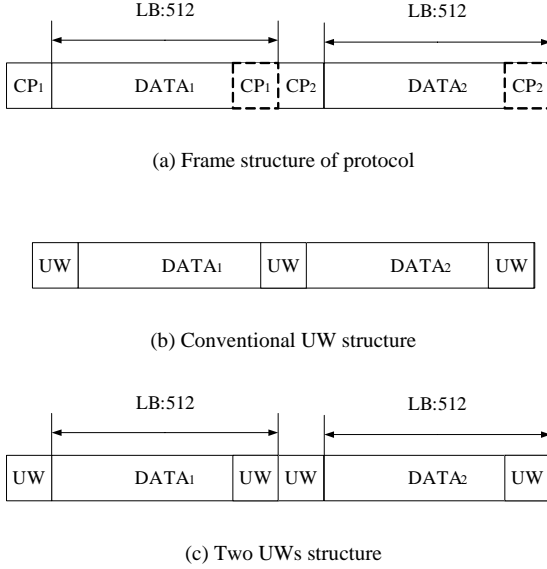


Fig.2.Long Block (LB) data structure

### III. ITERATIVE DFE WITH CYCLIC DETECTION

In this algorithm, the general frame structure shown in Fig.2 (a) is used. Fig. 3 shows the diagram of the novel equalizer.

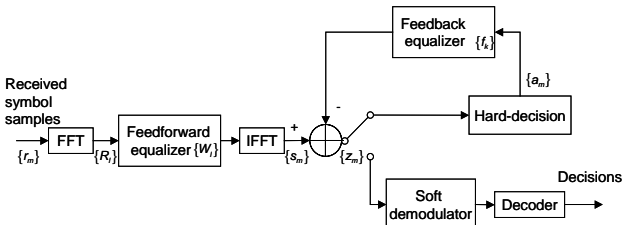


Fig.3.Iterative DFE with cyclic detection

Without loss of generality, only the first two interfering paths are considered for simplification. The first tap coefficient is corresponding to the first interfering path, so is the second.  $M$  received symbols are equalized simultaneously.

**Step 1:** Set the maximum iteration number as  $Iter\_Max$  and initialize the iteration counter  $Iter\_Counter=0$ .

**Step 2:** Calculate the forward equalization coefficients  $W$  and backward equalization coefficients  $f$  as in the conventional DFE equalizer.

**Step 3:** According to the input signal, calculate the symbol sequences  $s = \{s_1, s_2, \dots, s_M\}$ . Once per block, the  $M$  FFT output coefficients  $\{R_l\}$  are multiplied by the complex-valued  $M$  forward equalizer coefficients  $\{W_l\}$  (which compensate for the frequency selective channel's variations of amplitude and phase with frequency).

$$s_m = \sum_{l=1}^M W_l R_l \exp\left(j \frac{2p}{M} lm\right) \quad (2)$$

**Step 4:** Initialize  $[a]_{old} = s$ .

**Step 5:** If the maximum iteration number  $Iter\_Max$  has been reached, stop the iteration and go to the step 7. Otherwise, calculate the symbol sequences  $z = \{z_1, z_2, \dots, z_M\}$  after backward feedback equalization of  $s$ . In this process, a hard-decision, operated symbol by symbol, is involved and the feedback method is different due to the locations of symbols being processed in a data block. There are three cases:

(1) For the  $k_1$  head symbols, that is,  $m = \{1, 2, \dots, k_1\}$ , we have

$$z_m = \sum_{l=1}^M W_l R_l \exp\left(j \frac{2p}{M} lm\right) - f_1^* [a_{M+m-k_1}]_{old} - f_2^* [a_{M+m-k_2}]_{old} \quad (3a)$$

$$[a_m]_{new} = Q(z_m) \quad (3b)$$

Hereafter  $Q(\bullet)$  denotes the hard-decision.

(2) If the symbols locate between  $k_1$  and  $k_2$  that is,  $m = \{k_1 + 1, k_1 + 2, \dots, k_2\}$

$$z_m = \sum_{l=1}^M W_l R_l \exp\left(j \frac{2p}{M} lm\right) - f_1^* [a_{m-k_1}]_{new} - f_2^* [a_{M+m-k_2}]_{old} \quad (4a)$$

$$[a_m]_{new} = Q(z_m) \quad (4b)$$

(3) For the remaining symbols, that is,  $m = \{k_2 + 1, k_2 + 2, \dots, M\}$

$$z_m = \sum_{l=1}^M W_l R_l \exp\left(j \frac{2p}{M} lm\right) - f_1^* [a_{m-k_1}]_{new} - f_2^* [a_{m-k_2}]_{new} \quad (5a)$$

$$[a_m]_{new} = Q(z_m) \quad (5b)$$

**Step 6:** Update  $Iter\_Counter = Iter\_Counter + 1$  and compare the lattermost  $k_2$  information bits of  $[a]_{old}$  and that of  $[a]_{new}$ . If  $[a_m]_{old} = [a_m]_{new}$ ,  $m = \{k_2 + 1, k_2 + 2, \dots, M\}$  stop the iteration and transfer to step 7. Otherwise, go back to step 5.

**Step 7:** Execute demodulation.

#### IV. ALGORITHM ANALYSIS

##### A. Reliability

The gain of the UW based method is exactly derived from UW, a known sequence, which cuts down the error propagation. In our novel algorithm, iterative decision makes lattermost  $k_2$  bits much more reliable. Therefore, it has similar effects to the UW method whose length is  $k_2$ . So the novel algorithm has almost the same reliability as the UW method. Moreover, since the lattermost  $k_2$  information bits contain useful information in the new algorithm, higher bandwidth efficiency can be obtained, comparing with two-UW method whose UW part is not occupied by data in DFT-S OFDM system. Last but not least, our new algorithm does not need to change the frame structure.

##### B. Complexity

In our new algorithm, before the first detection, the forward equalization coefficients  $\mathbf{W}$  and backward equalization coefficients  $\mathbf{f}$  as well as the symbol sequences  $\mathbf{s}$  calculated through forward equalization process, have been obtained. Then they can be saved for the subsequent iterations and do not need to be recalculated. That is, the multiple iterations involve just a linear computation and a hard-decision. Therefore, it is more computationally intensive than the traditional DFE equalization but brings remarkable gain.

##### C. Convergence

As hard-decision is used in the iteration process, the new algorithm can converge very quickly. Generally, two iterations are enough for satisfactory convergence. Therefore, the algorithm has very little delay.

#### V. SIMULATION RESULTS

Fig.4 and Fig.5 show the probability of Block Error Rate (BLER) for four algorithms against the ratio of information bit energy  $E_b$  to noise variance. The simulation environments are based on the 3GPP LTE uplink enhancement DFT-S OFDM system. The system bandwidth is 5MHz and localized transmission type is used. The total number of sub-carrier is 512 while 300 are occupied in Fig.4 and 100 are occupied in Fig.5. QPSK modulation is investigated without channel coding. The channel model is SCM-C, a scenario of SCME (Spatial Channel Model Extended) [6-7], which extends 3GPP SCM (Spatial Channel Model) to wider bandwidth. Suppose that the mobile velocity is 30kmph and, according to reference [1], the length of CP is 31. Perfect knowledge of the channel is assumed.

The performance curves at the right side show the advantage of our new algorithm compared with other three algorithms. The new algorithm hold about 1dB gain compared with Frequency Domain Equalization algorithm for using ISI cancellation method. And the advantage will be expanded when more sub-carriers are occupied, especially at low  $E_b/N_0$ , since higher transmission rate when more sub-carriers are occupied by data symbols leads to higher ISI which can be suppressed effectively by our new algorithm.

It is observed that the conventional DFE which uses frequency domain filtering only for the forward filter part of the DFE, and conventional transversal feedback filter for the feedback part basically cannot be used in DFT-S OFDM system owing to its inherent error propagation. The error floor degrades the performance even below the linear equalization at high  $E_b/N_0$ .

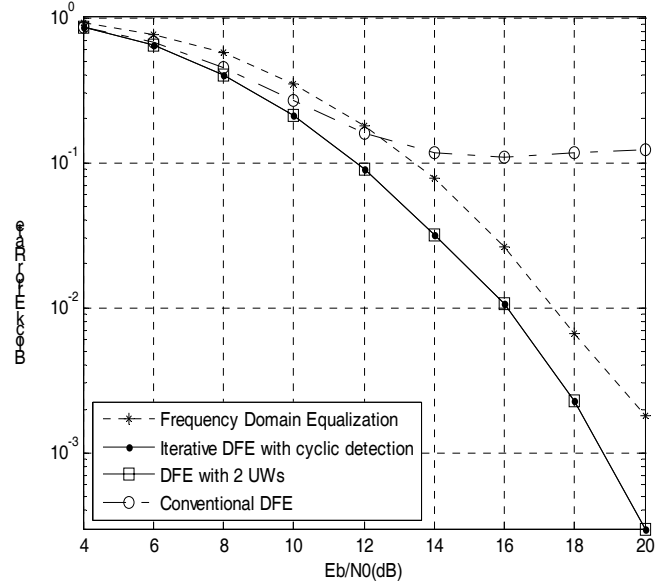


Fig.4. Performance comparison among several equalization algorithms (300 sub-carriers are occupied)

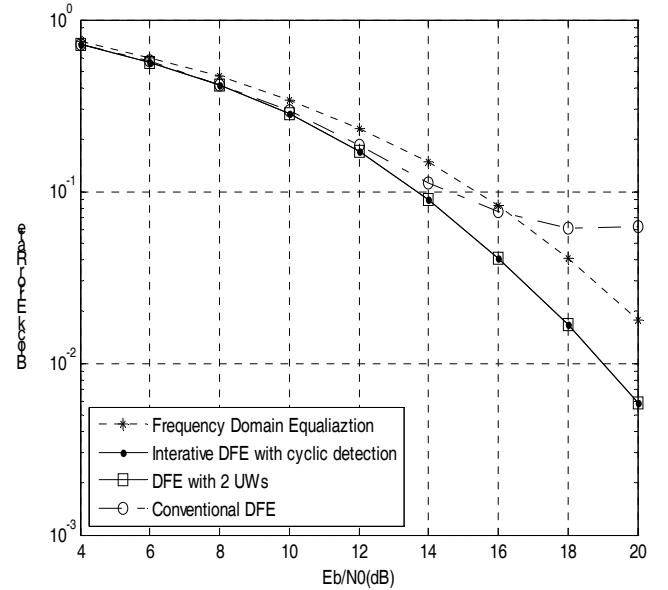


Fig.5. Performance comparison among several equalization algorithms (100 sub-carriers are occupied)

The iterative DFE with cyclic detection gives almost the same performance as a DFE using our two-UW method. The cyclic detection in the iterative DFE makes the estimation of the CP more accurate, and almost as though it was known, as in the two-UW structure. But, our iterative DFE offers two main

advantages. Firstly, the iterative DFE has much higher bandwidth efficiency than the two-UW structure. Secondly, the iterative DFE does not need to change the frame structure of existing protocols, as would be required for the two-UW structure.

## VI. CONCLUSIONS

We proposed a novel DFE algorithm with low complexity using iterative decision and cyclic detection for DFT-S OFDM system in E-UTRA uplink transmission. This DFE algorithm gives noticeable improvement since the error propagation was interdicted. And it needs not to change the frame structure of existing protocol.

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