

Spectrum Shaping Technique Combined with SC/MMSE Turbo Equalizer for High Spectral Efficient Broadband Wireless Access Systems

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Abstract— This paper proposes a spectrum shaping technique exploiting frequency clipping for frequency domain soft canceller with minimum mean square error (FD-SC/MMSE) based turbo equalization systems, of which aims to achieve high spectral efficiency and efficient energy transfer from a transmitter to a receiver, as well as to provide a novel spectrum division multiplexing strategy. In the proposed scheme, the water-filling strategy is modified to clip more spectrum components with lower signal to noise power ratio, and transmit power allocated to the clipped spectrum components are evenly distributed to the non-clipped components thereby concentrates transmit power on spectrum with higher channel gain. Furthermore, the proposed spectrum shaping technique is applied to user multiplexing in the frequency domain with higher spectral efficiency. Computer simulation including extrinsic information transfer (EXIT) analysis confirms that the proposed scheme is effective in enhancement of spectral efficiency with low power penalty.

I. INTRODUCTION

In broadband wireless access systems, compensation for frequency selective fading and efficient energy transfer from a transmitter to a receiver are the most important issues for enhancement of sum-rate capacity as well as the average user rate under transmit power limited conditions. Although orthogonal frequency division multiplexing (OFDM) has been extensively developed as a suitable wireless transmission technique under frequency selective fading conditions, single carrier broadband transmission is also becoming a realistic technique for broadband wireless access system due to development of frequency domain equalizer (FDE).

In the linear FDE, because its frequency transfer function is determined by the tradeoff between residual inter-symbol interference (ISI) after equalization and noise enhancement due to large weight multiplication at deeply faded frequency components, channel frequency transfer functions after equalization are still deviated. One of its solutions is pre-coding [1]-[3] in which residual ISI after equalization is reduced not by the receiver signal processing but by the transmitter encoding process. However, the pre-coding technique does not guarantee an efficient energy transfer from a transmitter to a receiver.

Theoretically, water-filling spectrum shaping [4] in which more energy is allocated to spectrum with higher channel gain and less or no energy is allocated to spectrum with lower channel gain, is optimum to promise the efficient energy

transfer. However, it would cause extra ISI in the transmitter, thereby residual ISI after equalization is potentially enhanced in the case of linear FDE.

Frequency domain soft-canceller with minimum mean square error (FD-SC/MMSE) based turbo equalizer [5] is another equalization technique suitable for broadband single carrier transmission. The most important features of the FD-SC/MMSE turbo equalizer is

- ISI after equalization is gradually suppressed in the iterative manner using a soft canceller, and if the turbo equalization process is successfully converged, residual ISI becomes negligible [6]-[9].
- All the effective desired signal components dispersed in the time domain can be efficiently combined by following matched filter combining basis even though the channel memory length is large [10].

These features suggest us that water-filling spectrum shaping is applicable when an FD-SC/MMSE turbo equalizer is equipped in the receiver, thereby requirements for broadband transmission, compensation of frequency selective fading and efficient energy transfer from a transmitter to a receiver, can be jointly satisfied. In this paper, no energy allocation to specific spectrum components will be called "clipping" in the following.

This spectrum shaping process is also considered as one of the pre-coding techniques. However, this concept can be considered as an approach contrary to the conventional pre-coding techniques aiming at suppressing residual ISI at the equalizer output in that it intentionally emphasizes ISI in the transmitter side for efficient energy transfer from a transmitter to a receiver.

In this paper, we will utilize such clipping process more actively to enhance spectral efficiency while minimizing degradation of power efficiency. In the proposed scheme, in addition to the frequency components clipped according to the water-filling process, a certain amount of spectrum components with lower channel gain is intentionally clipped to create vacancy in spectrum. With this process, spectral efficiency for a user is enhanced. For example, when 50% of spectrum is clipped, spectral efficiency for the user can be doubled. Moreover, if

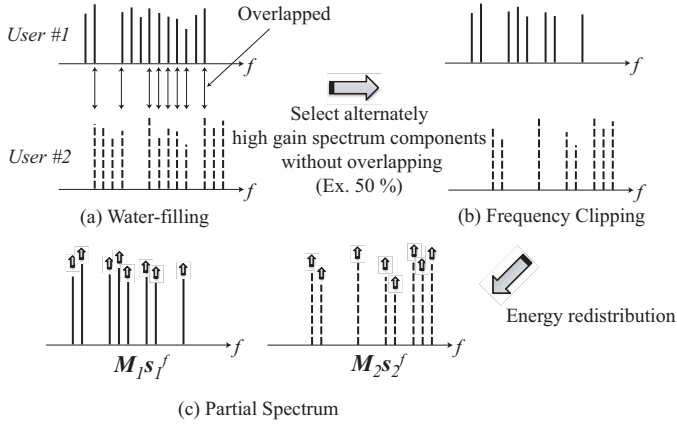


Fig. 1. Process of the proposed spectrum shaping

the created vacant spectrum is utilized by another user, the system capacity can also be enhanced.

In order to clarify the effectiveness, extrinsic information transfer (EXIT) analysis [11], [12] is conducted. The analysis provides deep understanding of the proposed strategy. Furthermore, frame error rate (FER) is evaluated by computer simulations. Connecting EXIT analysis with FER analysis, the cause-and-effect logic of the spectrum division multiplexing access is clarified.

The rest of this paper is organized as follows: In Section II, the proposed spectrum shaping scheme is described. Section III presents EXIT analysis to verify some advantages of the proposed strategy, and FER performance is evaluated in Section IV. Finally, this paper is concluded in Section V.

II. PROPOSED SPECTRUM SHAPING SCHEMES

A. Spectrum shaping module

Let us assume an uplink multi-user environment in which U users simultaneously communicate with a base station (BS) over single-input single-output (SISO) channels. When transmitted spectrum for u -th user is expressed as s_u^f , spectrum for the received signal r^f is given by

$$r^f = \sum_{u=1}^U \Xi_u M_u s_u^f + v^f, \quad (u = 1, 2, \dots, U) \quad (1)$$

where, M_u is a spectrum shaping matrix for u -th user that determines a weight of each spectrum component including spectrum clipping (weight = 0), and Ξ_u is a frequency domain channel matrix for u -th user. v^f is additive white Gaussian noise (AWGN) vector in the frequency domain composed by mutually independent complex Gaussian random variable with zero mean and variance of $N_0/2$. The channel model is described in Appendix. When M_u is multiplied by s_u^f , the transmitted spectrum can be transformed to an arbitrary shape. The spectrum shaping matrix M_u is given by

$$M_u = \text{diag}[m_u(1), m_u(2), \dots, m_u(K)] \quad (2)$$

where $\text{diag}[\mathbf{x}]$ denotes diagonal matrix having elements of vector \mathbf{x} , K is the number of spectrum components, and $m_u(k)$

is zero or a weighting factor for k -th spectrum component in the transmitted signal. In this process, the receiver should notify information about an optimum spectrum shape to the transmitter via feedback channels.

Assuming data symbol energy is normalized as one, the covariance matrix after spectrum shaping $\mathbf{\kappa}_u$ is given by

$$\begin{aligned} \mathbf{\kappa}_u &= \mathbb{E}\{M_u s_u^f s_u^{fH} M_u^H\} \\ &= M_u^2 \end{aligned} \quad (3)$$

where $\mathbb{E}\{\cdot\}$ and \cdot^H denote expectation and conjugate transpose, respectively. Thus, the total transmitted energy P_u is given by

$$\begin{aligned} P_u &= \text{tr}\{\mathbf{\kappa}_u\} \\ &= \sum_{k=1}^K m_u^2(k) \end{aligned} \quad (4)$$

where $\text{tr}\{X\}$ denotes summation of all elements in a diagonal matrix X . In this paper, transmitted energy per symbol E_s is given by P_u/K .

Using Eqs. (1) and (4), the received signal energy to noise density ratio (E_s/N_0) matrix $\mathbf{\rho}_u$ is expressed as

$$\begin{aligned} \mathbf{\rho}_u &= \frac{\Xi_u M_u^2 \Xi_u^H}{N_0} \\ &= \text{diag} \left[\frac{m_u^2(1)}{N_0} \Xi_u(1) \Xi_u^*(1), \frac{m_u^2(2)}{N_0} \Xi_u(2) \Xi_u^*(2), \right. \\ &\quad \left. \dots, \frac{m_u^2(K)}{N_0} \Xi_u(K) \Xi_u^*(K) \right] \end{aligned} \quad (5)$$

where \cdot^* denotes conjugate, and $\Xi_u(k)$ is the complex channel weight in frequency domain representation at a discrete frequency position k .

Figure 1 shows an example of the proposed spectrum shaping process in the case of two users. In the first step, each user independently exploits its optimum weight to be multiplied to each spectrum component based on water-filling theorem as shown in Fig. 1 (a). The optimum $m_u(k)$ for maximization of $\text{tr}\{\mathbf{\rho}_u\}$ can be obtained using Lagrange's method of undetermined multipliers as

$$m_u(k) = \left(\xi - \frac{N_0}{\Xi_u(k) \Xi_u^*(k)} \right)^+ \quad (6)$$

where $(x)^+$ denotes the positive part of x otherwise zero, and ξ is decided in power constraint given by Eq. (4).

In the second step, more number of spectrum components having smaller $m_u(k)$ are clipped. In other words, only a certain number of spectrum components are reserved and the others are clipped as shown in Fig. 1 (b). In this process, the BS alternately selects a spectrum component with higher channel gain for each user among non-selected spectrum components, where clipping rate for each user is assumed to be preliminarily determined. For example, when clipping rate is 0.25, 75% of spectrum components in the assigned bandwidth are allocated to each user. In the case of Fig. 1, clipping rate is 0.5. Due to independency of channel characteristics for each user, user diversity effect can be expected in the spectrum

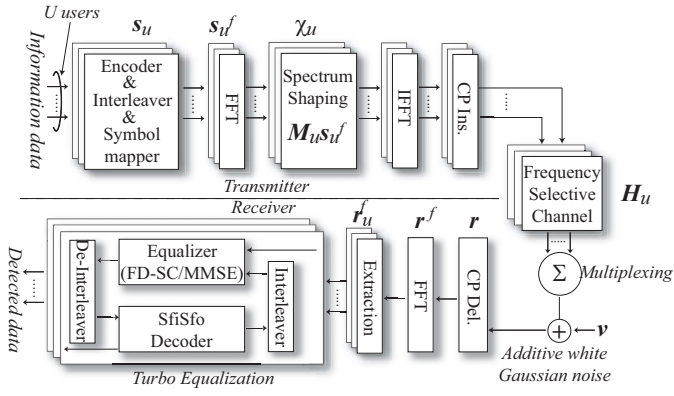


Fig. 2. Proposed system in SISO scheme

component selection process. That is, spectrum components with relatively higher channel gain tend to be allocated to each user. Finally, transmitted energy, which is allocated to the clipped spectrum components, is evenly distributed to the selected spectrum components so that total transmitted power given by Eq. (4) is equal to preliminarily determined level. As the result, spectrum shaping matrix M_u to be applied to each user is obtained as shown in Fig. 1(c).

B. Transmission process

Figure 2 shows a block diagram of transmitter and receiver. In the transmitter, information data for each user is independently encoded to construct a coded bit sequence of length $2K$, interleaved and mapped onto constellation points, according to Gray encoding of quadrature phase shift keying (QPSK). Consequently, the u -th user's data symbol vector s_u having data symbol length K is generated. Then an fast fourier transform (FFT) is employed to transform the data symbol into the original spectrum s_u^f . After multiplying M_u and s_u^f , a partially clipped transmitted spectrum χ_u is generated at the spectrum shaping module. After χ_u is transformed back into the time domain using an IFFT, a length P -symbol cyclic prefix (CP) is appended to the head of the transmitted vector to allow frequency domain discrete signal processing at the receiver. As a result, the transmitted spectrum for each user is multiplexed over wireless channels.

Figure 3 illustrates the signal processing at the receiver. After CP in the frame is removed, an FFT is employed to transform the received symbol vector r into the received spectrum vector r^f depicted in Fig. 3 (a).

Let, r^f be rewritten as

$$r^f = \sum_{u=1}^U \Theta_u s_u^f + v^f \quad (7)$$

where $\Theta_u = \Xi_u M_u$ denotes an equivalent complex channel matrix including the effect of spectrum shaping matrix. Then, a received spectrum r_u^f illustrated in Fig. 3 (b) is extracted from r^f , which is defined as

$$\begin{aligned} r_u^f &= M_u' r^f \\ &= [r_u^f(1), r_u^f(2), \dots, r_u^f(K)]^T \end{aligned} \quad (8)$$

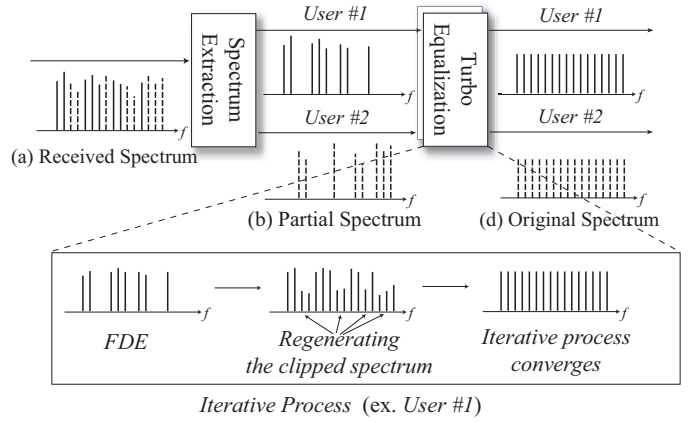


Fig. 3. Illustration on the signal processing at the receiver

where M_u' denotes an $K \times K$ diagonal matrix which aims to extract the u -th user's partial spectrum. For example, $M_u'[i, i]$ is one if i -th spectrum component is used, otherwise zero.

Then, FD-SC/MMSE turbo equalization is adopted to the received partial spectrum r_u^f . As described in Section I, turbo equalization is capable of suppressing ISI completely in the case where the behavior of the iterative detection is converged. Therefore, the original spectrum s_u^f can be derived by using turbo equalization as shown in Fig. 3 (c). In other words, turbo equalization can also regenerate the clipped spectrum components. Note that any other modification of FD-SC/MMSE algorithm is not required except that the channel matrix Ξ_u is replaced by Θ_u .

Hereinafter, the advantage of the proposed spectrum shaping is discussed from the standpoint of the frequency domain MMSE filter output. When E_s is set at one, the frequency domain MMSE filter output vector $z_u = [z_u(1), z_u(2), \dots, z_u(K)]^T$ is obtained as

$$z_u = (1 + \gamma_u \delta_u)^{-1} [\gamma_u \hat{s}_u + F^H \Phi_u \tilde{r}_u^f] \quad (9)$$

where

$$\Phi_u = \Theta_u^H (\Theta_u \Delta_u \Theta_u^H + N_0 I_K)^{-1}, \quad (10)$$

$$\gamma_u = \frac{1}{K} \text{tr} \{ \Phi_u \Theta_u \}, \quad (11)$$

$$\Delta_u = (1 - \delta_u) I_K, \quad (12)$$

and

$$\delta_u = \frac{1}{K} \sum_{k=1}^K |\hat{s}_u(k)|^2. \quad (13)$$

The vector $\tilde{r}_u^f = [\tilde{r}_u^f(1), \tilde{r}_u^f(2), \dots, \tilde{r}_u^f(K)]^T$ is referred to as an interference residual vector, which is given by

$$\tilde{r}_u^f = r_u^f - \Theta_u \hat{s}_u^f \quad (14)$$

$\hat{s}_u^f = [\hat{s}_u^f(1), \hat{s}_u^f(2), \dots, \hat{s}_u^f(K)]^T$ denotes an expected spectrum vector derived by a priori log-likelihood ratio (LLR) [11] fed back from the soft-input soft-output (SfiSfo) channel decoder

TABLE I
SIMULATION PARAMETERS

Modulation	QPSK
Channel coding	Convolutional Code Constraint length = 4 Coding rate = 1/2
Cyclic prefix length : P	64 symbols
Data symbols length : K	2048 symbols
Coded bit length : $2K$	4096 bits
Interleaver	Random
Num. of antennas	Tx : 1, Rx : 1
Symbol rate	20 Msymbols / sec
Channel model	2dB exponentially decaying 24-path Rayleigh model
Channel estimation	Perfect
Feedback	Perfect
Equalizer	Frequency domain SC/MMSE
Decoder	Max-Log-MAP w/ Jacobin logarithm

[13], which is expressed as

$$\begin{aligned}\hat{s}_u^f &= \mathbf{F}\hat{s}_u \\ &= \mathbf{F}[\hat{s}_u(1), \hat{s}_u(2), \dots, \hat{s}_u(K)]^T\end{aligned}\quad (15)$$

where

$$\hat{s}_u(k) = \mathbb{E}\{s_u(k)\}.\quad (16)$$

Note that $\hat{s}_u(k)$ in Eq. (16) can be obtained by following Refs. [11], [14]. Assuming that the MMSE filter output follows a Gaussian distribution, the equalizer output can be rewritten as

$$z_u(k) = \mu_u s_u(k) + \psi_u\quad (17)$$

where μ_u and ψ_u denote equivalent amplitude level and zero mean independent complex Gaussian noise with variance N_u , and μ_u and N_u are given by

$$\mu_u = (1 + \gamma_u \delta_u)^{-1} \gamma_u,\quad (18)$$

$$N_u = \mu_u - \mu_u^2.\quad (19)$$

Therefore, the averaged signal to noise power rate (SNR) after MMSE filtering Υ_u are expressed by

$$\begin{aligned}\Upsilon_u &= \frac{\mu_u^2}{N_u} \\ &= \frac{\mu_u}{1 - \mu_u}.\end{aligned}\quad (20)$$

Let us assume the two specific cases. The first one is the case when the turbo algorithm is successfully converged, which corresponds to the case of $\delta_u = 1.0$. The second one is the case for the first iteration. Since there are no a priori information, this corresponds to the case of $\delta_u = 0$. The first and second cases are hereinafter called *end-point* and *start-point*, respectively. In these two cases, substituting the value of δ_u to Eq. (18), Υ_u given by Eq. (20) is expressed as

$$\Upsilon_u = \begin{cases} \gamma_u & (\delta_u = 1 : \textit{end-point}) \\ \frac{\gamma_u}{1 - \gamma_u} & (\delta_u = 0 : \textit{start-point}). \end{cases}\quad (21)$$

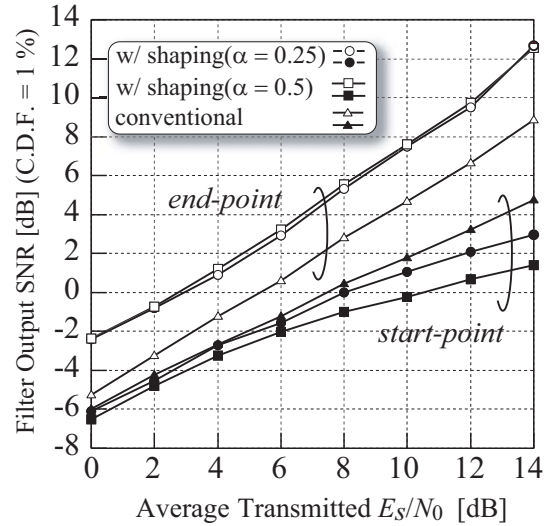


Fig. 4. C.D.F. 1% of the average SNR on MMSE filter output as a function of average transmitted E_s/N_0

Figure 4 shows that Υ_u at the cumulative distribution function (C.D.F.) of one percent for both *end-point* and *start-point* cases as a function of average transmitted E_s/N_0 . α denotes the frequency clipping rate. Parameters for this evaluation is summarized in Tab. 1. Note that parameters for all simulation results through this paper are based on Tab. 1. At the *end-point*, Υ_u with clipping rate $\alpha = 0.25$ and 0.5 achieve higher SNR at the FD-SC/MMSE filter output compared to the performance without the spectrum shaping. The performances without the spectrum shaping is, hereinafter, called the conventional. In contrast, Υ_u at the *start-point* case becomes lower. These phenomena indicate that, although the proposed clipping technique lowers equalizer output SNR at the *start-point* due to ISI by the clipping, it is effective in enhancement of the SNR at the *end-point* compared to the non spectrum shaping scheme. This tendency can be confirmed at any value of average transmitted E_s/N_0 .

III. EXIT ANALYSIS

In this section, EXIT analysis is conducted to verify some advantages of the proposed spectrum shaping. The mutual information (MI) I between the coded bits $\mathcal{S} \in \pm 1$ with equiprobable occurrence and LLR λ is given by

$$I = 1 - \int_{-\infty}^{\infty} p_{\lambda|\mathcal{S}}(x|1) \log_2(1 + e^{-x}) dx\quad (22)$$

where $p_{\lambda|\mathcal{S}}(x|b)$ is the probability density function (PDF) of LLR being x conditioned upon the coded bit b .

Both EXIT properties for the equalizer output and for the SfiSfo channel decoder output are depicted in Fig. 5. The average transmitted E_s/N_0 is set to 7 dB. Let I_{Eq}^E and I_{De}^E be the MI for the equalizer output and decoder output extrinsic LLRs, respectively. The SfiSfo decoder output MI I_{De}^E is calculated by substituting the measured extrinsic LLR histograms to Eq. (22).

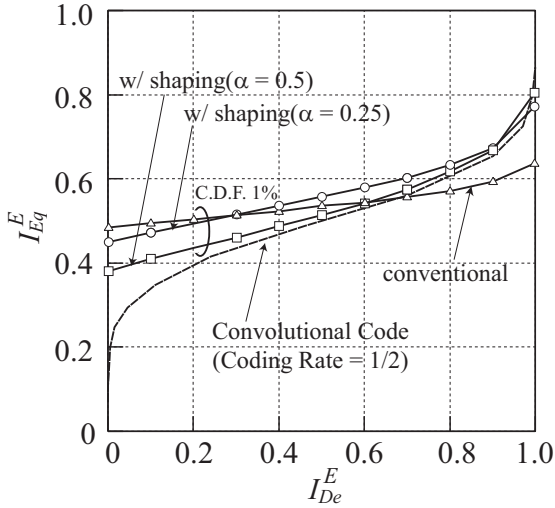


Fig. 5. Extrinsic information transfer characteristics of equalizer with different clipping rates ($\alpha = 0.25, 0.5$) and the *conventional* at average transmitted E_s/N_0 is 7 dB

As shown in semi-analytical calculation of I_{Eq}^E in Ref. [15], as long as the equalizer output can be regarded to be subject to a Gaussian random process, the equalizer output MI I_{Eq}^E can be approximately given by the J -function [16] without LLR measurements as following

$$I_{Eq}^E = J\left(\frac{1}{\Upsilon_u}\right) \approx \left(1 - 2^{H_1\left(\frac{1}{\Upsilon_u}\right)^{H_2}}\right)^{H_3} \quad (23)$$

The mapping-specific parameters are $H_1 = 0.3073$, $H_2 = 0.8935$ and $H_3 = 1.1064$ that are obtained by least-squared curve fitting [16]. Note that Υ_u depends on I_{De}^E , and the relationship is expressed as $\Upsilon_u = F(I_{De}^E, \Theta_u, E_s/N_0)$. From Eq. (23), I_{Eq}^E can be determined only by Υ_u shown in Fig. 4. For example, when we focus the *end-point* with $\alpha = 0.5$ at averaged transmitted $E_s/N_0 = 7$ dB in Fig. 4, the filter output Υ_u is approximately 4.5 dB. Substituting the value 4.5 dB into Eq. (23), the I_{Eq}^E becomes 0.8. This relationship can be seen at the *end-point* with $\alpha = 0.5$ in the Fig. 5.

A remarkable point in Fig. 5 is the MI at the equalizer output for clipping employed cases (both $\alpha = 0.25$ and 0.5) is higher than the *conventional* case at the *end-point*, although it is lower than the *conventional* at the *start-point*. As a result, SNR at the SC/MMSE filter output can also be enhanced as shown in Fig. 4. The most attractive observation in Fig. 5 is that the EXIT curves for $\alpha = 0.25$ and 0.5 cross with the curve for decoder at $I_{De}^E \approx 1$. This fact indicates that FER is negligibly low since almost perfect knowledge about transmitted information data can be obtained at the decoder. From a different perspective, EXIT curve for the *conventional* intersects with curve for the decoder at $I_{De}^E \approx 0.7$. This means that FER is not 0 since the perfect knowledge cannot be obtained by the iterative detection.

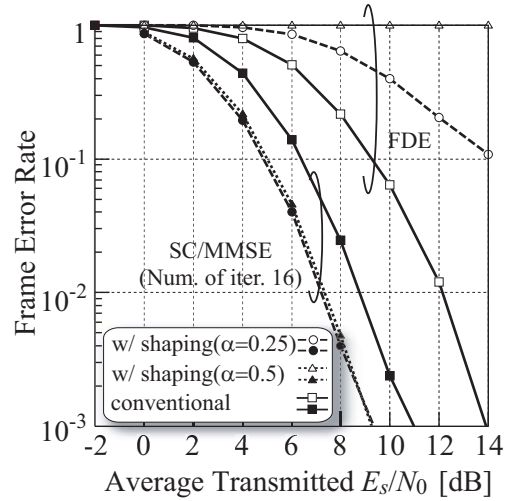


Fig. 6. FER results of the proposed spectrum shaping with FD-SC/MMSE Turbo Equalization (Number of iterations is 16) and FDE as a function of average transmitted E_s/N_0

IV. FER PERFORMANCES

A. Single User Case ($U = 1$)

The FER performances of the proposed spectrum shaping under single user environment ($U = 1$) is assessed by computer simulation. Figure 6 shows FER performances with different clipping rates ($\alpha = 0.25, 0.5$) and the *conventional* as a function of the average transmitted E_s/N_0 . At the FER = 10^{-2} , both $\alpha = 0.25$ and 0.5 cases achieve 2.0 dB gain compared to the *conventional*. In contrast, FDE without iterative detection deteriorates the FER performances for $\alpha = 0.25$ and 0.5 .

As discussed in section III, cross points of EXIT curves for $\alpha = 0.5$ and 0.25 is located at $I_{De}^E \approx 1$ at transmitted $E_s/N_0 = 7$ dB although those for the *conventional* is located at $I_{De}^E \approx 0.7$. As a result, FER of the spectrum shaping introduced scheme is lower than the *conventional*. In the case of FDE, the FER performance depends on I_{Eq}^E at the *start-point* since no knowledge about transmitted information data is fed back from the decoder. Therefore, FER performances with spectrum shaping is getting worse with higher clipping rate.

B. Multi User Case ($U = 2$)

The FER performances of a spectrum division multiple access exploiting the proposed spectrum shaping in multiuser environment ($U = 2$) are shown in Fig. 7 as functions of average transmitted E_s/N_0 . Frequency clipping rate is set at $\alpha = 0.5$, and performances for the *conventional* ($U = 1$) discussed in the previous section are depicted in same figure.

As can be seen in the figure, degradation of the FER performance caused by the multiplexing is less than 1 dB compared to the *conventional* scheme although the spectral efficiency in terms of bit/s/Hz for the proposed scheme is doubled compared to the *conventional* scheme, because half of the spectrum in each user is clipped. This degradation is due to sub-optimality of spectrum shaping. As explained in section II, the BS allocates available spectrum components for each user.

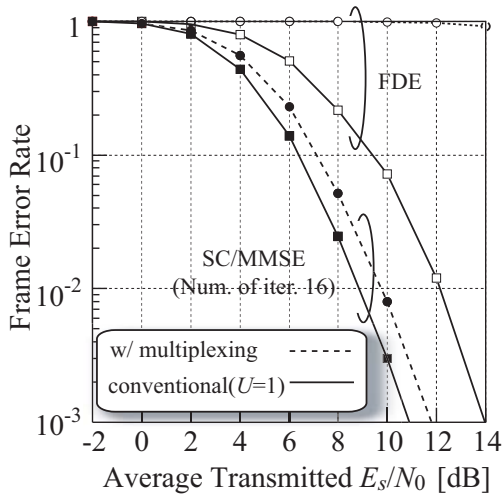


Fig. 7. FER results of a spectrum division multiple access exploiting the partial spectrum multiuser environment ($U = 2$) as a function of average transmitted E_s/N_0

Therefore, each user cannot select optimal frequency positions. However, 1 dB degradation can be considered to be acceptable because spectrum efficiency can be doubled with only 1 dB of power penalty. When 16QAM is applied instead of QPSK to double spectral efficiency, much larger power penalty is necessary. Thus, the proposed spectrum shaping technique is considered to be effective in enhancement of spectral efficiency in SISO transmission cases.

V. CONCLUSIONS

A spectrum shaping scheme exploiting frequency clipping for turbo equalization system has been proposed in this paper. In the proposed scheme, the transmitted spectrum components with lower channel gain is intentionally clipped to concentrate transmit power on spectrum with higher channel gain, and ISI caused by spectrum shaping in addition to the channel is compensated for using a FD-SC/MMSE turbo equalizer. Computer simulation confirms that the proposed scheme provides high spectrum efficiency with low power penalty.

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APPENDIX

System Model

In this paper, the channel model is assumed that uncorrelated frequency selective SISO system with U users. At the u -th user's transmitter, two independent binary coded sequences are input to a QPSK modulator, where the I- and Q-channels are independently modulated by two binary sequences. The channel is comprised of L independent fading path components that are separated in time by the symbol duration. The received signal is sampled once per symbol. The signal received by the received antenna can be expressed as

$$\begin{aligned} \mathbf{r} &= \sum_{u=1}^U \mathbf{H}_u \mathbf{F}^H \mathbf{M}_u \mathbf{F} \mathbf{s}_u + \mathbf{v}, \quad (n = 1, 2, \dots, U) \\ &= [r_u(1), r_u(2), \dots, r_u(K)]^T \end{aligned} \quad (24)$$

where $\mathbf{s}_u = [s_u(1), s_u(2), \dots, s_u(K)]^T$ and $\mathbf{v} = [v(1), v(2), \dots, v(K)]^T$ denote the u -th user's transmitted signal vector and the additive white Gaussian noise (AWGN) vector, whose element $v(k)$ is independent random Gaussian variable with zero mean and variance N_0 . \mathbf{F} is $K \times K$ discrete Fourier transform (DFT) matrix defined by

$$\mathbf{F} = \frac{1}{\sqrt{K}} \begin{bmatrix} W^{(0)(0)} & W^{(0)(1)} & \dots & W^{(0)(K-1)} \\ W^{(1)(0)} & W^{(1)(1)} & \dots & W^{(1)(K-1)} \\ \vdots & \vdots & \ddots & \vdots \\ W^{(K-1)(0)} & W^{(K-1)(1)} & \dots & W^{(K-1)(K-1)} \end{bmatrix} \quad (25)$$

in which W denotes a twiddle factor given by $W = \exp^{-j2\pi/K}$. Let a P -length CP be appended to the head of the transmitted

vector, the $K \times K$ channel matrix \mathbf{H}_u , which is constant over the frame, is given by

$$\mathbf{H}_u = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_K] \quad (26)$$

where \mathbf{H}_u is a circulant matrix based on a column vectors given by

$$\mathbf{h}_k = [h(1), h(2), \dots, h(L-1), \mathbf{0}_{K-L}]^T. \quad (27)$$

Note that $\mathbf{0}_x$ denotes all-zero vector with length x .

The transmitted spectrum and noise spectrum vector is given by

$$\begin{aligned} \mathbf{s}_u^f &= \mathbf{F} \mathbf{s}_u \\ &= [s_u^f(1), s_u^f(2), \dots, s_u^f(K)]^T, \end{aligned} \quad (28)$$

$$\begin{aligned} \mathbf{v}^f &= \mathbf{F} \mathbf{v} \\ &= [v^f(1), v^f(2), \dots, v^f(K)]^T. \end{aligned} \quad (29)$$

Therefore, the received spectrum vector can be expressed as

$$\begin{aligned} \mathbf{r}^f &= \mathbf{F} \mathbf{r} \\ &= \mathbf{F} \sum_{u=1}^U \mathbf{H}_u \mathbf{F}^H \mathbf{M}_u \mathbf{F} \mathbf{s}_u + \mathbf{F} \mathbf{v} \\ &= \sum_{u=1}^U \mathbf{\Xi}_u \mathbf{\chi}_u + \mathbf{v}^f \\ &= [r^f(1), r^f(2), \dots, r^f(K)]^T \end{aligned} \quad (30)$$

where, $\mathbf{\chi}_u$ denotes the shaped spectrum of u -th user, is given by

$$\mathbf{\chi}_u^f = \mathbf{M}_u \mathbf{s}_u^f = [\chi_u(1), \chi_u(2), \dots, \chi_u(K)]^T. \quad (31)$$

Moreover, $\mathbf{\Xi}_u$ denotes the $K \times K$ frequency domain channel matrix, is given by

$$\mathbf{\Xi}_u = \mathbf{F} \mathbf{H}_u \mathbf{F}^H = \text{diag}[\Xi_u(1), \Xi_u(2), \dots, \Xi_u(K)] \quad (32)$$

where $\Xi_u(k)$ is the complex channel weight of the frequency domain at symbol timing k .