Joint Power Allocation and Relay Selection in Network-coded Multi-unicast Systems

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Abstract—Physical-layer network coding (PNC) promises a significant gain in overall network throughput for multi-user cooperative communications. In a multi-unicast scenario, one drawback associated with PNC is an additional noise term, coined as network coding (NC) noise, which severely degrades the system data rate. In this paper, our contribution to address this challenging problem is source and relay power control with the objective of maximization of the minimum average achievable rate among all the source-destination pairs subject to a given total power constraint. We further develop a joint power allocation and relay selection scheme, which only rely on long-term channel statistics, to extend our results to general network topologies. We show that the joint optimization problem can be divided into two problems: optimal power allocation and optimal relay selection where the former is scalable and leads to a power assignment algorithm that exhibits the same optimization complexity for any number of sources in the network and the latter can be performed in a decentralized manner. By means of simulations, we validate our theoretical developments and verify the efficiency of our algorithm in improving the average achievable rate compared to a multi-unicast system with no power control or relay selection. We conclude that the proposed algorithm largely combats the adverse effects of NC noise while achieving near optimal fairness.

I. INTRODUCTION

Network coding (NC) [1], a technique originally developed to increase the capacity for lossless wireline networks, is a promising way to improve spectral efficiency of wireless cooperative networks. The key idea of NC is to encourage the intermediate nodes, known as relays, to forward the mixture of their observations either in digital (DNC [2]) or physical (PNC [3]) domain. There has been considerable effort in deploying PNC in various multi-user cooperative scenarios, referred to as network-coded cooperative communication (NC-CC). For example, in [4]–[8] efficient PNC protocols have been developed for the two-way and multi-way relay channels. Moreover, several cooperative multiple access transmission protocols based on PNC have been proposed in [9] and [10]. For such multi-way relaying and multiple access relay channels, it has been revealed that the use of NC can significantly improve the network performance, in terms of the system throughput and the reception reliability.

In this paper, however, we are interested in applying PNC to another fundamental building block of wireless communications known as multi-unicast scenario. There are limited works [11]–[14] that have studied NC-CC in this scenario. These works illustrate that bringing the promising benefits of NC to the multi-unicast NC-CC is not free of cost. In [11] it was shown that even with the perfect knowledge of channel information, there will be a non-negligible noise introduced during the signal extraction at destinations, which was coined NC noise. Moreover, the data rates of source-destination pairs are highly dependent on the NC noise and can even be lower than the conventional cooperative scheme [11].

To the best of our knowledge, [12]–[14] are the only works that made some efforts to deal with NC noise. The authors in [12] considered a multi-unicast NC-CC system with amplify and forward (AF) relaying and investigated the problem of group allocation and relay selection to maximize the weighted sum-rate. The analysis in [12] was developed without taking advantages of power control, which has long been a well-known technique for improving the wireless system performance. With multiple sources participating in PNC, it is still a challenging issue to design a joint optimal power allocation for relay and sources in NC-CC system. How multi-unicast NC-CC behaves when power control is employed remains largely an unaddressed problem and is one main goal of this paper. In [13] and [14], we addressed the NC noise problem by using power allocation at the relay. The main idea presented in [14] was to maximize the total data rate of single-relay multi-unicast NC-CC via optimal relay power allocation in which each received signal from a source is weighted by a power allocation coefficient. Optimal relay power allocation was shown to be more effective in helping alleviate the adverse effects of NC noise compared to equal power allocation in [11] and [12].

To implement the idea of [14] in a real NC-CC system, however, one is immediately confronted with fairness issue. Indeed, the proposed strategy in [14] maximizes the total data rate of the system, but not the data rate of individual users. In some practical applications there are fairness requirements, in which case the maximization of the minimum achievable rate of the system is the performance metric of interest. Moreover, the low achievable rate for some users may cause a bottleneck, which will unavoidably degrade the whole system performance.

Motivated by all above, in this paper we study and optimize a general multi-unicast NC-CC system when there are multiple relay nodes in a network. We present a joint power allocation and relay selection technique with the objective of maximization of the minimum average achievable rate among K source-destination pairs subject to a given total power constraint. The optimization problem can be divided into two problems: optimal power allocation and optimal relay selection. Following a similar approach as in [15], we show that the former with $K + 1$ transmission power variables,
can be reformulated to a single-variable optimization problem which can be easily solved by a simple numerical search of the single variable. The proposed optimization scheme is scalable and leads to a power assignment algorithm that exhibits the same optimization complexity for any number of sources in the network. Then we propose the optimal relay selection which combines the physical and medium access control (MAC) layer mechanisms to identify the best relay in a decentralized manner. The proposed power allocation and relay selection only rely on long term channel statistics. Therefore, we expect the additional costs of implementing them on NC-CC system in terms of computational complexity and overhead to be small. Numerical results show that the proposed power allocation and relay selection is promising in multi-unicast NC-CC as significant data rates gains, of up to 89.5% are observed compared with equal power allocation and random relay selection. Moreover, this scheme is able to largely combat the adverse effects of NC noise while achieving near optimal fairness.

II. SYSTEM MODEL AND PERFORMANCE ANALYSIS

Consider a multi-unicast wireless network where N mobile sources communicating with K mobile destinations with the help of the M fixed relay nodes using AF protocol and PNC in the presence of the source-to-destination links. We consider a NC-CC system with relay selection such that among M available relays, one of the them has been chosen as the best relay. Without loss in generality denote this relay by Ri, i ∈ {1, ⋯, M}. The reference multi-unicast NC-CC system is shown in Fig. 1. Let Si, denote the k-th source and Di the corresponding destination. Note that we use small subscripts s, d and r to refer to nodes S, D, and R respectively. We consider frequency-nonselective Rayleigh block fading channels. That is, the realization of the fading channel in each link stays constant during transmission of a block of symbols and changes to an independent value in the next block. We assume all nodes operate in half-duplex mode using time division multiplexing (TDM). Thus, transmissions from sources and relay occur in different time blocks. In each block, N symbols are transmitted in N symbol slots of duration Ts each.

The overall transmission can be divided into (K+1) blocks. In the first K blocks, each source Si, ℓ = 1, ⋯, K, sends its own block of N symbols with transmission power Ps to the destination Di, which is overheard by all other nodes, in a pre-assigned time block. The corresponding received signals by the destination node Di, k = 1, ⋯, K and the relay Ri, ℓ = 1, ⋯, K at the n-th time slot can be expressed, respectively, as

\[
y_{r_i,d_k} = h_{r_i,d_k} x_{\ell} + z_{r_i,d_k},
\]

(1)

\[
y_{s_i,r_i} = h_{s_i,r_i} x_{\ell} + z_{s_i,r_i},
\]

(2)

where z_{r_i,d_k} and z_{s_i,r_i} represent zero-mean complex-valued additive white Gaussian noise (AWGN) with variance σ^2_{r_i,d_k} and σ^2_{s_i,r_i} during the Si transmission at the destination Di and at the relay Ri, respectively. In addition, h_{r_i,d_k} and h_{s_i,r_i} denote the coefficients of the channels between the source Si and the destination Di and between the source Si and the relay Ri, respectively. We assume channel coefficients h_{r_i,d_k} and h_{s_i,r_i} follow a zero-mean complex Gaussian (ZMCG) distribution with variance σ^2_{r_i,d_k} = 1/d^2_{r_i,d_k} and σ^2_{s_i,r_i} = 1/d^2_{s_i,r_i}, respectively, where d_{r_i,d_k} and d_{s_i,r_i} are the Si to Di and Si to Ri distances and a is the path loss exponent [16]. This channel model includes both long-term path loss and short-term fading. Dropping the indices, the long-term path loss σ^2_a determines the strength of the short-term fading i.e., the variance of the fading channel h or the mean of |h|^2.

In the final block, i.e., the (K+1)-th block, relay R_i performs PNC by mixing the analog received signals. The processed signal at R_i can be expressed as

\[
x_{r_i} = \sum_{\ell=1}^{K} y_{s_i,r_i} = \sum_{\ell=1}^{K} (h_{s_i,r_i} x_{\ell} + z_{s_i,r_i}).
\]

(3)

The relay Ri amplifies x_{r_i} with an amplification factor

\[
A_i = \sqrt{\frac{P_i}{\sum_{\ell=1}^{K} |h_{s_i,r_i}|^2 + \sigma^2_{s_i,r_i}}},
\]

(4)

to maintain a constant power P_i at the relay output and then broadcasts the resulted signal to all destination nodes. The received signal at the destination Di can be written as

\[
y_{r_i,d_k} = A_h y_{r_i,d_k} x_{r_i} + z_{r_i,d_k} = A_h y_{r_i,d_k} y_{s_i,r_i} + A_h y_{r_i,d_k} \sum_{\ell=1,\ell \neq k}^{K} (h_{s_i,r_i} x_{\ell} + z_{s_i,r_i}) + z_{r_i,d_k},
\]

(5)

where z_{r_i,d_k} represents AWGN noise with variance σ^2_{r_i,d_k} at the destination Di during the R_i transmission and h_{r_i,d_k} is the fading channel between the relay R_i and the destination Di with ZMCG distribution and variance σ^2_{r_i,d_k} = 1/d^2_{r_i,d_k} where d_{r_i,d_k} is Ri to Di distance.

Accordingly, destination node Di receives one copy of signal x_k in the k-th block transmission. Further, it obtains another copy of x_k in the (K+1)-th block transmission. Using the overheard signals from Si, ℓ ≠ k, at Di, in (1), we can write

\[
x_{\ell} = \frac{y_{s_i,d_k} - z_{s_i,d_k}}{h_{s_i,d_k}}.
\]

Using this notation in the received signal from Si at R_i in (2), we can rewrite y_{r_i,d_k} in (5) as

\[
y_{r_i,d_k} = A_h y_{r_i,d_k} y_{s_i,r_i} + A_h y_{r_i,d_k} \sum_{\ell=1,\ell \neq k}^{K} \frac{h_{s_i,r_i}}{h_{r_i,d_k}} y_{s_i,d_k} - z_{s_i,d_k}
\]

\[+ A_h y_{r_i,d_k} \sum_{\ell=1,\ell \neq k}^{K} z_{s_i,r_i} + z_{r_i,d_k}.
\]

(6)

The multi-unicast NC-CC system requires that complete channel state information (CSI), i.e., h_{s_i,r_i}, h_{r_i,d_k} and h_{s_i,d_k}, ℓ = 1, ⋯, K be available at the destination node Di to cancel the unwanted terms [11]. Moreover, the relay nodes also require complete CSI to perform optimal relay selection.

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1Time index n is omitted in equations to simplify the notations.
Here, we assume that the mobility of the source and destination nodes is low, so that the channel conditions are stable for sufficiently long time, and therefore the frequency to update the channel information is low. However, the manner in which nodes obtain this information is beyond the scope of this paper. In order to remove $y_{s_k,d_k}$, $\ell = 1, \cdots, K$, $\ell \neq k$, $D_k$ multiplies signal $y_{s_k,d_k}$ by a factor $\frac{1}{h_{s_k,d_k}}$ and subtracts it from (6). Note that the destination node $D_k$ needs to repeat this cancelation for all the $(K-1)$ overheard signals $y_{s_k,d_k}$. The resulting signal at destination node $D_k$, denoted by $y_{r_k,d_k}^*$, is obtained as

$$y_{r_k,d_k}^* = y_{r_k,d_k} - Ah_{r_k,d_k} \sum_{\ell=1, \ell \neq k}^K h_{s_{\ell},d_{d_{\ell}}} y_{s_{\ell},d_{\ell}}$$

$$= Ah_{r_k,d_k} y_{s_k,r_k} - Ah_{r_k,d_k} \sum_{\ell=1, \ell \neq k}^K \frac{h_{s_{\ell},d_{\ell}}}{h_{s_k,d_k}} z_{s_k,d_k} + Ah_{r_k,d_k} \sum_{\ell=1, \ell \neq k}^K z_{s_{\ell},r_{\ell}} + z_{r_k,d_k}. \tag{7}$$

From (7), it is observed that instead of $z_{r_k,d_k}$, we now have a new noise term in this constructed signal as $z_{d_k}^{NC} = z_{d_k} - z_{r_k,d_k}$ where $z_{d_k}^{NC} = -Ah_{r_k,d_k} \sum_{\ell=1, \ell \neq k}^K \frac{h_{s_{\ell},d_{\ell}}}{h_{s_k,d_k}} z_{s_k,d_k} + Ah_{r_k,d_k} \sum_{\ell=1, \ell \neq k}^K z_{s_{\ell},r_{\ell}}$, is the NC noise at destination node $D_k$. It can be shown that $z_{d_k}^{NC}$ has a zero mean and variance

$$\sigma_{z_{d_k}}^2 = A^2 |h_{r_k,d_k}|^2 \sum_{\ell=1, \ell \neq k}^K \left( \left| \frac{h_{s_{\ell},d_{\ell}}}{h_{s_k,d_k}} \right|^2 \sigma_{s_{\ell},r_{\ell}}^2 + \sigma_{s_{\ell},d_{\ell}}^2 \right) + \sigma_{r_k,d_k}^2,$$

which is larger than the original noise variance $\sigma_{r_k,d_k}^2$.

### A. Achievable Rate

Now let us derive the mutual information for $S_k$-$D_k$ pair (when $R_i$ selected as the best relay) which is denoted by $T_{s_k,d_k}$. By substituting $y_{s_k,r_k}$ from (2) to (7) we have

$$y_{r_k,d_k}^* = Ah_{r_k,d_k} \left( h_{s_k,r_k} x_k + z_{s_k,r_k} \right) + z_{d_k}^{NC} \tag{8}$$

Now, (1) for $\ell = k$ and (8) present the appropriate channel model for NC-CC scheme with AF relaying and direct path from source $S_k$. As it was discussed in [17], the AF protocol with direct path provides an equivalent one-input, two-output complex Gaussian noise channel with different noise levels in the outputs. Therefore, it can be easily shown that for the above channel, $T_{s_k,d_k}$ is given by

$$T_{s_k,d_k} = \frac{W}{K+1} \log \left( 1 + \frac{P_s}{\sigma_{s_k,d_k}} \left| \frac{h_{s_k,d_k}}{h_{r_k,d_k}} \right|^2 \right)$$

$$+ \frac{P_s A^2 |h_{r_k,d_k}|^2}{\left( \sum_{\ell=1}^K \sigma_{s_{\ell},r_{\ell}}^2 + \sum_{\ell=1}^K \left| \frac{h_{s_{\ell},d_{\ell}}}{h_{s_k,d_k}} \right|^2 \sigma_{s_{\ell},d_{\ell}}^2 \right)} + \sigma_{r_k,d_k}^2 \tag{9}$$

where $W$ is the available bandwidth. Note that, in this paper, for the performance comparison we will use average mutual information, denoted by $\mathbb{E}\{T_{s_k,d_k}\}$, where $\mathbb{E}\{\cdot\}$ is the statistical expectation. Finding an exact expression for the average rate, i.e., the rate averaged over the fading channels, using (9) is difficult. Therefore, we numerically evaluate the average rate using Monte Carlo simulation in Section IV.

### III. Joint Power Allocation and Relay Selection

In this section, we design the joint optimal power allocation and relay selection for the multi-unicast NC-CC system, with NC noise, that maximizes the minimum average achievable rate among all source-destination pairs. We assume the network has a total power constraint $P_T$. The total transmit power, when the $i$-th relay is selected, $P_T^i$, can be written as $P_T^i = \sum_{k=1}^{K} P_{s_k} + P_{r_i}$. Accordingly, the main problem can be represented as

$$\max_{P_{s_k},P_{r_i},P_{r_i},\cdots,P_{r_i}} \min_{k \in \{1, \cdots, K\}} \mathbb{E}\{T_{s_k,d_k}\} \tag{10}$$

s.t.

$$P_T \leq P_T^{\max},$$

Solving (10) considering the difficulty of deriving $\mathbb{E}\{T_{s_k,d_k}\}$ is involved. Therefore, we have to use some approximations to find an acceptable power allocation and relay selection. Our approach is as follows. In (10), we substitute all channel gains $|h|^2$ (dropping the indices) in $T_{s_k,d_k}$ with their means $\sigma_{h_k}^2$ to obtain

$$\max_{P_{s_k},P_{r_i},\cdots,P_{r_i}} \min_{k \in \{1, \cdots, K\}} \mathbb{E}\{T_{s_k,d_k}\} \tag{11}$$

s.t.

$$P_T \leq P_T^{\max},$$

where

$$\mathbb{E}\{T_{s_k,d_k}\} = \frac{W}{K+1} \log \left( 1 + \frac{P_{s_k} \sigma_{h_k,d_k}^2}{\sigma_{s_k,d_k}^2} \right)$$

$$+ \frac{P_{s_k} A^2 \sigma_{h_k,d_k}^2}{\left( \sum_{\ell=1}^K \sigma_{s_{\ell},r_{\ell}}^2 + \sum_{\ell=1}^K \left| \frac{h_{s_{\ell},d_{\ell}}}{h_{s_k,d_k}} \right|^2 \sigma_{s_{\ell},d_{\ell}}^2 \right)} + \sigma_{r_k,d_k}^2 \tag{12}$$

and

$$\hat{A} = \frac{P_{r_i}}{\sum_{\ell=1}^K \left( \frac{P_{s_k} \sigma_{h_k,d_k}^2}{\sigma_{s_k,d_k}^2} + \sigma_{s_{\ell},d_{\ell}}^2 \right)} \tag{13}$$

Note that in (11) no expectation is required because $T_{s_k,d_k}^i$ is deterministic. We use the above optimization problem to provide power allocation and relay selection scheme. We will compare the results of (11) with optimum numerical results of (10) found by global search over $(K+1)$-dimension space in the next section.

The optimization problem (11) is equivalent to first optimizing over $P_{s_1}, P_{s_2}, \cdots, P_{s_k}, P_{r_i}$, and then optimizing over $i$ where the former is the optimal power allocation problem and the latter is the optimal relay selection problem. In the following we consider the two problems separately.

### A. Optimal Power Allocation

In this subsection, our goal is to obtain the optimum power assignment for relay and source nodes through the maximization of the smallest of the average mutual information $\hat{T}_{s_k,d_k}$ under a total power budget. The optimal power allocation problem can be expressed as

$$\max_{P_{s_k},P_{r_1},\cdots,P_{r_k}} u \tag{14}$$

s.t.

$$\hat{T}_{s_k,d_k} \geq u, \quad k = 1, \cdots, K,$$

$$P_T \leq P_T^{\max},$$

where $u$ is defined as $u = \min_{k \in \{1, \cdots, K\}} \hat{T}_{s_k,d_k}$. We note that for the above optimization problem at the optimum point, it is required that

$$\frac{P_{s_k}}{P_{s_k} + P_{r_i} + \cdots + P_{r_k} + P_{r_i}} = \frac{\sigma_{h_k,d_k}^2}{\sigma_{s_k,d_k}^2 + \sigma_{s_{\ell},d_{\ell}}^2}$$

and

$$\frac{P_{r_i}}{P_{s_k} + P_{r_i} + \cdots + P_{r_k} + P_{r_i}} = \frac{\sigma_{s_k,d_k}^2}{\sigma_{s_k,d_k}^2 + \sigma_{s_{\ell},d_{\ell}}^2},$$

for any $\ell \neq i$.

1115
\[ \hat{P}_{s_k,d_k} = \hat{P}_{s_2,d_2} = \cdots = \hat{P}_{s_K,d_K} = \hat{P}_r. \]  
(15)

Otherwise, if, for example, \( \hat{P}_{s_1,d_1} > \hat{P}_{s_2,d_2} \), then \( P_{s_1} \) can be reduced such that \( \hat{P}_{s_1,d_1} = \hat{P}_{s_2,d_2} \) and this reduction of \( P_{s_1} \) will not violate the total power constraint [8]. We note that our approach of power allocation is inspired by the work in [15], where some power assignment algorithms have been developed for a non-network coded single-relay multi-unicast network which allows simultaneous multi-source transmissions through nonorthogonal channels.

Now let us define an auxiliary parameter \( x \), which can be viewed as a normalized power factor at the relay, as follows:
\[
x \triangleq \hat{A}^2, \quad (16)
\]
where \( \hat{A} \) is the amplification factor specified in (13). The auxiliary parameter \( x \), as we shall see, will play a key role in the optimization procedure. Using (12) and (16) we can rewrite constraints \( \hat{P}_{s_k,d_k} \geq u, \ k = 1, \cdots, K \) in optimization problem (14) as
\[
P_{s_k} \geq \frac{2^{(K+1)u} - 1}{f_k(x)}, \quad k = 1, \cdots, K, \quad (17)
\]
where
\[
f_k(x) \triangleq \left( \frac{\sum_{i=1}^{K} \sigma_{h_{s_k,r_k}}^2 + x\sigma_{h_{s_k,r_k}}^2}{\sigma_{s_{s_k,d_k}}^2} \right). \]

We note that for any given value of the auxiliary parameter in (16), according to \( \hat{A} \) in (13), the transmission power at the relay can be determined as
\[
P_{s_k} = x \sum_{k=1}^{K} \left[ P_{s_k} \sigma_{h_{s_k,r_k}}^2 + \sigma_{s_{s_k,r_k}}^2 \right]. \quad (18)
\]

Thus, for any given \( x \geq 0 \), the optimization problem in (14) becomes
\[
\begin{align*}
\max_{x \geq 0} & \quad u(x) \\
\text{s.t.} & \quad \sum_{k=1}^{K} \left[ (x\sigma_{h_{s_k,r_k}}^2 + 1)P_{s_k} + x\sigma_{s_{s_k,r_k}}^2 \right] \leq P_{\text{max}}, \\
\text{s.t.} & \quad P_{s_k} \geq \frac{2^{(K+1)u(x)} - 1}{f_k(x)}, \quad k = 1, \cdots, K. 
\end{align*} \quad (19)
\]

Let for any \( x \geq 0 \) we denote the maximal value of the objective function in (19) as \( u(x) \). So, \( u(x) \) must satisfy
\[
P_{s_k} \geq \frac{2^{(K+1)u(x)} - 1}{f_k(x)}, \quad k = 1, \cdots, K. \quad (20)
\]

Substituting the above constraints into the total power constraint in (19), leads to
\[
\sum_{k=1}^{K} \left[ (x\sigma_{h_{s_k,r_k}}^2 + 1)\frac{2^{(K+1)u(x)} - 1}{f_k(x)} + x\sigma_{s_{s_k,r_k}}^2 \right] \leq P_{\text{max}}. \quad (21)
\]

Using (21), the optimization problem (19) can be written as
\[
\max_{x \geq 0} \quad u(x) \\
\text{s.t.} \quad \sum_{k=1}^{K} \left[ (x\sigma_{h_{s_k,r_k}}^2 + 1)\frac{2^{(K+1)u(x)} - 1}{f_k(x)} + x\sigma_{s_{s_k,r_k}}^2 \right] \leq P_{\text{max}}. \quad (22)
\]

We observe that for any given \( x \geq 0 \), the inequality constraint in (22) is a constant. Moreover, it is obvious that equality is achieved at the optimum point (i.e., \( P_{s_k} = P_{s_k}^{\text{max}} \)). Therefore, we have the following optimum power allocation problem
\[
\max_{x \geq 0} \quad u(x) = \frac{1}{K} \log \left( 1 + \frac{P_{\text{max}} \sum_{k=1}^{K} x\sigma_{h_{s_k,r_k}}^2}{\sum_{k=1}^{K} x\sigma_{h_{s_k,r_k}}^2 + 1} f_k(x) \right), \quad (23)
\]
which can be easily solved by a simple numerical search for the optimal value of the parameter \( x \).

The above discussion shows that we are able to convert the optimization problem (14) over a \((K+1)\)-dimension space to the maximization problem in (23), which depends only on one variable \( x \), i.e., over a one-dimension space. Using the optimal value of \( x \), denoted by \( x^* \), we can obtain the corresponding optimal source power \( P_{s_k}^* \) as
\[
P_{s_k}^* = \frac{2^{(K+1)u(x^*)}}{f_k(x^*)}, \quad k = 1, \cdots, K, \quad (24)
\]
and then by substituting \( x^* \) and \( P_{s_k}^* \) into (18) the corresponding relay power is obtained as
\[
P_{r_i}^* = x \sum_{k=1}^{K} \left[ P_{s_k}^* \sigma_{h_{s_k,r_i}}^2 + \sigma_{s_{s_k,r_i}}^2 \right]. \quad (25)
\]

B. Relay Selection

Using the optimal power allocation found in (24) and (25), the optimization problem (11) reduces to the following relay selection problem.
\[
\max_{i \in \{1, 2, \cdots, M\}} \quad \hat{P}_r^i. \quad (26)
\]
where \( \hat{P}_r^i \) is the approximated maximum average achievable rate, achieved by selecting the \( i \)-th relay, and is obtained by substituting \( P_{s_k}^* \) and \( P_{r_i}^* \) into (12).

It is desirable to develop a decentralized scheme to choose the best relay node which results in the maximum \( \hat{P}_r^i \). In our proposed algorithm, relays use similar carrier sensing scheme [18] and go through a back-off period before sending the combination of received signals to the destination nodes. The optimal relay selection can be accomplished as follows. Each candidate relay node \( R_i, i = 1, \cdots, M \) solves (23), calculates \( \hat{P}_r^i \) and then sets a back-off time which is proportional to the \( \hat{P}_r^i \). The back-off period for each relay is chosen such that the higher the \( \hat{P}_r^i \), the shorter the back-off time is. After the first back-off time expired, the best relay with the largest \( \hat{P}_r^i \) and the smallest back-off timer can occupy the channel first. It will broadcast an acknowledgment to other relay nodes which will quit the competition and refrain from forwarding. At the same time power adjustments will also be sent along with the same acknowledgement to the sources.

In our proposed scheme, power allocation and relay selection are performed in advance based on long-term channel statistics. Indeed, it maintains transmission powers of relay and source nodes and the index of the best relay node throughout multiple block transmissions as long as channel statistics and the network topology are unchanged. Hence, we expect that the complexity for searching power allocation and relay selection and the required overhead to update this information to be small.
The relay power allocation proposed in [14] to combat the NC noise is only concerned with maximizing the total data rate. In particular, each received signal from a source at the relay is weighted by a power allocation coefficient $\alpha_k$ where $0 \leq \alpha_k \leq 1$. Consequently, relay may allocate $\alpha_k = 0$ to a user and does not forward data for it. In this paper, however, we take the fairness issue among all the users into consideration and adopt the max-min average achievable rate based power allocation scheme because of the fact that the worst user usually affects the network performance significantly. To have a quantitative measure of fairness, different schemes and formulas might be considered. The authors in [19] discussed the desired properties of a fairness index and suggested a very useful statistical formula to measure the fairness, which will be used in this paper. Using the notations in this paper, the index proposed by [19] can be written as

$$f = \frac{\left( \sum_{k=1}^{K} \mathbb{E}\{T_{s,k,d_k}\} \right)^2}{K \sum_{k=1}^{K} \mathbb{E}\{T_{s,k,d_k}\}}. \quad (27)$$

Clearly, solving the optimal max-min optimization problem (10) will lead to the optimal fairness index $f = 1$, since at the optimum point $\mathbb{E}\{T_{s,k,d_k}\} = \cdots = \mathbb{E}\{T_{s,K,d_k}\}$. In the proposed scheme using (11) the above equality, however, may not necessarily hold. This is due to the replacing square of magnitudes of channels with their means. Nevertheless, the proposed scheme maintains a fairness index very close to 1 as evidenced by numerical results in the next section.

### IV. NUMERICAL RESULTS

In this section, we study the efficiency of the proposed joint power allocation and relay selection scheme, OPA-ORS, in a multi-unicast system. We consider 100 randomly generated network instances. For each instance, $K$ source-destination pairs are randomly distributed (with a uniform distribution) in a two-dimensional rectangular region of size $450 \text{m} \times 450 \text{m}$. For all simulations we assume that the path-loss exponent is $\alpha = 4$ and without loss of generality, assume that all the noise variances are equal to $10^{-10} \text{ W}$. We consider a carrier frequency of 2.5 GHz and a bandwidth of $W = 10 \text{ kHz}$, which is suitable for mobile WiMAX, i.e., IEEE 802.16e. In our numerical results, correlated mobile-to-fixed channel coefficients, i.e., sources to relays and relays to destinations channels, are generated according to Clarke’s model [20]. Besides correlated mobile-to-mobile channel coefficients, i.e., sources to destinations channels, are generated using the method of exact Doppler spread (MEDS) [21]. We use normalized Doppler frequency (i.e., $f_D T_S$ where $f_D$ is the Doppler frequency shift and $T_S$ is the symbol duration) of 0.003 corresponding to mobile speed of 13 km/h. First we compare OPA-ORS with two other schemes; EPA-ORS: In this scheme optimum relay selection is used while power is equally allocated to relay and source nodes. The problem formulation in this case is a simplified version of the formulation given in (11) as $\max_{x \in \{1, \ldots, M\}} \min_{k \in \{1, \ldots, K\}} \frac{\mathbb{E}\{T_{s,k,d_k}\}}{T_{s,k,d_k}}$, where $\mathbb{E}\{T_{s,k,d_k}\}$ is obtained by substituting $P_{s_i} = \cdots = P_{s_m} = P_s = \frac{P_{T}^{\text{max}}}{(K + 1)}$ in (12). EPA-RRS: In this scheme equal power allocation and random relay selection is used.

In Fig. 2 we plot the highest average achievable rate versus the maximum total transmit power $P_T^{\text{max}}$ in a NC-CC system with 5 source-destination pairs and 9 relays. We also present results for EPA-RRS scheme without considering the NC noise, represented by “EPA-RRS, Ideal”. Three observations follow from this figure are: 1) NC-CC system with NC noise suffers rate loss. In particular, the rate loss of the EPA-RRS to the EPA-RRS, Ideal is around 13.5%. However, schemes based on optimum power allocation and/or relay selection significantly combat the adverse effects of NC noise. Indeed, the highest achievable rates of the OPA-ORS and EPA-ORS considering NC noise are higher than those of the EPA-RRS, Ideal. 2) The highest average achievable rates of the proposed scheme OPA-ORS are much higher than those of the rest. Compared with EPA-ORS, an improvement of approximately 33.8% – 58.7% is achieved via optimum power allocation. Compared with EPA-RRS an improvement of approximately 52.6% – 89.5% is achieved via joint optimum power allocation and relay selection. 3) The less total available power, $P_T^{\text{max}}$, the more rate enhancement of the optimum power allocation and relay selection.

Fig. 3 shows the highest average achievable rate of NC-CC with power allocation and relay selection against $P_T^{\text{max}}$ using two optimization schemes; proposed OPA-ORS and optimal scheme (10) which has been solved by global numerical search over $(K + 1)$-dimension space. The latter scheme is complex and time consuming. Moreover, the complexity of searching over $(K + 1)$-dimension grid increases exponentially for increasing the number of sources in the network, as opposed to our scheme which exhibits the same optimization complexity for any number of sources. Note that, accordingly,
It was shown that an improvement of up to 89.5% is achieved by using the proposed optimal power allocation and relay selection over that of a non-power controlled NC-CC system with random relay selection. Simulation results also indicated that as compared to the relay power allocation technique of [14], our proposed scheme can provide a notable improvement in fairness performance.

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REFERENCES


V. CONCLUSION

In this paper, we jointly leveraged power control and relay selection to deal with NC noise issue in a multi-unicast NC-CC system. Our approach was based on maximizing the minimum average achievable rate among K source-destination pairs subject to a given total power constraint. We reformulated the main power allocation problem with (K + 1) power variables to a problem which can be easily solved by numerical search over one-dimension space. Combining MAC and physical layer mechanisms, we also developed a decentralized relay selection algorithm. The accuracy of the proposed methods and analysis have been verified numerically. It was shown that an improvement of up to 89.5%