Error Propagation in a Multi-way Relay Channel

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Abstract—This paper studies the error propagation phenomenon for a multi-way relay channel based on binary phase shift keying (BPSK) modulation and network coding in the physical layer. Though physical layer network coding enhances the throughput of a multi-way relay system by reducing the number of time slots, its error performance can be highly degraded since decision about a user’s message depends on previous decisions. In this paper, the error probability of a multi-way relay system is examined through both analytical and numerical methods. We show that the system performance is significantly controlled by signal power and number of users accessing the single relay. Based on numerical analysis, the critical number of users to limit the error rate within an acceptable upper bound is found.

Index Terms—Error propagation, functional decode and forward, maximum a posteriori (MAP) detection, multi-way relay, physical layer network coding.

I. INTRODUCTION

A. Motivation and Background

Relay systems are receiving more attention day-by-day due to their larger capacity [1] and energy efficiency [2] compared to other wireless systems. The reasons behind this are the spatial diversity, achieved through node cooperation [3] and extended coverage [1]. Earlier studies on relay channels mainly concentrated on the impact of a relay on the system capacity [4] through different coding mechanisms. Laneman et al. [3] proposed three types of relay cooperation protocols: Amplify and Forward [3], [5]–[8], Decode and Forward [3], [8]–[10] and Compress and Forward [11], [12].

Studies on relaying started with the classical relay channel model [3], where the relay contributes to successful transmission of information from source to destination. This concept of unidirectional relay channel has been extended further to bidirectional or two-way relay channel [13]–[19] that can provide effective cooperation between two source nodes through simultaneous exchange of information. Straightforward implementation of the classical relay model for bidirectional operation would require four time slots, resulting in a poor spectral efficiency. To increase the capacity from that of classical relay channel models, network coding schemes are implemented in [13]–[15], [19]. Such schemes include digital [19] and analog network coding [13]–[15]. Digital network coding is performed through XOR operations on bit streams from the two users [15]. This scheme requires three time slots, two slots for the sources to transmit their own bit stream and one for the relay to transmit the XOR of these two bit streams [15], [19]. Thus, network coding improves the spectral efficiency of the relay channel. The performance can be further improved by implementing network coding in the physical layer, termed as physical layer network coding or analog network coding. It is based on the additive nature of physical electromagnetic waves [15]. It requires only two time slots for successful exchange of information [13]–[15]. In the first time slot, both the users transmit and the relay receives the sum. Then the relay decodes [14], [15] or amplifies [13] the sum and retransmits to both the users and each user detects the other user’s information by subtracting its own information from the relay transmission. Thus analog network coding offers a larger capacity than digital network coding, whereas error performance degrades [13] as the number of time slots for information exchange is reduced.

The concept of two-way relaying can be extended to multi-user case, termed as multi-way relay channel, where multiple users exchange information with the help of a single relay terminal [20], [21]. Gündüz et al. [20] have considered Gaussian multi-way relay channel with decode and forward strategy, whereas Ong et al. [21], [22] implemented functional decode and forward, where the relay decodes a function of the users’ messages. In case of multiple users, analog network coding cannot be implemented in a straightforward manner like a two-way relay channel. If all the users transmit at the same time, then no user can detect other users’ messages using only its own information. So, the relay decodes functions of message pairs, which are simple XOR operations, using time-division multiple-access [21]. Each user receives the functions from the relay and decodes them sequentially through the same mechanism as in a two-way relay to retrieve all the other users’ messages. It has been shown in [21] that this functional decoding approach can achieve the common rate capacity for a multi-way relay channel.

B. Approach and Contribution

Previous works on multi-way relay channel have focused primarily on the capacity and achievable rate regions and the issue of error performance has not been considered to the best of our knowledge. In the coding strategy used in
In this paper, we have considered the following aspects of a multi-way relay channel:

1) The problem of error propagation in a multi-way relay channel and the resulting serious performance degradation are investigated. This problem can cause a large increase in error rate for small deviations in signal to noise ratio.

2) We consider the effect of total number of users on error performance and show that as the number of users increase, the problem of error propagation becomes more severe.

3) Also, we have studied numerically the maximum number of users to meet a certain upper bound on error for a certain signal to noise ratio. According to our analysis, to maintain the same system performance as that of a standard BPSK system, the number of users should be limited to a small value of 5 users.

The rest of the paper is organized in the following manner. The system model is described in Section II. Analytical equations regarding to error performance of the systems under consideration have been derived in Section III. Section IV provides the simulation results and compares the analytical and numerical solutions. We conclude the paper in Section V.

II. SYSTEM MODEL

In this paper, we consider binary transmission over a Gaussian multi-way relay channel with $L$ users, exchanging their information through a single relay and without any direct link in between them. Fig. 1 shows the channel model under consideration. The channel model is similar to that of [21]. The complete information exchange among all the users is performed in two phases. In the multiple access phase, the users transmit in a pairwise manner and the relay receives the sum of signals. In the broadcast phase, the relay broadcasts the decoded message and all the users receive and store it. After receiving all the information from the relay, the users subtract their own information to retrieve messages from other users. The only difference in the channel model from that in [21] is that instead of using abstract binary symmetric channel, we are dealing with a realistic physical channel model. The relay network can be considered to be composed of $L−1$ two-way relay channels, with the first $L−1$ steps in multiple access phase and the next $L−1$ steps in broadcast phase. For example, in the $\ell$th step of multiple access phase, only the users, $\ell$ and $\ell + 1$ participate in the two-way relaying operation. In the following step, users, $\ell + 1$ and $\ell + 2$ transmit and so on. Each user is denoted by $i$. Let, the $i$th and $(i + 1)\ell$th user transmit binary messages, $W_i$ and $W_{i+1}$ which are BPSK modulated to $X_i$ and $X_{i+1}$ respectively. The relay receives the signal

$$r_{i,i+1} = X_i + X_{i+1} + n$$

where $n$ is the zero mean additive white Gaussian noise (AWGN) with noise variance $\frac{N_0}{2}$. The distribution of the received signal is shown on Fig. 2. The relay then decodes the signal using MAP criterion, where the optimum threshold is $\gamma_r$, as denoted in Fig. 2. The true network coded symbol transmitted by the sources are given by [21]:

$$V_{i,i+1} = W_i \oplus W_{i+1}.$$  

That is, in a noise-free environment, if both the users transmit '+' or '-1', the absolute value of the received signal would be above the threshold and the relay would detect $V_{i,i+1} = 1$. If the users have transmitted different signals, then the absolute value of the received signal would be below the threshold and the relay would detect $V_{i,i+1} = 0$.

However, in a noisy environment, the symbol detected by the relay is given by $V_{i,i+1}$. Then the relay again modulates the decoded signal through
BPSK and transmits to the users. The signals received at the $i^{th}$ and $(i+1)^{th}$ users are denoted by $Y_i$ and $Y_{i+1}$, respectively, where

$$Y_i = Z_{i,i+1} + n$$

Here, $Z_{i,i+1}$ is the BPSK modulated signal from the relay. The users then detect the received signal through MAP criterion. The detected symbol is denoted by $\hat{V}_{i,i+1}$, which represents an estimation of the network coded information of the user pair consisting of the $i^{th}$ and $(i+1)^{th}$ users. When $i^{th}$ user has estimated the network coded information of all such user pairs, it performs XOR operation between its own information bit and the detected symbol and thus estimates the information of the $(i+1)^{th}$ user. When this information is available, $i^{th}$ user utilizes it to obtain the information of the $(i+2)^{th}$ user in the same manner and the process is continued until the message of $L^{th}$ user has been decoded. In a similar manner, $i^{th}$ user can decode the message of $(i-1)^{th}$ user and utilize it to obtain the information of all the users which have transmitted prior to it. The decoding process is shown in equations (4a)-(4i).

$$\hat{W}_{i+1} = \hat{V}_{i,i+1} \oplus W_i$$

$$\hat{W}_{i+2} = \hat{V}_{i+1,i+2} \oplus \hat{W}_{i+1}$$

$$\hat{W}_{i+3} = \hat{V}_{i+2,i+3} \oplus \hat{W}_{i+2}$$

$$\hat{W}_{i+4} = \hat{V}_{i+3,i+4} \oplus \hat{W}_{i+3}$$

$$\vdots$$

$$\hat{W}_L = \hat{V}_{L-1,L} \oplus \hat{W}_{L-1}$$

$$\hat{W}_{i-1} = \hat{V}_{i-1,i} \oplus W_i$$

$$\vdots$$

$$\hat{W}_4 = \hat{V}_{4,4} \oplus W_4$$

$$\hat{W}_2 = \hat{V}_{2,3} \oplus W_3$$

$$\hat{W}_1 = \hat{V}_{1,2} \oplus W_2$$

$(i+1)^{th}$ user follows the same process and detects all the messages. The whole process is repeated for all the users and after the $2(L-1)^{th}$ step, each user has exchanged information with all other users.

### III. Theoretical Error Performance Analysis

We consider error performance of a two-way relay channel first and then combine the results for such $(L-1)$ channels to analyze the performance of a $L$-user multi-way relay channel.

#### A. $Pr(\hat{V}_{i,i+1} \neq V_{i,i+1})$

If both the relay and the user make an error, they cancel each other and the detected symbol is correct. Thus, error in the overall system occurs if the relay transmits wrong message and the user correctly detects the transmission from relay or the relay correctly decodes the information and the user makes an error. The overall error probability, that is, the probability that the $i^{th}$ user makes error about $(i+1)^{th}$ user is:

$$P_{V_i} = Pr(\hat{V}_{i,i+1} \neq V_{i,i+1})$$

$$= \frac{1}{8} \left[ \text{erf} \left( \frac{\gamma - 1}{\sqrt{N_0}} \right) \left( \text{erf} \left( \frac{\gamma + 2}{\sqrt{N_0}} \right) + \text{erf} \left( \frac{\gamma - 2}{\sqrt{N_0}} \right) \right) + \text{erfc} \left( \frac{1 - \gamma}{\sqrt{N_0}} \right) \left( \text{erf} \left( \frac{\gamma + 2}{\sqrt{N_0}} \right) + \text{erf} \left( \frac{\gamma - 2}{\sqrt{N_0}} \right) \right) + \text{erfc} \left( \frac{\gamma - 1}{\sqrt{N_0}} \right) \left( \text{erf} \left( \frac{\gamma - 2}{\sqrt{N_0}} \right) + \text{erf} \left( \frac{\gamma} {\sqrt{N_0}} \right) \right) + \text{erfc} \left( \frac{\gamma + 1}{\sqrt{N_0}} \right) \left( \text{erf} \left( \frac{\gamma} {\sqrt{N_0}} \right) + \text{erf} \left( \frac{\gamma - 2}{\sqrt{N_0}} \right) \right) \right]$$

This probability also refers to the error probability of incorrectly estimating the network coded information $V$. That is why, the subscript is chosen to be $V$. In this equation, $\gamma$ is the optimum threshold for MAP detection of message of the relay ($\hat{V}_{i,i+1} \neq V_{i,i+1}$ and $\hat{V}_{i,i+1} \neq \hat{V}_{i,i+1}$). The probability that the relay has made an error is given by [15]:

$$Pr(\hat{V}_{i,i+1} \neq V_{i,i+1})$$

$$= \frac{1}{4} \left[ \text{erf} \left( \frac{\gamma}{\sqrt{N_0}} \right) + \frac{1}{4} \text{erf} \left( \frac{\gamma + 2}{\sqrt{N_0}} \right) + \frac{1}{4} \text{erf} \left( \frac{\gamma - 2}{\sqrt{N_0}} \right) \right]$$

$$= \frac{1}{2} \text{erf} \left( \frac{\gamma}{\sqrt{N_0}} \right) + \frac{1}{2} \text{erf} \left( \frac{\gamma + 2}{\sqrt{N_0}} \right) + \frac{1}{2} \text{erf} \left( \frac{\gamma - 2}{\sqrt{N_0}} \right)$$

(5)
Fig. 3. Possible cases for exactly one error when the $i^{th}$ user is decoding. Here the wide tilde above the hat and double hat indicates an error in estimation. The figure shows both the cases of end and middle user at the same time.

### B. Error Propagation in Decoding Users’ Messages

The results for two-way relay channel can be extended to multi-user case to investigate the effect of error propagation. This extension is possible because probability that any user makes wrong decision about the network coded information of any user pair is the same for all the users. Our analysis is based on the following facts about the channel model under consideration:

(a) Decision about each user depends on decision about previous users. If an error occurs in the decoding process, the error continues until another error is made. For example, in the equation set (4)(a) and (4)(b), if error occurs in the detection of $W_{i+1}$, then the detection of $W_{i+2}$ will also be erroneous, provided $V_{i+1,i+2}$ is correct.

(b) If error is made about one of the end users, then the error cannot propagate through the decisions about other users. For example, in the equations (4)(e) and (4)(i), if error occurs in $W_L$ or $W_1$, it does not affect the other users.

(c) If $i^{th}$ user is decoding, then decisions about users transmitting after $i^{th}$ user and before $i^{th}$ user are independent from each other. That is, from equation (4)(a) and (4)(f), an error in the $(i+1)^{th}$ user does not affect the decision about the $(i-1)^{th}$ user and vice versa.

1) Probability of No Error: For an entirely error-free system, each user needs to correctly decode all the other users. To be more specific, in an $L$-user system, each user has to correctly decode $(L-1)$ users (since it knows its own information). Thus, probability of zero error in a multi-way relay is:

$$P_i(0) = (1 - P_V)^{L-1} \quad \forall i = 1, ..., L \quad (10)$$

2) Probability of One Error: We identify two cases: either the error must occur about one of the end users so that it cannot propagate or two consecutive incorrect estimations of the network coded information are needed for users other than the end users. As an example of the first case, if an error occurs in $V_{1,2}$, then the $1^{st}$ user will be incorrectly decoded (see (4)(i)).

As an example of the second case, if error occurs in both $V_{1,1} + 1$ and $V_{1,1} + 2$, then only the $(i + 1)^{th}$ user will be incorrectly decoded (see (4)(a) and (4)(b)). These two cases are shown in Fig. 3. Note that two consecutive middle errors can happen anywhere and do not have to coincide with $V_{i+1}$. For example, $V_{i+2,i+3}$ and $V_{i+3,i+4}$ can be in error resulting in wrong decoding of the $(i + 2)^{th}$ user’s message (see (4)(c) and (4)(d)).

Let us define:

$$A = \text{Probability of one error in users except the end user} = (1 - P_V)^{L-3} P_V^2 \quad (11a)$$

$$B = \text{Probability of one error in one of the end users} = (1 - P_V)^{L-2} P_V \quad (11b)$$

Thus, probability of exactly one error is:

$$P_i(1) = \begin{cases} (L - 3)A + 2B & i \neq 1 \text{ and } i \neq L \\ (L - 2)A + B & i = 1 \text{ or } i = L \end{cases} \quad (12)$$

The first equation is valid when any user except the end users is decoding information. In this case, the two end users are considered as separate terms in the second part of the equation. Since the end users are considered separately and the decoding user knows its information, $(L - 3)$ terms are possible for the first part. The second equation describes the error probability when the end users decode others’ information. Here, the $1^{st}$ (or $L^{th}$) user is taken as a separate term in the second part of the equation and the $L^{th}$ (or 1st) user knows its information. Thus $(L - 2)$ terms remain in the first part of the equation.

3) Probability of Two Errors: In case of exactly two errors, the errors can be either consecutive or non-consecutive. When the errors are consecutive, there are two subcases: two consecutive errors involving an end user, or two consecutive errors involving users in the middle. We can verify that if there is an error in $V_{1,2}$, but no error in $V_{2,3}$, then both the $1^{st}$ and the $2^{nd}$ user will be incorrectly decoded (see (4)(h).
and (4)(i)). A similar statement applies to the decoding of the last two messages. As an example of consecutive errors in the middle, imagine that error occurs in \( \hat{V}_{i+1} \) and \( \hat{V}_{i+2} \) with a correct detection about \( \hat{V}_{i+1} \). Then the \((i+1)\)th and \((i+2)\)th users’ messages will be incorrectly estimated (see (4)(a), (b) and (c)). Thus there should be wrong detection about users other than the end users. (13b) In this case, the two decisions must be separated by a correct detection so that error can propagate to the next user.

For non-consecutive errors there are three subcases: non-consecutive errors involving one end user, non-consecutive error involving both the end users or non-consecutive errors involving middle users. As an example of the first subcase, if there are errors in \( \hat{V}_1, \hat{V}_3 \) and \( \hat{V}_{3,4} \), the \( 1^{st} \) and the \( 3^{rd} \) users’ message will be incorrectly decoded (see (4)(g), (h) and (i)). A similar statement applies to the decoding of the last three messages. Thus, if one of the two non-consecutive errors is about an end user, then one error about \( \hat{V}_{i+1} \) of the end user and two errors about \( \hat{V}_{i+1} \) of the preceding or following users are enough for two errors. When both the end users are involved, for example, if the errors are in \( \hat{V}_{L-1}, \hat{V}_1 \), then they do not affect the other users but the \( 1^{st} \) and the \( L^{th} \) user. That is, if the errors are about only the end users, then two wrong detection about \( \hat{V}_{i+1} \) of the end users result into two errors (equations (4)(e) and (i)). For non-consecutive errors involving middle users, there must be four wrong detection of the network coded information, two for each user. For example, if \( \hat{V}_1, \hat{V}_1, \hat{V}_2, \hat{V}_2, \hat{V}_3, \hat{V}_3, \hat{V}_4, \hat{V}_4 \) all are incorrectly estimated, then error occurs only in the messages of the \((i+1)^{th}\) and the \((i+3)^{th}\) user (see (4)(a), (b), (c) and (d)). That is, there should be error in both steps in which each of the two users (whose messages are incorrectly decoded) transmits. Fig. (4) illustrates our discussion.

Let us define,

\[ C = \text{Probability that two consecutive errors are made about users other than the end users} \]
\[ = (1 - P_V)(1 - P_Y^2) \quad (13a) \]

\[ D = \text{Probability that two consecutive errors are made when one of the users is an end user} \]
\[ = (1 - P_V)P_Y \quad (13b) \]

\[ E = \text{Probability that two non-consecutive errors are made about users other than the end users} \]
\[ = (1 - P_V)P_Y^4 \quad (13c) \]

\[ F = \text{Probability that two non-consecutive errors are made when one of the users is an end user} \]
\[ = (1 - P_V)(1 - P_Y)P_Y^3 \quad (13d) \]

\[ G = \text{Probability that two non-consecutive errors are made when the users are end users} \]
\[ = (1 - P_V)(1 - P_Y^2)P_Y^2 = C \quad (13e) \]

Considering all these cases, probability of two errors is given as follows:

\[ P_i(2) = \begin{cases} 
(L - 4)C + D + \sum_{m=2}^{L-4}(L - 3 - m)E+ \\
2(L - 4)F + C,\quad i = 2 \text{ or } i = L - 1 \\
(L - 5)C + 2D + \sum_{m=2}^{L-4}(L - 3 - m)E+ \\
2(L - 4)F + C,\quad i = 3 \text{ or } i = L - 2 \\
(L - 3)C + D + \sum_{m=2}^{L-3}(L - 2 - m)E+ \\
(L - 3)F, \quad i = 1 \text{ or } i = L \\
(L - 5)C + 2D + \sum_{m=2}^{L-2}(L - 4 - m)E+ \\
\sum_{m=1}^{L-i-1}(L - 3 - m)E+ \sum_{m=L-i}^{L-3}(L - 2 - m)E+ \\
2(L - 4)F + C, \quad i \notin \{1, 2, 3, L - 2, L - 1, L\} 
\end{cases} \]

(14)

Here, \( m \) indicates decoding order difference for non-consecutive errors. For example, if the errors occur about the \( 2^{nd} \) and the \( 4^{th} \) user, then decoding order difference of the two users is 2. Since, \( i^{th} \) user knows its information and the
end users are taken as separate terms, \((L - 3)\) users remain. If at least one pair needs to be formed from these \((L - 3)\) users, the decoding order difference between them can be at most \((L - 4)\). Thus \(m\) can be no less than 2 and no more than \((L - 4)\). If one of the users is an end user, then it knows its information and the other end user is taken as a separate term. Thus, \((L - 2)\) users remain, from which, only one pair can be formed if the decoding order difference is \((L - 3)\). Thus, in this case, \(m\) is limited between 2 and \(L - 3\).

4) **Probability of \((L - 1)\) Errors:** The probability that all messages are incorrectly decoded depends on the decoding user. If the decoding user is not an end user, then two errors at specific locations are needed. For example, if user \(i\) is decoding and the error occurs only in \(\hat{V}_{i,i+1}\) and \(\hat{V}_{i-1,i}\), then the \(i^{th}\) user will make an error about every other user. Thus, if the decoding user makes an error to detect the network coded information in both the steps in which it has transmitted, then it will incorrectly decode all users’ messages. On the other hand, if the \(1^{st}\) (or \(L^{th}\)) user is decoding and a single error occurs in \(\hat{V}_{1,2}\) (or \(\hat{V}_{L-1,L}\)), then it will incorrectly decode information of all the users. That is, in case of end users, if error occurs in \(\hat{V}_{i,i+1}\) of the user just preceding or just following it, then all the users’ messages will be incorrectly estimated (see (4)(i) or (e)). These cases are shown in Fig. 5.

Probability for exactly \((L - 1)\) errors is given by:

\[
P_i(L - 1) = \begin{cases} 
  (1 - P_V)^{(L-3)} P_V^2 & i\neq 1 \text{ and } i\neq L \\
  (1 - P_V)^{(L-2)} P_V & i = 1 \text{ or } i = L 
\end{cases}
\] (15)

Analytical solutions for probability of other errors is more complex and will be examined through numerical simulation.

### IV. NUMERICAL RESULTS

This section provides the numerical simulation verifications of the analytical results and an upper bound to the maximum number of users that can be served by a single relay has been investigated. In all the cases, the signal to noise ratio (SNR) is assumed to be SNR per bit of the message. Fig. 6 shows the error performance of a two-way relay channel. From the figure, it can be clearly observed that analytical results ((5) and (8)) are in complete agreement with those of simulation. Total error of the system is slightly larger than that in the relay because error can occur either in the multiple access phase from source to relay or in the broadcast phase from relay to source. However, the overall system performance is almost similar to that of a standard BPSK system. A wide range of SNR has been chosen to show the complete system performance, since very low SNRs are not unusual in satellite relay channels.

Error probability for no error and exactly one error cases for \(L = 10\) users can be observed in Fig. 7. Probability that a user correctly decodes all the other users increases with SNR and reaches to 1 above 5 dB SNR. Thus, increasing the SNR to 5 dB or more assures almost no error. Probability that the \(1^{st}\) or \(L^{th}\) user makes an error is slightly smaller than that of the \(2^{nd}\) or other users. For exactly one error, either the error must occur at the end users so that it cannot propagate or two consecutive errors need to occur. For the end users, they know their own information. So, when one of the end users is decoding, the probability of making exactly one error is slightly less than the case of any other user. All the analytical results (see (10) and (13)) have been verified through simulation.

Fig. 8 shows the probability of two consistent errors at the \(1^{st}\) and \(3^{rd}\) user for a total of \(L = 10\) users. In this case, also, the end users have a smaller probability to make errors than the \(3^{rd}\) user. Since error cannot propagate from end users, there is no chance to correct an error made in the detection of the end users by others through another subsequent error. However, the end users know their information and thus, are not affected by the end of error propagation. The analytical results of equation (14) have been shown for verification purpose.

Fig. 9 shows the probability of exactly \((L - 1)\) errors at the \(1^{st}\) and the other users. At low SNRs, this probability is small. Then almost all the information is wrongly detected by any user. Since two consecutive wrong decisions about the network coded information result into incorrect detection of one user, many of the users will be correctly decoded, not because of themselves, but due to error in the previous step. As SNR increases, error in detection decreases and all the phases are
not erroneous. Then the chance of decoding a user correctly through error in the previous step gets smaller, resulting in a larger probability of exactly \((L-1)\) errors. At high SNRs, probability of zero error is very large, as can be seen in Fig. 7. So probability of exactly \((L-1)\) errors decreases with high SNR. For exactly \((L-1)\) errors, any user except the end users needs to make error about both the users that transmit immediately before and immediately after that user. This probability gets smaller as SNR increases. In the case of end users, there is no user transmitting before (after) the 1st (Lth) user and hence, the error performance of the end users does not improve like others in the high SNR region. The analytical results from equation (15) has been shown for verification.

Fig. 10 shows the probability of different range of errors for different number of users with 2 dB SNR. The error performance is more or less similar for the first user and the Lth user. Though in case of exactly 1, 2 or more errors there is small difference in their performance levels, it is not significant when an error range is considered. Error probability increases with number of users. For a large number of users, the problem of error propagation becomes more serious. Chance of 10% wrong decisions has higher probability than 20% and 50% wrong decisions.

Fig. 11 provides an idea about the optimum number of users to operate within a certain tolerance on probability of at least 1 error. For an SNR of 10 dB, to decrease the error probability to less than \(10^{-4}\), number of users must be limited to less than 10, as can be clearly seen from the inset. If the number of users is increased, error probability becomes so high that it would
be unacceptable for any application. Another interesting point is that a drop of 1 dB in SNR reduces the optimum number of user from 1000 to 140 for the same error tolerance of $10^{-2}$. Thus, the system is highly vulnerable to potential signal power changes due to fading or shadowing.

V. CONCLUSION

The problem of error propagation, studied in this paper, is likely to be a serious limitation of the multi-way relaying. Even when the SNR is high, the system performance is worse than a standard BPSK system for a large number of users. To overcome this, a more complex coding strategy and higher level of modulation schemes would be needed. The analysis can be easily extended to fading environment. Ju et al. [23] examined the fading effects on a BPSK modulated two-way relay channel. Under the assumption of independent block fading channel for the user transmissions during the multiple access phases and relay transmissions during the broadcast phases, the equations for the multi-way relay channel derived in this paper can be combined with the derivations in [23] to analyze the error propagation problem in a fading channel.

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