# Robustness of Compressive Sensing under Multiplicative Perturbations: The Challenge of Fading Channels

Daniel H. Chae, Parastoo Sadeghi, and Rodney A. Kennedy
Research School of Information Sciences and Engineering
The Australian National University, Canberra ACT 0200, AUSTRALIA
Emails: {daniel.chae,parastoo.sadeghi,rodney.kennedy}@anu.edu.au

***Abstract*—We investigate the robustness of Compressive Sensing (CS) as a direct signal acquisition and reconstruction method at the wireless receiver in fading channels. The wireless channel introduces additive as well as multiplicative random perturbations to the received signal. The original CS theory considers only additive and bounded perturbations for signal reconstruction with an a priori known basis. However, the impact of multiplicative perturbations, which manifests itself as basis mismatch, on the CS reconstruction performance is largely unknown. In this paper we first formulate such multiplicative perturbations due to wireless fading channel in the CS acquisition and reconstruction problem. We then show that these perturbations can result in significant errors in signal reconstruction if the basis is not properly adjusted. Furthermore, we will propose a method for adjusting the elements of basis to the fading channel coefficients and discuss possible improvements in the signal reconstruction.**

## I. Introduction

In general, direct sampling of a Radio Frequency (RF) signal requires the sampling rate of analog to digital converter (ADC) to be at least twice the maximum signal frequency. In high-frequency applications, the speed of ADC should be increased significantly to meet the Nyquist rate. Despite recent advances in semiconductor technology, there is a physical limit in increasing the sampling rate of an ADC, and this is the reason why down-conversion of frequency is widely used.

To overcome the need for high sampling rate of ADCs, random sampling [1]–[4] was introduced. Furthermore, the recent technique of Compressive Sensing (CS) [5]–[8] has emerged.

CS theory states that an $s$-sparse signal $\mathbf{x}$ of length $N$ can be reconstructed with an overwhelming probability if the number of measurements taken is of order $s \log N$. This means that we can recover the signal with very few data samples provided that the signal is sparse in a certain basis domain.

The original CS theory can be applied to reconstruct sparse signals even in the presence of *additive* bounded measurement noise [7]. However, a more serious problem may arise when there are other types of perturbations in measurements. Most notably, CS theory works as expected under the assumption of perfect match between the sparse signal basis and the assumed basis (or simply CS dictionary) for reconstruction. Due to a variety of reasons, some level of basis mismatch can be present. Basis mismatch results in *multiplicative* perturbations by introducing an unknown or noise term that is dependent on the signal itself. In general, multiplicative perturbations are much more difficult to deal with and are more detrimental to the reconstruction result than additive perturbations. The authors in [9] derive a bound on the recovery error in terms of Restricted Isometry Constant (RIC) when the signal is suffering from additive and multiplicative perturbations, and [10] also expresses a bound on the recovery error versus the magnitude of basis mismatch. Although these works aim to understand the robustness of CS to such perturbations, devising methods for minimizing perturbations deserves investigation.

In this paper, we are interested in the applications of CS in wireless communications to acquire the Radio Frequency (RF) signal directly with a low-rate ADC. The possibility to acquire and reconstruct the original signal with a low-rate ADC is already shown in [11], [12]. However, since the RF signal goes through the wireless channel, it is not free from perturbation noise. Additive white Gaussian noise (AWGN) and multi-path fading channel will behave as additive and multiplicative perturbations of CS, respectively. The impact of AWGN may be tolerable provided that signal to noise ratio (SNR) is under control. But, in the channel, multi-path fading will cause more severe errors than AWGN if it is not properly dealt with.

In this context, we use random sampling as an acquisition method. (Other sampling techniques can be used, but random sampling is used for simplicity.) We then apply CS to random sampling under the assumption that the signal is sparse in some basis-domain as shown in [13]. We proceed to analyze the perturbation effects through numerical examples. The contributions of this paper are three-fold: 1) we formulate multiplicative fading perturbations in the CS acquisition and reconstruction problem, 2) we study the impact of these perturbations in terms of CS-reconstruction errors, and 3) we propose a method for adjusting the elements of the CS dictionary to the fading channel coefficients and discuss the foreseen improvements in the signal reconstruction.

## II. Problem Formulation

In the original CS problem formulation, an $s$-sparse vector $\mathbf{x}$ of length $N$ is acquired through an $M \times N$ sensing matrix $\mathbf{A}$ to give $\mathbf{y} = \mathbf{A}\mathbf{x}$. The reconstruction problem is ill-posed

978-1-4244-8179-8/10/$26.00 ©2010 IEEE 

in general when $M < N$. There are many candidates for $\mathbf{x}$ for which we have $\mathbf{Ax} = \mathbf{y}$. However, reconstruction is possible with high probability using $\ell_1$-minimization if 1) $\mathbf{x}$ is sufficiently sparse and 2) the requirement on minimum number of measurements is met so that all columns of the sensing matrix $\mathbf{A}$ are nearly orthogonal [6]. The reconstructed vector $\mathbf{x}^\star$ is the solution to the Basis Pursuit (BP) problem, or $\ell_1$-minimization problem, which can be cast as the following linear program [14], [15]

$$\mathbf{x}^\star = \arg\min \|\mathbf{x}\|_{\ell_1} \quad \text{s.t. } \mathbf{y} = \mathbf{Ax}. \tag{1}$$

If there is noise in the measurements, (1) is relaxed to

$$\mathbf{x}^\star = \arg\min \|\mathbf{x}\|_{\ell_1} \quad \text{s.t. } \|\mathbf{Ax} - \mathbf{y}\|_{\ell_2} \leq \varepsilon, \tag{2}$$

where $\varepsilon$ is a bound on noise.

In this paper, we investigate a situation where there is uncertainty about the sensing matrix $\mathbf{A}$ itself. In particular, we will show that if the uncertainty in $\mathbf{A}$ due to fading channels is not dealt with properly, we are faced with the following reconstruction problem

$$\mathbf{x}^\star = \arg\min \|\mathbf{x}\|_{\ell_1} \quad \text{s.t. } \mathbf{y}^{perturbed} = \text{Re}\big[\mathbf{Ax}\big], \tag{3}$$

where $\mathbf{A}$ is predefined, but the acquired signal is actually given by

$$\mathbf{y}^{perturbed} = \text{Re}\big[\mathbf{AHx} + \mathbf{A}^{noise}\mathbf{x}\big]. \tag{4}$$

where both $\mathbf{H}$ and $\mathbf{A}^{noise}$ are unknown a priori at the receiver. We will then show a way to alleviate this problem by estimating a more accurate sensing matrix $\hat{\mathbf{A}}^{\mathbf{h}}$ which incorporates our estimate of the fading and therefore, the reconstruction problem becomes of the form

$$\mathbf{x}^\star = \arg\min \|\mathbf{x}\|_{\ell_1} \quad \text{s.t. } \mathbf{y}^{perturbed} = \text{Re}\big[\hat{\mathbf{A}}^{\mathbf{h}}\mathbf{x}\big]. \tag{5}$$

### *A. Reconstruction of Randomly-Sampled Signals with CS*

The wireless system model that we consider is composed of $N$ transmitters and one receiver. Each transmitter is allocated a carrier at the frequency $f_i$ and aims to communicate the complex data symbol $x_i$ to the receiver. It is assumed that only a small fraction of transmitters are active at a given time and hence, the CS theory can be applied. Our model is applicable to distributed or co-located transmitters and can be easily generalized to include other communication scenarios.

In the absence of the wireless channel, the received signal $r(t)$ can be expressed as follows

$$r(t) = \text{Re}\big[\sum_{i=1}^{N} x_i \psi_i(t)\big], \tag{6}$$

where $\psi_i(t) = e^{j2\pi f_i t}$ is the $i^{th}$ carrier. This can be rewritten in matrix form as

$$r(t) = \text{Re}\big[\mathbf{\Psi}(t)\,\mathbf{x}\big], \tag{7}$$

where $\mathbf{\Psi}$(t) is the vector of carriers

$$\mathbf{\Psi}(t) = \big[\psi_1(t),\, \psi_2(t),\, \cdots,\, \psi_N(t)\big],$$

and $\mathbf{x}$ is the transmitted data of size $N \times 1$

$$\mathbf{x} = [x_1,\, x_2,\, \cdots,\, x_N]^T.$$

Let us assume that measurements of the signal are performed at appropriate random sampling times, $t_1, ..., t_M$. The measurements can be written as

$$\mathbf{y} = \text{Re}\big[\mathbf{Ax}\big], \tag{8}$$

where $\mathbf{A}$ is the matrix of size $M \times N$ whose element in row $m$ and column $n$ is defined as

$$A_{m,n} = \psi_n(t_m).$$

Our aim is to find the coefficient vector $\mathbf{x}$ from the given measurements. To estimate $\mathbf{x}$, in general, the number of measurements, $M$, should be greater than or equal to the number of bases, $N$. But if $\mathbf{x}$ is a sparse vector, whose elements are mostly zero, we can estimate $\mathbf{x}$ with a number of measurements much smaller than $N$ by solving

$$\mathbf{x}^\star = \arg\min \|\mathbf{x}\|_{\ell_1} \quad \text{subject to } \|\text{Re}\big[\mathbf{Ax}\big] - \mathbf{y}\|_{\ell_2} \leq \varepsilon, \tag{9}$$

where $\varepsilon$ is an adequate relaxation to the solution. This is the reconstruction of a randomly-sampled signal using CS.

### *B. Problem of Multiplicative Perturbations in CS*

In a wireless environment, the signal is experiencing the wireless channel which is modeled with multi-path, Doppler effect, pathloss, etc. This wireless channel will behave as multiplicative perturbation in signal reconstruction.

If the delay spread of channel is smaller than the inverse of signal bandwidth, the received signal of (6) can be modified as

$$r^{perturbed}(t) = \text{Re}\big[\sum_{i=1}^{N} x_i h_i(t) \psi_i(t)\big], \tag{10}$$

where $h_i(t)$ is the channel coefficient between the $i^{th}$ transmitter and the receiver.

The time-varying channel coefficient $h_i(t)$ [16] is given by

$$h_i(t) = \sum_{p=1}^{P_i(t)} c_i^p(t) e^{-j\phi_i^p(t)}, \tag{11}$$

where $P_i(t)$ is the number of resolvable multipath components, $c_i^p(t)$ and $\phi_i^p(t)$ are amplitude and phase of the $p^{th}$ path for the $i^{th}$ transmitter, respectively. The phase of the $p^{th}$ path of the $i^{th}$ transmitter can be approximated as $\phi_i^p(t) = 2\pi f_i \tau_i^p - 2\pi f_i^{D_p} t - \phi_i^0$. The unknowns in this expression are: the path delay $\tau_i^p$, the Doppler frequency $f_i^{D_p}$, and the phase constant $\phi_i^0$. Then, the channel coefficient $h_i(t)$ is re-expressed as

$$h_i(t) = \sum_{p=1}^{P_i(t)} H_i^p(t) e^{j2\pi f_i^{D_p} t}, \tag{12}$$

where $H_i^p(t)$ is the Doppler-extracted channel coefficient of the $p^{th}$ path for the $i^{th}$ transmitter defined as $H_i^p(t) = c_i^p(t) e^{-j(2\pi f_i \tau_i^p - \phi_i^0)}$.

We can decompose (10) into two parts as

$$r^{perturbed}(t) = \text{Re}\big[\sum_{i=1}^{N} x_i\psi_i(t) \sum_{p=1}^{P_i(t)} H_i^p(t)e^{j2\pi f_i^{D_p} t}\big]$$
$$= r^{signal}(t) + r^{noise}(t), \quad (13)$$

where $r^{signal}(t)$ is the signal of interest which is not affected by Doppler effect and $r^{noise}(t)$ is the signal perturbed by Doppler effect. $r^{signal}(t)$ and $r^{noise}(t)$ are given by

$$r^{signal}(t) = \text{Re}\big[\sum_{i=1}^{N} x_i\psi_i(t) \sum_{p=1}^{P_i(t)} H_i^p(t)\big], \quad (14)$$

and

$$r^{noise}(t) = \text{Re}\big[\sum_{i=1}^{N} x_i\psi_i(t) \sum_{p=1}^{P_i(t)} H_i^p(t)[e^{j2\pi f_i^{D_p} t} - 1]\big], \quad (15)$$

respectively. The Doppler-extracted channel coefficient $H_i^p(t)$ and the number of paths $P_i(t)$ can be regarded as constants $H_i^p$ and $P_i$ if the channel varies very slowly during the time-duration of measurement. Then, the measurements of (13) are expressed in matrix form as

$$\mathbf{y}^{perturbed} = \text{Re}\big[\mathbf{AHx} + \mathbf{A}^{noise}\mathbf{x}\big], \quad (16)$$

where $\mathbf{A}^{noise}$ is the sensing matrix of the signal deviant from the exact carrier frequency whose elements are defined as

$$A_{m,n}^{noise} = \psi_n(t_m) \sum_{p=1}^{P_n} H_n^p[e^{j2\pi f_n^{D_p} t_m} - 1],$$

and $\mathbf{H}$ is an $N \times N$ diagonal channel matrix, with diagonal elements given by

$$H_n = \sum_{p=1}^{P_n} H_n^p.$$

Comparing two measurements of (8) and (16), we observe two differences. One is that the channel response of the each carrier is included in $\mathbf{AHx}$. This means that during reconstruction we can recover $\mathbf{Hx}$, the product of the channel response and the transmitted symbol $\mathbf{x}$, instead of $\mathbf{x}$. And the other is the term due to the fading channel, $\mathbf{A}^{noise}\mathbf{x}$, added in the measurements. This term will behave like noise in the measurements and cause a serious problem when applying CS to the wireless receiver. Unfortunately, the adverse effect of the term $\mathbf{A}^{noise}\mathbf{x}$ will increase as the Doppler frequency increases. In the absence of any knowledge about $\mathbf{H}$ and $\mathbf{A}^{noise}$, the signal is estimated using

$$\mathbf{x}^\star = \arg\min \|\mathbf{x}\|_{\ell_1} \quad \text{s.t. } \|\text{Re}\big[\mathbf{Ax}\big] - \mathbf{y}^{perturbed}\|_{\ell_2} \le \varepsilon. \quad (17)$$

## III. A Method to Reduce the Effects of Multiplicative Perturbations

The noise term $\mathbf{A}^{noise}\mathbf{x}$ in the measurements of the previous section is wholly due to the mismatch between the basis of the signal and the basis of the CS receiver. In general, the basis refers to the elements of the CS dictionary $\mathbf{\Psi}(t)$, or each column vector of the sensing matrix $\mathbf{A}$. The wireless channel causes distortions to the basis of the signal. In this section, a method to reduce the basis mismatch will be introduced.

### *A. Basis Matching By Adjusting CS Dictionary*

With the CS dictionary $\mathbf{\Psi}(t)$ and its predefined sensing matrix $\mathbf{A}$, the CS receiver of wireless channel solves the problem given in (17). It is already described that the term $\mathbf{A}^{noise}\mathbf{x}$ due to the basis mismatch causes noise in the reconstruction.

But if we knew the channel state, the CS dictionary could be adjusted as follows:

$$\mathbf{\Psi}^h(t) = \big[h_1(t)\psi_1(t),\, h_2(t)\psi_2(t),\, \cdots,\, h_N(t)\psi_N(t)\big],$$

then the received signal is expressed as

$$r^{perturbed}(t) = \text{Re}\big[\sum_{i=1}^{N} x_i h_i(t)\psi_i(t)\big]$$
$$= \text{Re}\big[\mathbf{\Psi}^h(t)\,\mathbf{x}\big]. \quad (18)$$

The measurements of (18) in matrix form can be written as:

$$\mathbf{y}^{perturbed} = \text{Re}\big[\mathbf{A}^{\mathbf{h}}\mathbf{x}\big], \quad (19)$$

where $\mathbf{A}^{\mathbf{h}}$ is the sensing matrix whose element is defined as

$$A_{m,n}^h = h_n(t_m)\psi_n(t_m).$$

Comparing (16) and (19), we can see that there is no noise term in (19) which is caused from basis mismatch between the received signal and the receiver. Now, the problem of (17) can be recast as:

$$\mathbf{x}^\star = \arg\min \|\mathbf{x}\|_{\ell_1} \quad \text{s.t. } \|\text{Re}\big[\mathbf{A}^{\mathbf{h}}\mathbf{x}\big] - \mathbf{y}^{perturbed}\|_{\ell_2} \le \varepsilon. \quad (20)$$

### *B. Least Square Method of Adjusting CS Dictionary*

In practical situations, $\mathbf{A}^{\mathbf{h}}$ is not available and an estimate of $\mathbf{A}^{\mathbf{h}}$ denoted by $\hat{\mathbf{A}}^{\mathbf{h}}$ has to obtained. To adjust the CS dictionary to the channel, the channel coefficient should be estimated with the given measurements. We can estimate the channel with the given measurements if a known training signal $\mathbf{x}$ is sent from the transmitter periodically.

It is assumed that the channel coefficient $h_i(t)$ can be approximated as a polynomial within the time-duration of measurement. The channel coefficient $h_i(t)$ can be modeled as linear provided that the time-duration is sufficiently short. A first-order polynomial will be used for simplicity. This can be an accurate approximation for very slow to slow fading channels.

An approximation of a first-order polynomial of $h_i(t)$ can be expressed as

$$h_i(t) \approx a_i t + b_i + j(c_i t + d_i). \quad (21)$$

Using (21), the $m$-th element of the measurements in (19) can be rewritten as

$$y_m^{perturbed} = \sum_{k=1}^{K} [\alpha_{i_k}(t_m)(a_{i_k} t_m + b_{i_k}) - \beta_{i_k}(t_m)(c_{i_k} t_m + d_{i_k})], \quad (22)$$

where the subscript $i_k$ is the carrier index of the $k^{th}$ active carrier of total $K$ active carriers,

$$\alpha_{i_k}(t_m) = \psi_{i_k}^I(t_m) x_{i_k}^I - \psi_{i_k}^Q(t_m) x_{i_k}^Q,$$

and

$$\beta_{i_k}(t_m) = \psi_{i_k}^Q(t_m) x_{i_k}^I + \psi_{i_k}^I(t_m) x_{i_k}^Q.$$

Rewriting the measurements of (22) in matrix form, we obtain

$$\mathbf{y}^{perturbed} = \left[ \begin{array}{cccc} \mathbf{W} & \mathbf{X} & \mathbf{Y} & \mathbf{Z} \end{array} \right] \left[ \begin{array}{cccc} \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} \end{array} \right]^T, \quad (23)$$

where $\mathbf{W}$, $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{Z}$ are the channel matrices of size $M \times K$ whose element in row $m$ and column $k$ is defined as

$$W_{m,k} = t_m \alpha_{i_k}(t_m), \quad X_{m,k} = \alpha_{i_k}(t_m),$$

$$Y_{m,k} = -t_m \beta_{i_k}(t_m), \quad Z_{m,k} = -\beta_{i_k}(t_m),$$

respectively, and $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ are vectors of coefficients of the first-order approximation of the channel. For example,

$$\mathbf{a} = \left[ \begin{array}{c} a_{i_1}, a_{i_2}, \cdots, a_{i_K} \end{array} \right].$$

The approximation of channel can be calculated from (23) using the least square (LS) method [17]:

$$\left[ \begin{array}{cccc} \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} \end{array} \right]^T = (\mathbf{\Lambda}^T \mathbf{\Lambda})^{-1} \mathbf{\Lambda}^T \mathbf{y}^{perturbed}, \quad (24)$$

where

$$\mathbf{\Lambda} = \left[ \begin{array}{cccc} \mathbf{W} & \mathbf{X} & \mathbf{Y} & \mathbf{Z} \end{array} \right].$$

It is assumed in this paper that the known pilot signals are transmitted simultaneously by active transmitters before transmitting traffic symbols. There can be schemes to send the pilot signals efficiently which minimizes the redundancy. But in this paper, the feasibility to reduce the basis mismatch rather than the scheme to send the signal is investigated.

## IV. Simulation Results

Comparisons of reconstruction are shown in this section between the results of CS receiver which uses the pre-defined CS dictionary and the channel-adjusted CS dictionary. Results are obtained by convex programming [15] which is based on [14].

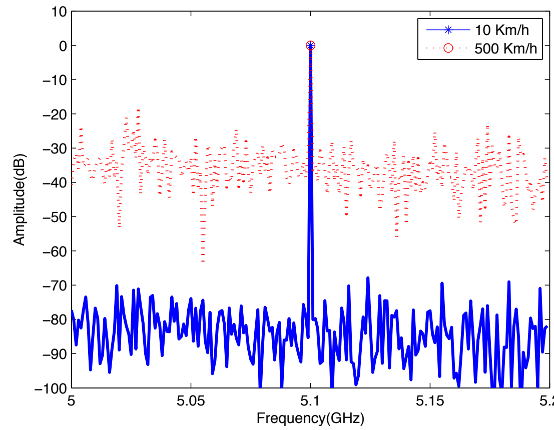


Fig. 1: CS-reconstruction of single-tone signal perturbed by the channel.

### A. Reconstruction with Pre-defined CS Dictionary

*1) An Example of Single-tone Signal:* It is assumed in this example that only one transmitter is active among 201 transmitters which use frequencies from 5.000GHz to 5.200GHz with the increment of 1MHz. The frequency of the active transmitter is 5.100GHz.

In the CS receiver, the signal is captured with 100 measurements which are sampled randomly within each time window of 10$\mu$s, i.e., the average sampling rate is 10MHz. The reconstruction is performed every 10$\mu$s with these 100 measurements. CS dictionary of the receiver is composed of frequencies of all transmitters. The time-sequence specifying random samples is chosen among randomly-generated time-sequences such that it results in the smallest condition number for the measurement matrix [4]. The inverse of the smallest interval of time sequence is set to 20% of the maximum frequency of the carrier.

Fig. 1 shows one snapshot of reconstruction result when the signal is experiencing a Rayleigh fading channel. Spatial Channel Model (SCM) [18] is used in generating fading channel. We can see that the noise level of the reconstructed signal becomes larger as the moving speed increases. Noise relative to signal level is increased from about $-80$dB to $-30$dB as the moving speed is increased from 10km/h to 500km/h. Doppler frequency which is calculated with the speed and the carrier frequency,

$$f_i^D \approx \frac{f_i * \text{speed}}{1000 \cdot 60 \cdot 60 \cdot 3 * 10^8}$$

is increased from 47Hz to 2.36kHz in this example. And the frequency offset ratio defined as

$$\text{Offset\_Ratio} = \frac{f_i^D}{\text{Resolution of CS dictionary}} \times 100(\%)$$

is 0.236% at the speed of 500km/h.

Fig. 2 shows the Signal to maximum Noise Floor Ratio (SNFR), the power ratio of the reconstructed signal of the

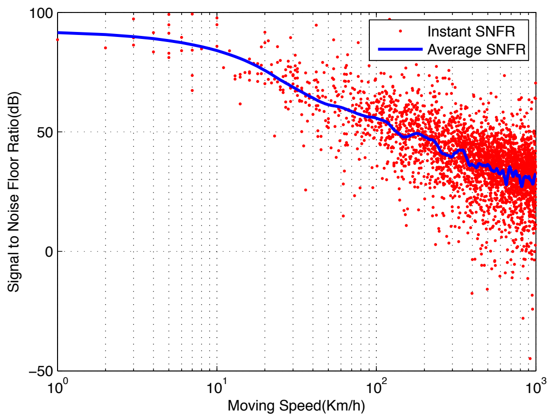


Fig. 2: SNFR of the CS reconstructed single-tone signal versus moving speed.

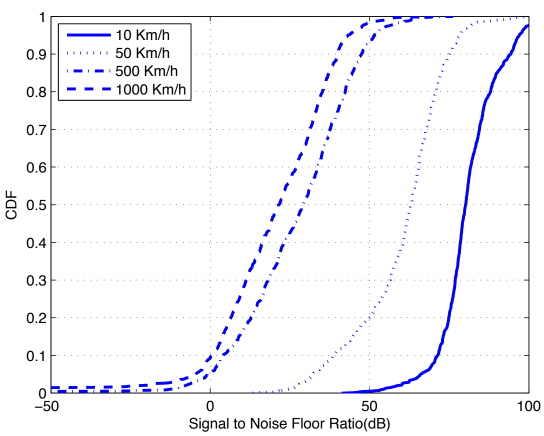


Fig. 3: Cumulative distribution function of the SNFR.

transmitted carrier and the maximum of the reconstructed carrier other than the transmitted carrier, versus the moving speed. We can see that SNFR decreases as the speed increases. And there are some SNFRs which are below 0dB. SNFR below 0dB means CS receiver failed in finding the active carrier. The failure rate increases with Doppler frequency.

Fig. 3 shows the cumulative distribution function (CDF) of SNFR at different moving speeds of 10km/h, 50km/h, 500km/h and 1,000km/h. We can see that about 10% of the reconstructions fail in finding the active carrier at the speed of 1,000Km/h, the usual speed of the civilian airliner. Further, the SNFRs of the recovered signal are less than 25dB at the rate of about 50%. This means that the possibility of failure in finding the active carrier is more than 50% if the power difference of multiple active transmitters is more than 25dB.

*2) An Example of Multi-tone Signal:* The assumption is the same as the previous example except that the received signal has 10 active carriers out of 201 carriers. Fig. 4 is the reconstruction result of multi-tone signal whose active carriers are experiencing the wireless channel with different pathloss.

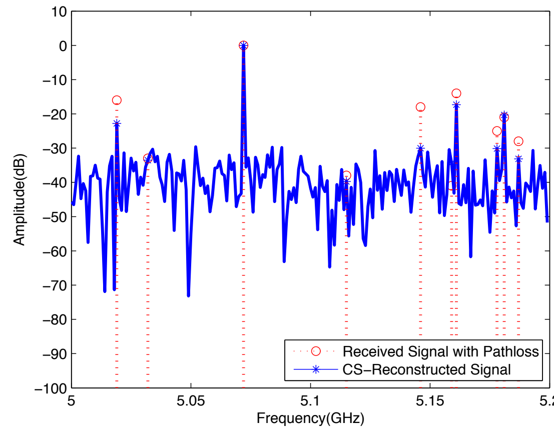


Fig. 4: CS reconstruction of a multi-tone signal experiencing a speed of 500km/h through the wireless channel.

The moving speed is assumed to be 500km/h. We can see that it failed in finding even the carriers which are active. This is because the leakage noise from basis mismatch of the strong carrier is bigger than the weak carriers themselves.

### *B. Reconstruction with CS Dictionary Adjusted to Channel*

It is of interest to understand the potential benefits that estimating the channel state can provide in terms of reconstruction performance. To this end, one approach is to consider an oracle-based method which assumes that active carriers are known for the purpose of channel estimation (and not for the signal reconstruction). We recognize that this is a hypothetical assumption in the context of CS (since by knowing the active carriers other reconstruction methods become more relevant) and even impractical in many situations. Nevertheless, the oracle-based method will serve as a limit as to what performance improvements are achievable. One more practical approach would be to iteratively estimate active carriers using the measurements. But, description and evaluation of this method is beyond the scope of the current paper.

*1) An Example of Single-tone Signal:* Fig. 5 shows the single-tone reconstruction result with CS dictionary adjusted to the channel. This is the same recovery problem as in Fig. 1 at the speed of 500km/h. Noise level is significantly decreased from about −30dB to −80dB. Fig. 6 compares the result of SNFR with 1) pre-defined CS dictionary, 2) CS dictionary adjusted with a first-order channel approximation, and 3) CS dictionary adjusted with a perfect channel state.

We see that CS dictionary adjusted to the channel even with a simple approximation can improve SNFR more than 30dB at the speed of 1,000km/h. And still there is a possibility to improve the reconstruction better to see the result whose CS dictionary is adjusted with the perfect channel state.

*2) An Example of Multi-tone Signal:* We can also see in Fig. 7 that all of the active carriers are exactly recovered even though the signal is experiencing the multiplicative perturbations.

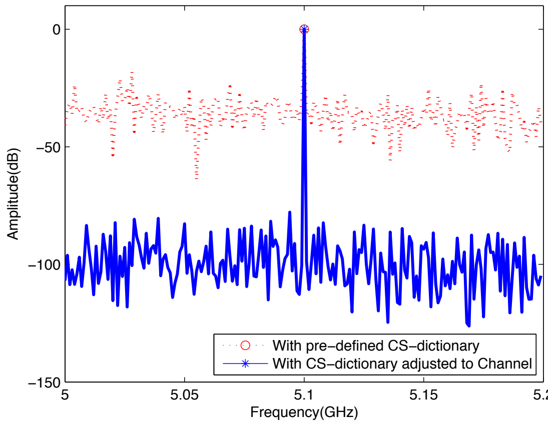


Fig. 5: CS reconstruction of a single-tone signal experiencing a speed of 500km/h through the wireless channel.

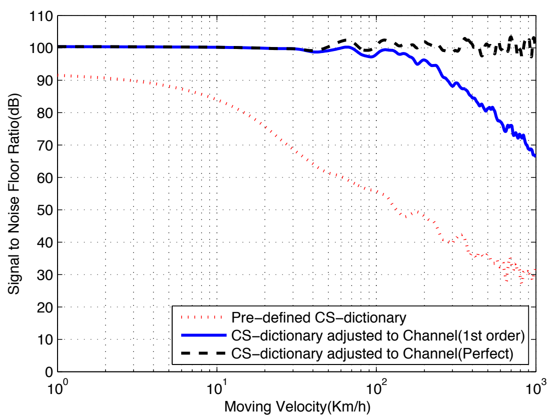


Fig. 6: SNFR of the CS reconstructed single-tone signal when the CS dictionary is adjusted to the channel.

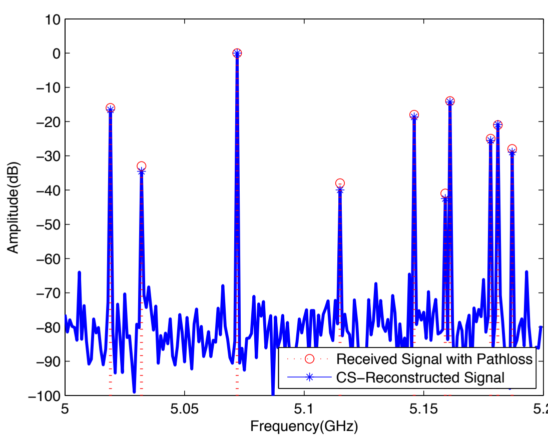


Fig. 7: CS reconstruction of a multi-tone signal with the moving speed of 500km/h when the CS dictionary is adjusted to the channel.

## V. Conclusion

In this paper, we showed the perturbation effects of the channel in reconstructing wireless signals with CS and proposed a method for reducing the effects of perturbations.

Our results show that perturbations become larger as the Doppler frequency increases. Doppler frequency causes a mismatch between the signal basis and the one used at the receiver. To reduce the basis mismatch, CS dictionary can be adjusted to the channel. Adjusting CS dictionary to the channel is very similar to the channel equalization of the conventional receiver. While the purpose of channel equalization is to reduce intersymbol interference (ISI), the aim of adjusting CS dictionary is to reduce the noise from basis mismatch.

Interesting problems for future work include mitigating ISI in a CS receiver and more practical methods for channel estimation with unknown active carriers.

## References


[1] J. J. Wojtiuk, “Randomised sampling for radio design,” *PhD Thesis of University of South Australia*, 2000.

[2] F. Papenfuss and D. Timmermann, “Alias-free periodic signal analysis using efficient rate nonuniform sampling sets,” in *Proc. of Intl. Conf. on Acoustics, Speech, and Signal Processing (ICASSP)*, vol. 3, Hawaii, U.S., Apr. 2007, pp. 1497–1500.

[3] D. M. Bechir and B. Ridha, “Non-uniform sampling schemes for RF bandpass sampling receiver,” in *Proc. of 2009 Intl. Conf. on Signal Processing System*, Singapore, May 2009, pp. 13–17.

[4] R. F. Bass and K. Gröchenig, “Random sampling of multivariate trigonometric polynomials,” *SIAM J. Math. Anal.*, vol. 36, no. 3, pp. 773–795, 2004.

[5] E. J. Candès, “Compressive sampling,” in *Proc. of Int. Congress of Mathematicians*, vol. 3, Madrid, Spain, Aug. 2006, pp. 1433–1452.

[6] E. J. Candès and M. Wakin, “An introduction to compressive sampling,” *IEEE Signal Process. Mag.*, vol. 25, pp. 21–30, Mar. 2008.

[7] E. J. Candès, J. K. Romberg, and T. Tao, “Stable signal recovery from incomplete and inaccurate measurements,” *Communications on Pure and Applied Mathematics*, vol. 59, pp. 1207–1223, 2006.

[8] D. L. Donoho, “Compressed sensing,” *IEEE Trans. Inform. Theory*, vol. 52, pp. 1289–1306, Apr. 2006.

[9] M. A. Herman and T. Strohmer, “General deviants : An analysis of perturbations in compressed sensing,” *IEEE J. Select. Topics Signal Processing*, vol. 4, pp. 342–349, 2010.

[10] Y. Chi, A. Pezeshki, L. Scharf, and R. Calderbank, “Sensitivity to basis mismatch in compressed sensing,” in *Proc. of Intl. Conf. on Acoustics, Speech, and Signal Processing (ICASSP)*, Texas, U.S., Mar. 2010, pp. 3930–3933.

[11] J. N. Laska, S. Kirolos, M. F. Duarte, T. S. Ragheb, R. G. Baraniuk, and Y. Massoud, “Theory and implementation of an analog-to-information converter using random demodulation,” in *Proc. of Circuits and Systems, IEEE International Symposium on*, L.A., U.S., May 2007.

[12] M. Mishali, Y. C. Eldar, and J. A. Tropp, “Efficient sampling of sparse wideband analog signals,” in *Proc. 2008 IEEE Conv. Electrical and Electronic Engineers in Israel (IEEEI)*, Eilat, Israel, Dec. 2008.

[13] H. Rauhut, “Random sampling of sparse trigonometric polynomials,” *Applied and Computational Harmonic Analysis*, vol. 22, no. 1, pp. 16–42, 2007.

[14] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[15] E. J. Candès and J. K. Romberg, “L1 magic : Recovery of sparse signals via convex programming,” available online under `http://www.acm.caltech.edu/l1magic`.

[16] A. Goldsmith, *Wireless Communication*. Cambridge University Press, 2005.

[17] Å. Björck, *Numerical Methods for Least Squares Problems*. SIAM, 1996.

[18] “Spatial channel model for multiple input multiple output (MIMO) simulations,” *ETSI Technical Report 125.996 V8.0.0*, 2009.