Statistical Properties of Amplify and Forward Relay Links with Channel Estimation Errors

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Abstract—This paper studies the statistical properties of the signal-to-noise ratio (SNR) of the relay link in a cooperative wireless communication system with fixed gain relay in presence of channel estimation error. The SNR expression is derived and three different analytic approaches with different approximate assumptions are used to obtain the probability distribution function of the SNR and outage probability of the relay link. The first approach is the most accurate one in many cases, however it does not have a closed-form expression. The other two approaches result in closed-form expressions for the outage probability. Therefore, the third approach has been used to find a sub-optimum power allocation scheme for the source and the relay. The numerical results compare the accuracy of the analytic approaches and simulation results for different positions of the relay between the source and the destination. It is shown that the second approach has the most accurate closed-form expression and the third one leads to the simplest closed-form result suitable for power optimization purposes.

I. INTRODUCTION

Cooperative relaying strategies introduce distributed spatial diversity into wireless networks and enable energy-efficient transmission of information from remote users to the desired destination via relays [1–3]. However, for the purpose of signal detection, almost all existing works assume that the perfect channel state information (CSI) is known, at least to the destination node [2–5]. In practical situations, the wireless channel may only be partially known at the receiving nodes and for acquiring such knowledge, a fraction of the power has to be spent for channel estimation using pilot symbols [6].

The relaying strategy which is used in cooperative systems can be DF (decode and forward) or AF (amplify and forward). We consider AF mode that imposes less processing load on the relay and can be preferable in practice [7, 8]. Also, the amplifying gain of the relay can be fixed or variable. Variable gain requires knowledge of the instantaneous channel realization at the relay, while fixed gain relays only require long-term statistics of the channel; therefore, the relay amplifies the received signal regardless of the fading amplitude of the first hop. Several authors have provided performance analysis of AF systems in terms of their bit-error-rate (BER) and outage probability under different assumptions of amplifier gain [7–9].

In this work, we assume that the source or the relay do not possess any CSI and the destination estimates the channel using pilot symbols. Therefore, we consider fixed gain relay in our scheme and find the outage probability of the relay link that is useful to find the overall outage probability in multiple relay networks using selection diversity strategy [3, 10].

In order to find outage probability, we first study the instantaneous behavior of SNR of the relay link at the destination. The relay fading channel is often modeled as a double Gaussian channel (i.e., the product of two complex Gaussian channels) whose distribution is available [11]; however, when it is multiplied by the input signal and added with complex Gaussian noise at the destination, the key difficulty in deriving the statistics of the signal at the destination is to obtain the statistics of the combination of products and sums of Gaussian random variables. In addition, the noise term in the received signal is not Gaussian and depends on the channel between the relay and the destination and the estimation error. These facts make the SNR distribution more complicated. In this paper, we provide three analytical approaches with different simplifying assumptions to find the outage probability of the relay link. In the first approach, only the estimation error is substituted with its mean; however, this approach does not lead to a closed-form expression for the outage of the relay link. Therefore, we will apply more simplifying approximations in the second and third approaches. In the second approach, we use the statistics of the channel instead of its estimate and in the third approach, we model the overall received noise with worst-case Gaussian noise. Both of these approaches result in closed-form expressions for the outage probability. However, the latter has a simpler form which can be used for power optimization. Therefore, we use the result of third approach to find a sub-optimum power assignment for this scheme.

The rest of the paper is organized as follows: In Section II, we introduce the system model and the channel estimation method. In Section III, SNR derivation and the outage probability of the relay link are provided and three analytical approaches with different assumptions are presented. The power assignment for the pilot and data of the source and relay is described in Section IV. The numerical results that compare the accuracy of our analytical approaches with Monte-Carlo simulations are given in Section V and finally, the paper is concluded in Section VI.

1Dual-hop link: source-relay-destination.
II. SYSTEM AND CHANNEL MODEL

We consider a wireless cooperative diversity network with a source, relay and a destination. We assume that all channels are Rayleigh block fading channels and there is no prior CSI available at the source, relay or destination. However, for coherent detection at the destination, we need to know the fading channel coefficients. Therefore, we use pilot symbol transmission in the first time slot of each block. We assume a block duration of $T$ seconds with $N$ time slots. In each time slot we transmit one symbol with duration of $T_s = T/N$. In the first block, the source sends the pilot and data symbols to the destination and the relay. The received signal during pilot transmission could be used to estimate the channel between source and the relay, however, we consider that the relay does not estimate the channel and only amplifies and forwards the signal. In the following analysis we concentrate on the statistics of the relay link and provide a suitable model for the SNR probability distribution function and the outage probability of the relay link.

In the first time block, the received signal at the relay is given by

$$Y_1 = H_1X + Z_1,$$

where during data transmission $X$ is the channel input with power $P_{D0}$ and during pilot transmission, it is the pilot symbol with the power of $P_{P0}$, $Z_1$ is zero-mean complex-valued additive white Gaussian noise (AWGN) with variance $N_0$. We assume that the channel fading gain $H_1$ follows a complex-valued, zero-mean Gaussian distribution with variance $\sigma_{H_1}^2 = 1/d_1^2$, where $d_1$ is the distance between the source and the relay and $a$ is the path-loss exponent.

In the second time block, the relay sends the amplified signal. The received signals at the destination during pilot and data retransmission, respectively are given by

$$Y_{2,P} = H_2A_P(H_1X + Z_1) + Z_2,$$

$$Y_{2,D} = H_2A_D(H_1X + Z_1) + Z_2,$$

where $H_2$ is the fading gain of the channel between the relay and the destination with zero-mean complex Gaussian distribution with variance $\sigma_{H_2}^2 = 1/d_2^2$, where $d_2$ is the distance between the relay and the destination. $Z_2$ is zero-mean complex-valued AWGN with variance $N_0$. $A_P$ and $A_D$ are the relay gains during pilot and data transmission, respectively

$$A_P = \sqrt{\frac{P_{P1}}{P_{P0}\sigma_{H_1}^2 + N_0}},$$

$$A_D = \sqrt{\frac{P_{D1}}{P_{D0}\sigma_{H_1}^2 + N_0}},$$

where $P_{P1}$ and $P_{D1}$ are the relay transmitting powers during pilot and data transmission, respectively. Note that the relay amplifies the signal with a fixed gain value, i.e., the relay gain does not depend on the instantaneous channel. Therefore, the relay only needs to know the variance ($\sigma_{H_1}^2$) of the source-relay fading channel. Throughout the paper, we assume that all wireless fading gains and all receiver noises are independent of each other. Furthermore, we assume a block fading model in which the realization of the fading channel in each link stays constant during a block and changes to an independent value in the next block.

It can be observed from (3) that the destination needs to estimate $H_1H_2$ for the data detection. We assume that the destination estimates $H = H_1H_2$ using linear minimum mean square error (LMMSE) method [12]

$$\hat{H} = KY_{2,P},$$

$$K = E\{HY_{2,P}\}E^{-1}\{Y_{2,P}\}$$

$$= \frac{\sigma_{H_1}^2\sigma_{H_2}^2A_P\sqrt{P_{P0}}}{A_P^2P_{P0}\sigma_{H_1}^2 + A_P^2\sigma_{H_2}^2N_0 + N_0},$$

where $\hat{H}$ is the channel estimate and $E\{}$ is the statistical expectation. Therefore, $H = \hat{H} - \bar{H}$ is the estimation error and its variance is given by

$$\sigma_{\hat{H}}^2 = E\{|\hat{H}|^2\} - E\{HY_{2,P}\}E^{-1}\{Y_{2,P}\}E\{Y_{2,P}H^*\}$$

$$= \frac{\sigma_{H_1}^2\sigma_{H_2}^2(A_P^2\sigma_{H_2}^2N_0 + N_0)}{A_P^2P_{P0}\sigma_{H_1}^2 + A_P^2\sigma_{H_2}^2N_0 + N_0}.$$ 

We note from (6) that the destination needs to know the variances of the source-relay and the relay-destination channels to find the value of $K$.

III. OUTAGE PROBABILITY OF THE RELAY LINK

Outage probability is defined as the probability that the instantaneous SNR $\rho$ falls below a predetermined threshold such as $\rho_{th}$

$$P_{out} = P(\rho < \rho_{th})$$

$$= \int_{0}^{\rho_{th}} f_\rho(\rho)\,d\rho,$$

where $f_\rho(\rho)$ is the probability distribution function (pdf) of the instantaneous SNR of the relay link. Using (3), during the data transmission, we have

$$Y_{2,D} = HADX + HADX + HADZ1 + Z2.$$ 

Therefore, the instantaneous SNR of the relay link at the destination is

$$\rho = \frac{|\hat{H}|^2A_P^2P_{D0}}{|\hat{H}|^2A_P^2P_{D0} + |H|^2A_D^2P_{D0} + N_0}$$

$$= \frac{K^2|Y_{2,P}|^2A_P^2P_{D0}}{|\hat{H}|^2A_P^2P_{D0} + |H|^2A_D^2P_{D0} + N_0},$$

where $K$ is the estimation coefficient in LMMSE estimator given in (6). As it can be observed from (11), the pdf of SNR is complicated mainly due to the presence of $|H|^2$ and $|\hat{H}|$ in the denominator. Furthermore, from (2) it is observed that no closed-form expression exists for the distribution of $|Y_{2,P}|^2$. This is because of the Gaussian noise signal ($Z_2$) which is added to a double Gaussian random variable. Note that during
pilot transmission in which the pilot symbol is deterministic, 
\( H_1X + Z_1 \) is Gaussian, therefore \( A_PH_2(H_1X + Z_1) \) is a
double Gaussian random variable. Also, the random variables
in the numerator and the denominator of \( \text{SNR} \) in (11) are
not independent. Therefore, we have to use some simplifying
assumptions in order to find the pdf of the instantaneous \( \text{SNR} \).
In the following, we will explain and formulate three different
approaches for this purpose.

A. Approach-1

In this approach, we consider the most general case with
minimum approximations. Due to the complexity of the dis-
tribution of \( |H|^2 \) which appears in the denominator of \( \text{SNR} \)
in (11), we substitute it with its mean given in (7). Moreover,
substitution of (2) in (11) gives
\[
\rho = \frac{K^2 A_P^2 P_D |A_P \sqrt{P_0 D H_2 H_1} + A_P H_2 Z_1 + Z_2|^2}{\sigma_H^2 A_P^2 P_D + |H_2|^2 A_P^2 N_0 + N_0}
\]
\[
= C_1 \left| A_P \sqrt{P_0 D H_2 H_1} + A_P H_2 Z_1 + Z_2 \right|^2
= C_1 Z_2, \quad (12)
\]
where \( C_1 \triangleq K^2 P_D / N_0 \), \( C_2 \triangleq (\sigma_H^2 A_P^2 P_D + N_0) / (A_P^2 N_0) \) and
\( Z \triangleq Y_{2,p}^2 / (|H_2|^2 + C_2) \). Therefore, we have to find
the distribution of \( Z \). Referring to (2), we observe that given
\( |H_2| \), \( Y_{2,p} \) is a complex Gaussian random variable and as a
result, \( Y_{2,p}^2 \) is exponentially distributed. Note that the phase
of \( H_2 \) is combined with the phases of \( H_1 \) and \( Z_1 \) in \( Y_{2,p} \). Let
\( U \triangleq |H_2|^2 + C_2 \) and \( V \triangleq |H_2|^2 \), we have
\[
f_Z(z) = \int_{-\infty}^{\infty} f_{Z|U}(z|u) f_U(u) du
\]
\[
= C_2 f_{Z|U}(z) f_U(u) du, \quad (13)
\]
where \( f_{Z|U}(z|u) \) is an exponential distribution with variance
\[
\sigma_{Z|U}^2 = A_P^2 P_D \sigma_{H_2}^2 \frac{(u - C_2)}{u} + \frac{A_P^2 N_0 (u - C_2)}{u} + \frac{N_0}{u} \quad (14)
\]
Using (13) and (14) and substituting \( f_U(u) \) in (13) results in
\[
f_Z(z) = \int_{C_2}^{\infty} \frac{u}{B u + D} \exp \left( - \frac{E u^2 + F u + G}{B u + D} \right) du
\]
where
\[
B \triangleq \frac{A_P^2 \sigma_H^2 (P_0 \sigma_H^2 + N_0)}{\sigma_{H_2}^2}, \quad D \triangleq \frac{A_P^2 \sigma_H^2 N_0 - B C_2}{\sigma_{H_2}^2},
E \triangleq \frac{B}{\sigma_{H_2}^2}, \quad F \triangleq \frac{z \sigma_{H_2}^2}{\sigma_{H_2}^2} + \frac{D - BC_2}{\sigma_{H_2}^2},
G \triangleq \frac{DC_2}{\sigma_{H_2}^2} \quad (16)
\]
To the best knowledge of the authors, integral in (15) does not
have a closed-form solution and therefore it should be evalu-
ated numerically. From (8) and (12), the outage probability of
the relay link is
\[
P_{out} = \int_{0}^{Z_{th}} f_Z(z) dz, \quad (17)
\]
where \( Z_{th} \triangleq \rho_{th} / C_1 \). Among the variables defined in (16),
only \( F \) depends on \( z \) and the integration of \( f_Z(z) \) versus \( z \)
can be easily computed to obtain the outage probability of the
relay link in the following form
\[
P_{out} = 1 - \int_{C_2}^{\infty} \frac{1}{\sigma_{H_2}^2} \exp \left( - \frac{E u^2 + Z_{th} \sigma_{H_2}^2 u + G}{B u + D} \right) du.
\]
Again (18) does not have a closed-form expression and should
be computed numerically. Therefore, we will use some more
simplifying assumptions to find a closed-form for \( \text{SNR} \) dis-
tribution in the following approaches.

B. Approach-2

Because of the complexity of the distribution of \( \text{SNR} \) and
as a result, the complexity of \( H \), here we approximate \( |H|^2 \) with \( H \) in addition to the approximation used in approach-1.
This assumption is more accurate at high \( \text{SNR} \) in which
there is less estimation error. Therefore, we can rewrite the
SNR expression in (10) as
\[
\rho = \frac{A_P^2 P_D |H_1|^2 |H_2|^2}{|H_2|^2 A_P^2 N_0 + N_0 + \sigma_{H_2}^2 A_P^2 P_D}
= \frac{\rho_{01} \rho_{12}}{\rho_{12} + C' \triangleq \frac{P_{D1} \sigma_{H_2}^2 A_P^2 P_D}{N_0 A_P^2}}, \quad (19)
\]
where \( \rho_{01} \) and \( \rho_{12} \) are the instantaneous \( \text{SNR} \) of the source-
relay and relay-destination links, respectively. Following [8],
the outage probability of the relay link can be found in closed-
form as
\[
P_{out} = 1 - 2 \sqrt{C' / \rho_{01} \rho_{12}} \exp \left( - \frac{C'}{\rho_{01} \rho_{12}} \right) K_1 \left( \sqrt{C' / \rho_{01} \rho_{12}} \right), \quad (20)
\]
where \( K_1(\cdot) \) is the first-order modified Bessel function of the
second kind and \( \rho_{01} \) and \( \rho_{12} \) are the average \( \text{SNR} \) of each hop
during data transmission and given by
\[
\rho_{01} = \frac{P_{D0} \sigma_{H_2}^2}{N_0}, \quad \rho_{12} = \frac{P_{D1} \sigma_{H_2}^2}{N_0}, \quad (21)
\]
C. Approach-3

Similar to the previous approach, we assume the same
distribution for \( |H|^2 \) and \( |H|^2 \). In addition, we consider all
the noise terms in (10) as worst case Gaussian noise. Similar
assumptions have been considered in [7] for BER analysis to
obtain an upper bound on the BER of the system with channel
estimation error. Therefore, we only need the distribution of
\( |H|^2 \) to obtain a closed-form expression for the \( \text{SNR} \)
distribution and outage probability.

Since \( H = H_1 H_2 \) is the product of two complex Gaussian
random variables, \( |H| \) is the product of two Rayleigh random

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variables and its pdf is given by [11]

\[ f(|H| = r) = \frac{4r}{\sigma_H^2 \sigma^2_H} K_0 \left( \frac{2r}{\sigma_H^2 \sigma^2_H} \right). \] (22)

where \( K_0(\cdot) \) is the zeroth-order modified Bessel function of the second kind. From the above assumptions and (10), we have

\[
\rho = \frac{A_D^2 P_D|H|^2}{\sigma_H^2 A_D^2 N_0 + N_0 + \sigma_H^2 A_D^2 P_D} = C_4 |H|^2, \quad C_4 = \frac{A_D^2 P_D}{\sigma_H^2 A_D^2 N_0 + N_0 + \sigma_H^2 A_D^2 P_D}.
\] (23)

Using (22) and (23) yields

\[
f_\rho(\rho) = \frac{2}{C_4 \sigma_H^2 \sigma_H^2} K_0 \left( \frac{\rho}{C_4 \sigma_H^2 \sigma_H^2} \right). \] (24)

Then the outage probability can be computed as

\[
P_{out} = \int_0^{\rho_{th}} f_\rho(\rho) d\rho = 1 - 2 \sqrt{\frac{\rho_{th}}{C_4 \sigma_H^2 \sigma_H^2}} K_1 \left( 2 \sqrt{\frac{\rho_{th}}{C_4 \sigma_H^2 \sigma_H^2}} \right). \] (25)

IV. POWER ASSIGNMENT

Assume that the average power available per symbol is \( P_s \); therefore, we have a total energy of \( 2NPT_s \) over two consecutive blocks where \( N \) and \( T_s \) were defined in Section II. Considering that the pilot symbol is sent in the first time slot and data symbols are sent in the following \( N - 1 \) time slots, the energy conservation for this problem dictates that

\[
P_{PO} T_s + P_{P1} T_s + P_{D0} (N - 1) T_s + P_{D1} (N - 1) T_s = 2N T_s P. \] (26)

Since the symbol duration \( (T_s) \) can be dropped from two sides of (26), we can consider (26) as a power constraint. We define \( \gamma \) as the fraction of power allocated to the transmission of pilot symbols, i.e.,

\[ P_{PO} + P_{P1} = \gamma 2NP. \] (27)

Furthermore, we assume that a fraction of \( \beta_1 \) of this power is allocated to the transmission of pilot symbol at the source and the rest to the transmission of pilot symbol at the relay, i.e.,

\[ P_{PO} = \beta_1 \gamma 2NP, \quad P_{P1} = (1 - \beta_1) \gamma 2NP. \] (28)

Similarly, we assume \( \beta_2 \) for the power allocation of data symbols per transmission

\[ P_{D0} = \beta_2 (1 - \gamma) \frac{2NP}{N - 1}, \quad P_{P1} = (1 - \beta_2) (1 - \gamma) \frac{2NP}{N - 1}. \] (29)

In the following, we will introduce a sub-optimum power allocation for this system and compare the outage probability performance of the relay link in Section V.

A. Sub-optimum Power Assignment

In many practical systems, it is easier for the transmitters to fix the power of output to have simpler implementation of power amplifiers. Therefore, we consider the pilot and data power are the same during each block. However, the source and relay powers do not have to be the same and we can optimize them. In this case, we have

\[ P_{D0} = P_{D1} = P_{P1} = P_s. \] (30)

Note that this assumption is equivalent to setting \( \gamma = 1/N \) in (27) and \( \beta_1 = \beta_2 = \beta \) in (28) and (29) that results in

\[ P_0 + P_1 = 2P. \] (31)

As discussed in Section II, we only have closed-form expression for outage probability in the approaches 2 and 3. In addition, it can be shown that taking the derivative of (20) with respect to power allocation ratios does not lead to a simple closed-form expression for the power allocation parameters [5]. Therefore, we consider Approach-3 and derive the optimum power allocation coefficient \( \beta \) to minimize the outage probability. It is shown in the Appendix that the optimum value of \( \beta \) is

\[ \beta_{opt} = \frac{\sigma_H^2}{\sigma_H^2 + \sigma^2_H}. \] (32)

V. Numerical Results

In this section, we study the accuracy of different analytical approaches introduced for the outage probability of the relay link in Section III and compare them with exact simulation results for different positions of relay between the source and the destination.

We assume that the source, relay and destination are placed on a straight line and affected by the same shadowing environment [5]. This assumption is only needed for shadowing effect that has not been considered in this model. In other words, it does not affect the accuracy of our approaches. The normalized distance between the source and the destination is \( d_2 = 1 \). Fig.1 shows the outage probability for the case that the relay is closer to the destination, i.e., the distance between the source and the relay is \( d_3 = 0.75 \) and the relay-destination distance is \( d_2 = 0.25 \). The path-loss exponent is \( a = 3 \). Block length is \( N = 50 \) symbols and the power \( P \) varies from 0 to 20 dB. Noise variance \( N_0 \) is normalized to unity and the threshold level for SNR is \( \rho_{th} = 0 \) dB. Similar system parameters have been used in [4–6, 8]. Sub-optimum power assignment introduced in Section IV has been used. For simulation results, we have used Monte-Carlo simulation and considered the exact expression for SNR given in (11). Approach-1 in which we approximated \(|H|^2\) with its mean \((\sigma^2_H)\) is the most accurate one, i.e., the closest one to the simulation results for moderate to high SNR. As it is observed from Fig.1, Approach-2 outperforms Approach-3 in approximating the outage probability, especially at high SNR. As the power increases, the estimation error decreases and substitution of \( H \) with \( \hat{H} \) causes less errors. However, considering the worst
we can observe that for this case the analytical results are closer to the Monte-Carlo simulation results in comparison with Fig. 1. In other words, using the mean of $|\hat{H}|^2$ causes less error, because of smaller variance of this random variable. Note that for both relay-destination distance cases the mean of $|\hat{H}|^2$ ($\sigma_{\hat{H}}^2$) is the same, however the variance of $|\hat{H}|^2$ is higher for the relay closer to the destination (Fig. 1), i.e., the estimation error is less than the mean value at most conditions and the outage probability is less than the reverse position (Fig. 2).

VI. Conclusions

The SNR distribution and the outage probability of the relay link in a wireless relay network with a fixed gain relay in the presence of channel estimation error has been studied and different approximations have been used to find closed-form expression for the outage probability. Considering the estimation error and other noise terms in SNR as a worst case noise results in an upper bound for moderate to high SNR; however, it has the simplest close form suitable for optimization purposes. Also, as the relay-destination distance increases all of the analytic approaches have a good performance.

APPENDIX

The Outage probability is an increasing function of $\rho_{th}$ and in (25), it is divided by $C_4\sigma_{\hat{H}}^2, \sigma_{\hat{H}}^2$. Therefore, it is a decreasing function of $C_4\sigma_{\hat{H}}^2, \sigma_{\hat{H}}^2$, and here we maximize $C_4$ defined in (23) subject to (31). After algebraic manipulations on $C_4$ and considering $A_P = A_D = A$ (equal power for pilot and data), we can observe that maximizing $C_4$ is equivalent to minimizing below function

$$\frac{1}{f_1 + \frac{1}{\sigma_{\hat{H}}^2}} + \frac{f_1}{f_2}$$

where

$$f_1 \triangleq A^2 \sigma_{\hat{H}}^2 N_0 + N_0, \quad f_2 \triangleq A^2 P_0.$$ 

are functions of $P_0$ and $P_2^3$. It can be concluded from (33) that we should minimize $f_1/f_2$ with the constraint given in (31). Substituting (4) and (30) in (34) and considering $P_1 = \alpha P_0$ yields

$$f_1 = \frac{\alpha P_0 \sigma_{\hat{H}}^2 N_0 + P_0 \sigma_{\hat{H}}^2 N_0 + N_0^2}{\alpha P_0^2 N_0^2},$$

$$P_0 = \frac{2P}{1 + \alpha}.$$ 

This can be verified by swapping $H_1$ and $H_2$ and changing $\alpha$ to $1/\alpha$ in (35) and (36) in the appendix and noting that $\sigma_{\hat{H}}^2 = \frac{1}{2 + \frac{1}{2\sigma_{\hat{H}}^2} + \alpha^2}.$

Note that $\alpha$ depends on both $P_0$ and $P_1$ in (4) and (5).

We have defined $\alpha$ for simpler mathematical calculation, however, the equations can be also written in terms of $\beta$.  

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**Fig. 1.** The outage probability of the relay link for different analytical approaches and simulation vs. average power per symbol, $d_1 = 0.75, d_2 = 0.25$.

**Fig. 2.** The outage probability of the relay link for different analytical approaches and simulation vs. average power per symbol, $d_1 = 0.25, d_2 = 0.75$.
Using (36) in (35) and taking derivative of the resulting function with respect to $\alpha$ gives the optimum value of $\alpha$ as follows

$$\alpha_{\text{opt}}^2 = \frac{2\sigma_H^2 P + N_0}{2\sigma_H^2 P + N_0} \sim \frac{\sigma_H^2}{\sigma_H^2},$$

(37)

where the second equality is more accurate at high SNR values. Therefore, the optimum value of $\beta$ is given by

$$\beta_{\text{opt}} = \frac{1}{\alpha_{\text{opt}}} + 1 = \frac{\sigma_H^2}{\sigma_H^2 + \sigma_H^2}.$$

(38)

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REFERENCES


