In this lecture, we will learn:

- about some of the people and the important steps in the development of modern logic
- Aristotle, Leibniz, Boole, Frege, Russell, Hilbert, Gödel, Turing, Tarski and Milner
- the language of quantified logic QL

Aristotle, about 350BC, Greece

- saw the need for a general account of correct reasoning
- codified valid argument forms called “syllogisms”
  - All men are mortal.
  - Aristotle is a man.
  - Therefore Aristotle is mortal.

Leibniz, about 1680, Germany

- “Wonderful idea” - a symbolic language for concepts, allowing problems to be solved by mechanical calculation

The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us calculate [calculemus], without further ado, to see who is right. (The Art of Discovery 1685, W 51)

Boole, about 1840, England

- developed a form of algebra (now called boolean algebra), where letters stand for concepts instead of numbers
- equivalent to our logic PL and elementary set theory
- the beginning of logic as mathematics
Frege, 1879, Germany

- “Concept Script” (1897) the first formal system to allow multiple quantification (everybody loves somebody)
- strange graphical notation, no longer used
- sought logical foundations for mathematics, e.g., definition of number - was Julius Caesar a number?
- almost unknown in his lifetime, and came to think his work useless (see next slide on Russell), now recognised as the father of modern logic

Russell, 1910, England

- discovered Russell's paradox “The set of all sets which are not members of themselves” which arises in Frege’s system
- with Alfred North Whitehead, wrote the “Principia Mathematica” - in volume II they prove 1+1=2
- introduced the theory of types to avoid paradoxes, and theory of descriptions to avoid the problem of referential failure (The king of France is bald)
- arguably the most important philosopher of the 20th century

Hilbert, 1920, Germany

- Hilbert wanted to save mathematics from the constructivists, who rejected important mathematical methods
- to do this, “Hilbert’s program” aimed to derive all maths from a single axiom system which can prove itself consistent

Gödel, 1931, Austria

- Gödel proved that all systems capable of expressing arithmetic are either inconsistent or incomplete, and thus can not prove themselves consistent, destroying Hilbert’s programme
- that is, any strong deductive system has statements which can neither be proved nor disproved
Turing, 1936, England

- proposed a class theoretical computing machine (now called a Turing machine), and showed that one machine of this type could be programmed to do the work of any of them - the universal machine
- extended Gödel’s work to show that some problems were uncomputable (the Halting problem)
- helped win WWII by breaking German “Enigma” code at Bletchley Park

Tarski, 1936, Poland

- developed the semantics of QL and the study of these structures: model theory
- proved that truth for a language can not be defined within that language
- integrated modern logic with modern abstract algebra

Milner, 1972, England

- introduced a style of computer proof checkers, and theorem provers whose reliability depends only on a very small “core”
- the expressive “higher order logic” and combination of human and computer proof has made formal verification practical
- many important formal/theoretical contributions, including formal systems for concurrent processes ($\pi$-calculus)

Exercises

In Class
- Chapter 5, Exercises 1, 2, 3, 4 odds

Homework Assignment
- Chapter 5, 1.2, 2.2, 3.2, 4.2