

# Characterization of 3D Spatial Wireless Channels

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# Motivation/Background

- ◇ MIMO theory makes weak connection with space
  - independence result
  - correlated gaussian result
- ◇ Have bounds but what can really be achieved? May be over-optimistic.

- Typical spatial channel models
  - discrete scatterers around Tx and Rx
  - contrived geometries, regular rings
  - linear arrays
- This paper has quite different approach
  - few assumptions about scattering
  - arrays → regions (no antennas)
  - **parsimonious** model

# Parsimony

- Multipath modelled with (the actual) multiple paths is rarely parsimonious.
- An accurate multipath model can do more harm than good.

Why?

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- Millions of multipath implies millions of parameters
- AIC hostile
- Excess parameters degrade estimation performance

Obvious that the finite resolving power of an array (in angle) implies we can reduce the number of multipaths we need to model.  
(Actually this has very little to do with arrays.)

# Messages

1. Reducing the number of multipaths is not just desirable but really necessary.



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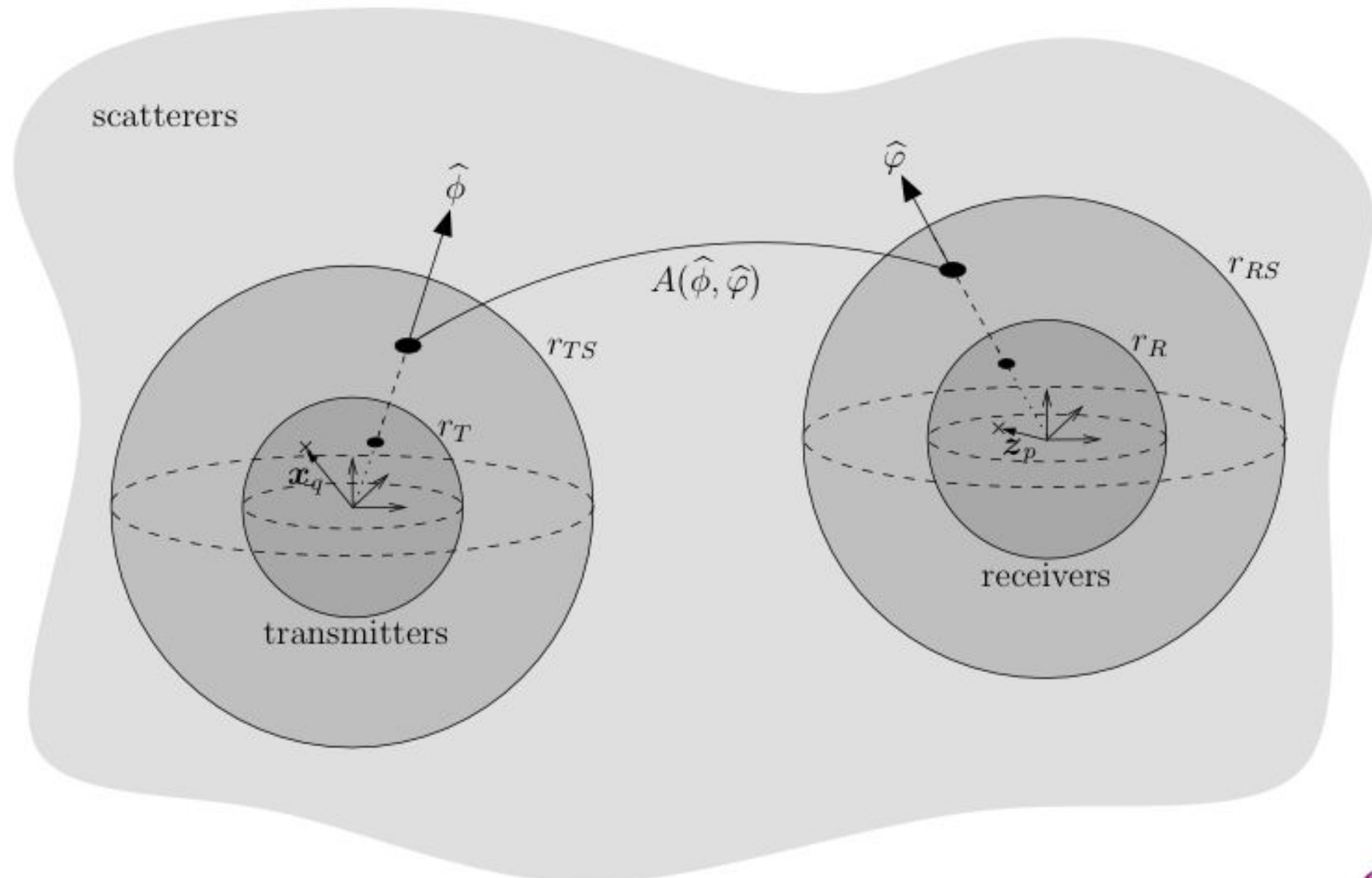
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# Messages

1. Reducing the number of multipaths is not just desirable but really necessary.
2. Array theory tells us this but actually we don't need arrays.
3. OK we do need arrays... Arrays as:
  - implementation
  - spatial sampling

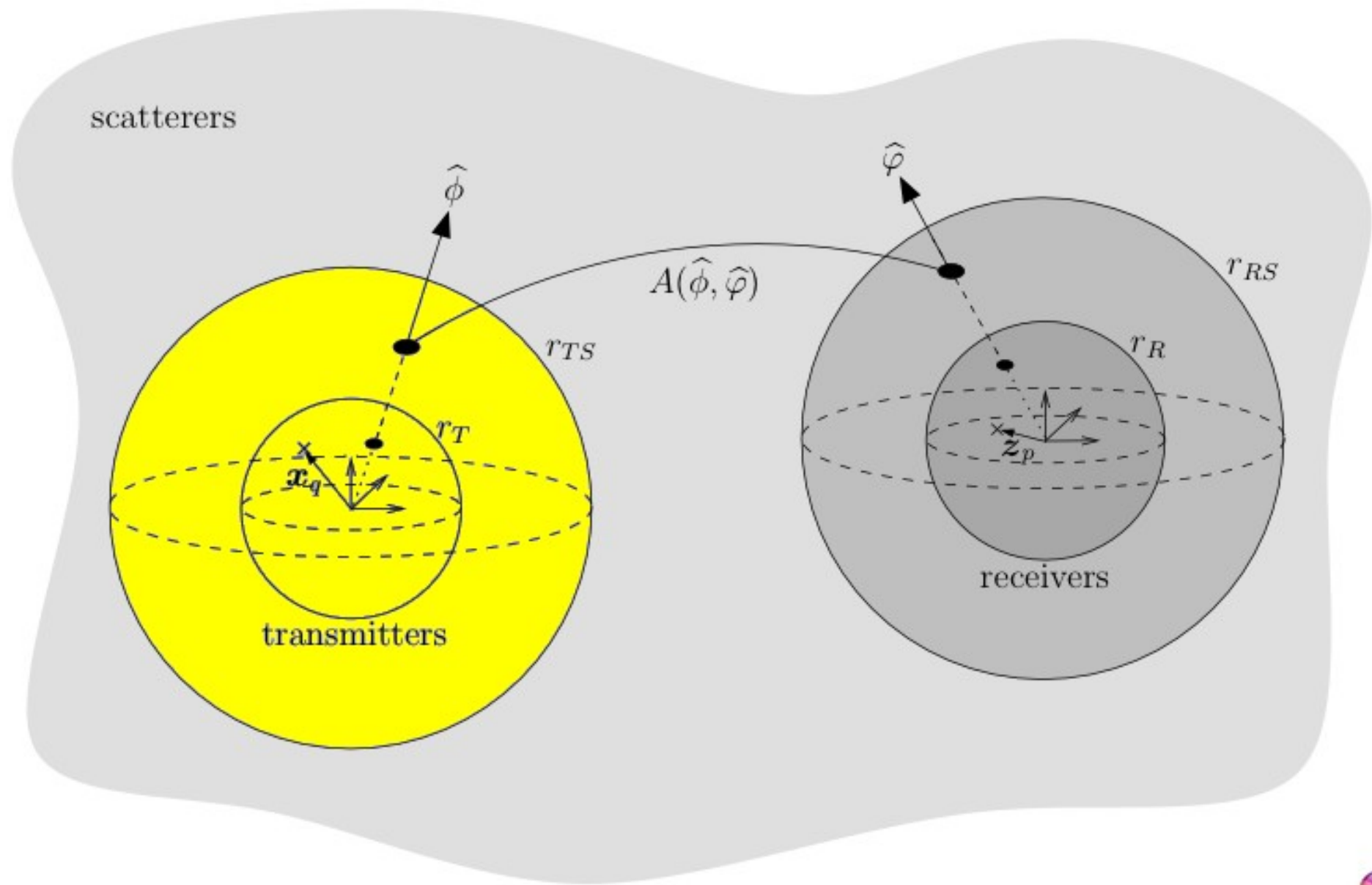
# 3D Model of Multipath



# Model Form – Three Spatial Regions

- source free ball about Tx





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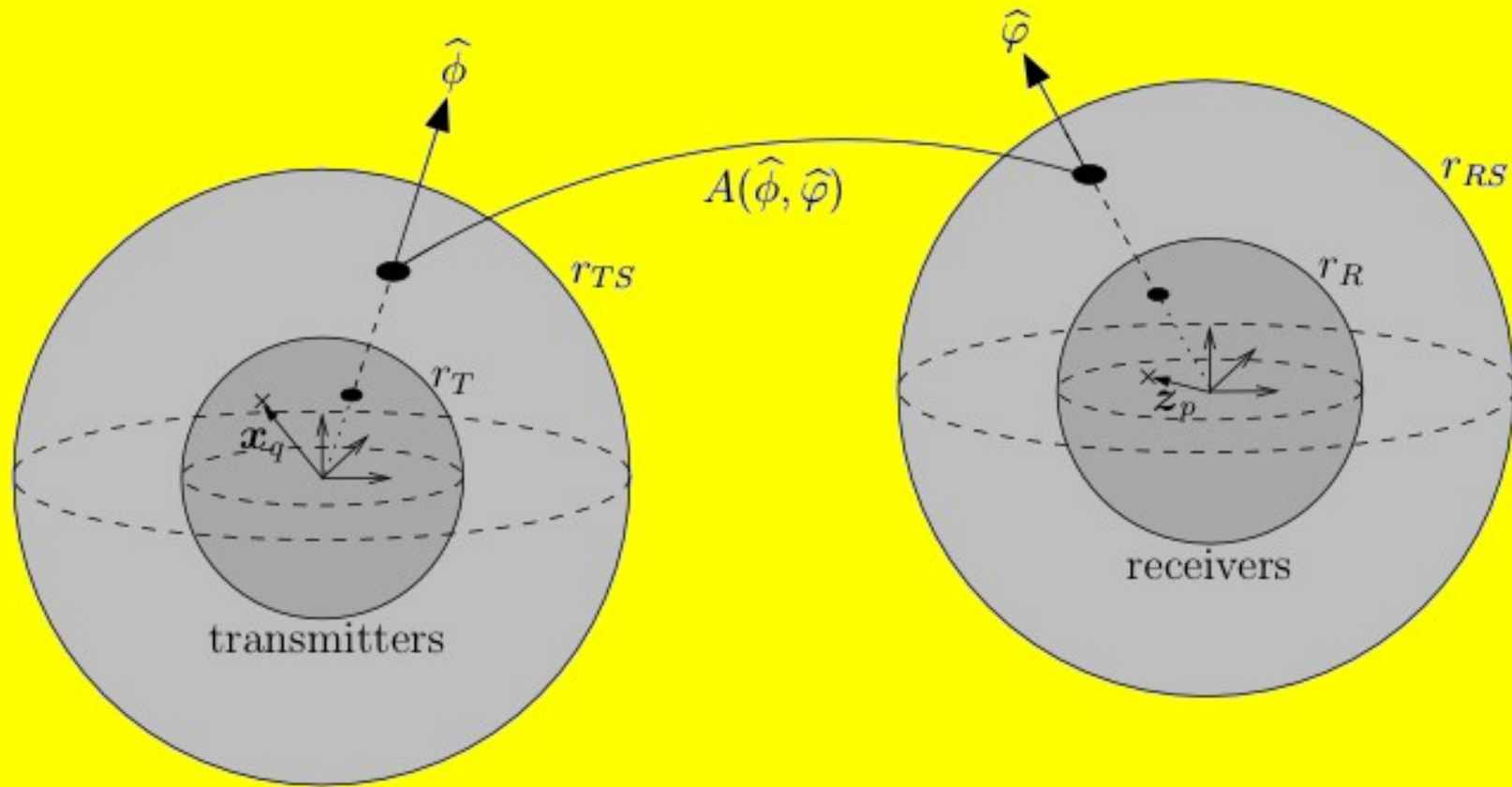


# Model Form – Three Spatial Regions

- source free ball about Tx
- complex scattering field



scatterers



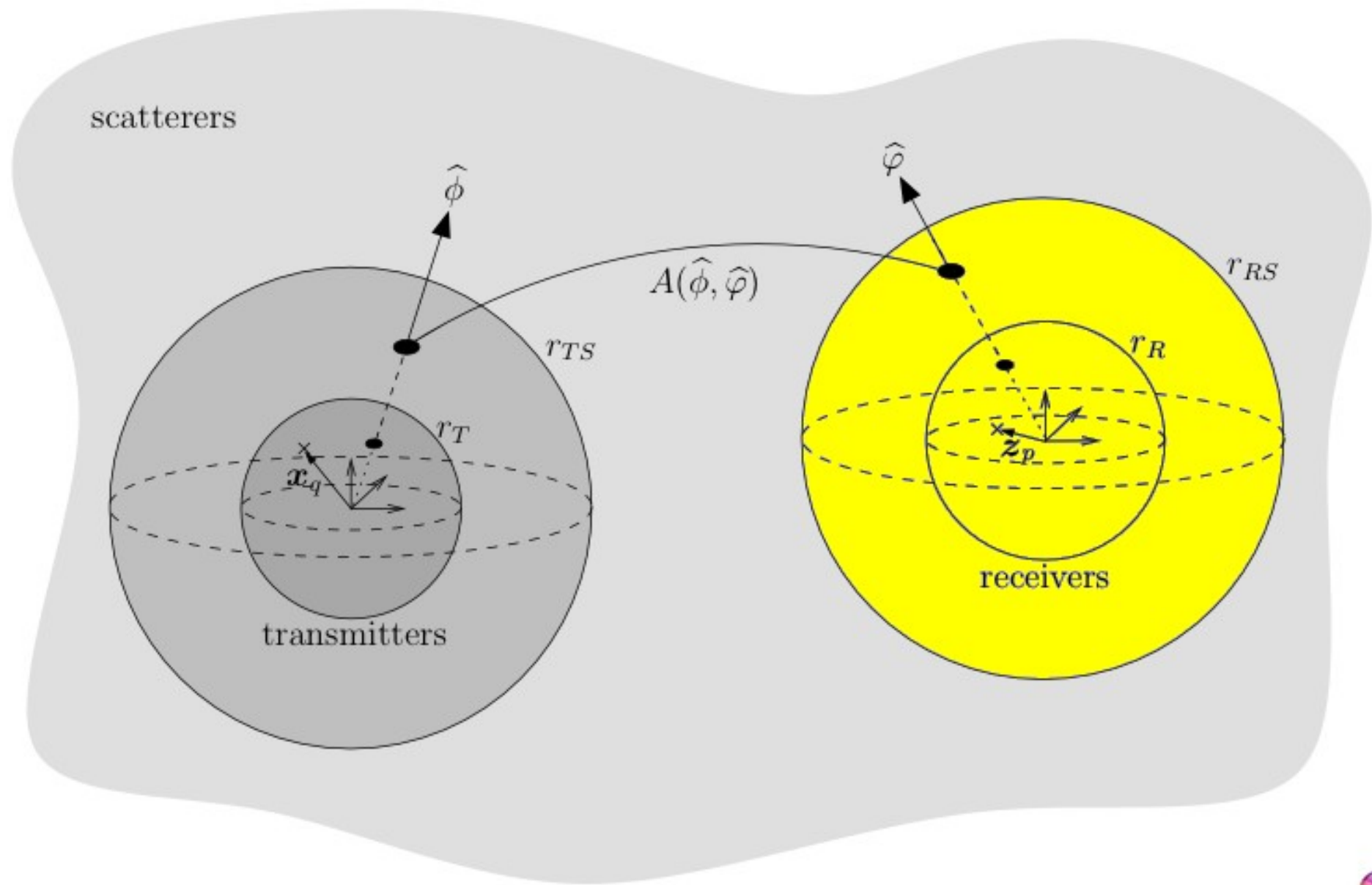
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# Model Form – Three Spatial Regions

- source free ball about Tx
- complex scattering field
- source free ball about Rx

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# Model Form – Three Spatial Regions

- source free ball about Tx
- complex scattering field
- source free ball about Rx

Generalizes to other shaped regions (holding the antennas).

# MIMO Model – Matrices

- multi Tx antennas
- **matrix** channel
- multi Rx antennas

**Matrices are highly desirable**

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# New Model – Matrices also

- orthonormal functions in Tx ball
- bounded operator
- orthonormal functions in Rx ball

**Bounded operators have a matrix representation (infinite).**

# Parsimony

- Use the most efficient orthonormal expansion in balls
- Bound the essential dimension (ugly)
- Given radius  $R$ , the essential dimension is  $(N + 1)^2$  where  $N \approx \pi e R / \lambda$ .
- Different expansions depending on the shape of the region — eigenfunctions.

# Channel Matrix Model

$$\mathbf{H} = \mathbf{J}_R \mathbf{H}_s \mathbf{J}_T^\dagger, \quad (1)$$

$$\mathbf{J}_R: P \times (N_R + 1)^2$$

$$\mathbf{H}_s: (N_R + 1)^2 \times (N_T + 1)^2$$

$$\mathbf{J}_T: (N_T + 1)^2 \times Q$$

$\mathbf{J}_R, \mathbf{J}_T$  — antenna  $\leftrightarrow$  scattering operator (matrix)

$\mathbf{H}_s$  — scattering operator (matrix)

# Sampling Matrices

Entries of  $\mathbf{J}_T$  look like

$$4\pi i^n j_n(k\|\mathbf{x}_q\|) \overline{Y_{nm}(\hat{\mathbf{x}}_q)}, \quad (2)$$

where  $\mathbf{x}_q$  is the location of the  $q$ th antenna.

Mapping from spatial sample points to an orthonormal function indexed by  $m$  and  $n$ . (This choice of orthonormal function is optimal in a energy concentration sense.)

# Key Points

$J_T, J_R$

- hold the antenna specific info only

- deterministic

$H_s$

- all scattering

- random

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# Flip Side

Conventional MIMO matrix  $H$

- antenna specific and scattering info is mixed up

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# In the Paper

- $H_s$  is also expressed in terms of the scattering gain function of angles

$$A(\hat{\phi}, \hat{\varphi})$$

- Linear array case reworked
- $\text{rank}\{\mathbf{H}\} = \min\{\text{rank}\{\mathbf{J}_T\}; \text{rank}\{\mathbf{J}_R\}; \text{rank}\{\mathbf{H}_s\}\}$

# Conclusions

The channel matrix of a MIMO system can be factored into fixed (and known) and random matrices where the deterministic portion depends on receiver and transmitter antenna configurations.

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