Beamforming methods for downlink channel

Jinho Choi
School of Electrical Eng. & Telecomm.
The University of New South Wales
j.choi@unsw.edu.au

October 28, 2003

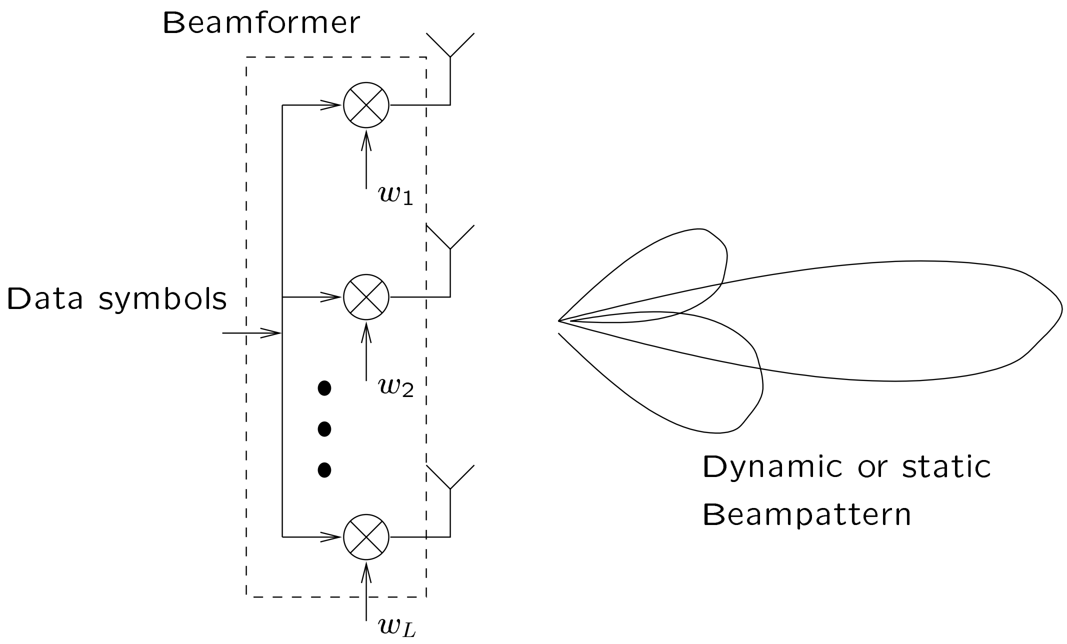
Outline

1. Introduction
2. Optimal and blind beamforming
3. Eigenbeamforming
4. Eigenbeamforming for OFDM
5. Conclusions

1. Introduction

Downlink beamforming can have two different aims:

- Spatial multiplexing
- Improving the BER performance



- When the beamformer is used for improving performance, there are two different gains that can be achieved:
  - Beamforming gain
  - Diversity gain (for a fading channel)
- For the beamforming, we need to know the channel state information (CSI).
- Depending on the type of the CSI, different beamforming methods can be used.
- In this talk, we mainly consider the eigenbeamforming that can provide both diversity gain and beamforming gain and its application to OFDM systems.

2. Optimal and blind beamforming

Let
L: the number of transmit antennas (at the base station)
M: the number of receive antennas (at the mobile station)

The $M \times 1$ received signal vector over a flat fading channel is given by

$$\mathbf{r}(t) = \sqrt{\gamma}\mathbf{H}(t)\mathbf{w}(t)s(t) + \mathbf{n}(t)$$

where
γ: the input SNR
$\mathbf{H}(t)$: the channel matrix
$\mathbf{w}(t)$: the beamforming vector (normalized such as $||\mathbf{w}(t)||^2 = 1$)
$s(t)$: the data symbol
$\mathbf{n}(t)$: the AWGN vector ($E[\mathbf{n}(t)] = \mathbf{0}$ and $E[\mathbf{n}(t)\mathbf{n}^H(t)] = \mathbf{I}$)

Optimal beamforming

For given $\mathbf{H}(t)$, the instantaneous SNR at the receiver is written as

$$\gamma_{rec}(t) = \gamma \mathbf{w}^H(t)\mathbf{H}^H(t)\mathbf{H}(t)\mathbf{w}(t).$$

The optimal beamforming vector that maximizes the *instantaneous* SNR can be obtained as follows:

$$\mathbf{w}_{opt}(t) = \arg \max_{||\mathbf{w}||^2 \leq 1} \mathbf{w}^H \mathbf{R}_{\mathbf{H}}(t)\mathbf{w},$$

where $\mathbf{R}_{\mathbf{H}}(t) = \mathbf{H}^H(t)\mathbf{H}(t)$.

It is time-varying and requires the instantaneous CSI. The mobile station needs to feed back $\mathbf{w}_{opt}(t)$ and update as soon as the CSI changes.
⇒ A larger amount of feedback is required for faster fading channel.

Blind beamforming

Suppose that the instantaneous channel matrix is not available. In addition, we assume that the beamforming vector is time-invariant (or very slowly varying). In this case, the beamforming vector can be obtained by maximizing the *mean* SNR as follows:

$$\begin{aligned}\mathbf{w}_{ms} &= \arg \max_{||\mathbf{w}||^2 \le 1} E[\gamma_{rec}(t)] \\ &= \arg \max_{||\mathbf{w}||^2 \le 1} \mathbf{w}^H \mathbf{R}_{\mathbf{H}} \mathbf{w},\end{aligned}$$

where $\mathbf{R}_{\mathbf{H}} = E[\mathbf{R}_{\mathbf{H}}(t)] = E[\mathbf{H}^H(t)\mathbf{H}(t)]$.

The $L \times L$ matrix $\mathbf{R}_{\mathbf{H}}$ is called the spatial covariance matrix.

Blind beamforming (cont'd)

The eigendecomposition of $\mathbf{R_H}$ is given by

$$\mathbf{R_H} = \mathbf{E}\mathbf{\Lambda}\mathbf{E}^H,$$

where $\mathbf{\Lambda} = diag(\lambda_1, \lambda_2, \ldots, \lambda_L)$ and $\mathbf{E} = [\mathbf{e}_1 \;\; \mathbf{e}_2 \;\; \ldots \;\; \mathbf{e}_L]$. Here, $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_L$ are the eigenvalues and $\mathbf{e}_m$ is the eigenvector corresponding to the eigenvalue λ_m. Hence, the beamforming vector is given by

$$\mathbf{w}_{ms} = \mathbf{e}_1.$$

Hence, only a constant vector $\mathbf{e}_1$ can be fed back.

Using the channel reciprocity, the spatial covariance matrix, $\mathbf{R_H}$, can be estimated by using uplink measurements.
⇒ No feedback is required. Hence, it is called the blind method.

Performance analysis

To simplify, let $M = 1$ (i.e., single receive antenna). Let

$$\mathbf{g}^T(t) = \mathbf{H}(t).$$

Then, the received signal is given by

$$r(t) = \sqrt{\gamma}\mathbf{g}^T(t)\mathbf{w}(t)s(t) + n(t).$$

Furthermore, assume that $\mathbf{g}(t)$ is a complex Gaussian random vector with

$$E[\mathbf{g}(t)] = \mathbf{0} \text{ and } E[\mathbf{g}^*(t)\mathbf{g}^T(t)] = \mathbf{R}_\mathbf{H}.$$

Performance analysis (cont'd)

The optimal beamforming:

$$\mathbf{w}(t) = \mathbf{g}^*(t)/||\mathbf{g}(t)||$$

The blind beamforming:

$$\mathbf{w} = \mathbf{e}_1.$$

Then, we have the received signals as follows

$$r(t) = \sqrt{\gamma}||\mathbf{g}^T(t)||s(t) + n(t) \quad \text{(optimal beamforming)}$$

and

$$r(t) = \sqrt{\gamma}\mathbf{g}^T(t)\mathbf{e}_1 s(t) + n(t) \quad \text{(blind beamforming).}$$

Note that

$||\mathbf{g}^T(t)||^2$: a χ^2 random variable with $2L$ degrees of freedom

$||\mathbf{g}^T(t)\mathbf{e}_1||^2$: a χ^2 random variable with 2 degrees of freedom

Performance analysis (cont'd)

The average BER's are bounded by

$$P_b(opt) \leq \prod_{l=1}^{L} (1 + \gamma\lambda_m)^{-1}$$

and

$$P_b(blind) \leq (1 + \gamma\lambda_1)^{-1}$$

Therefore, the optimal beamforming can provide the best performance when

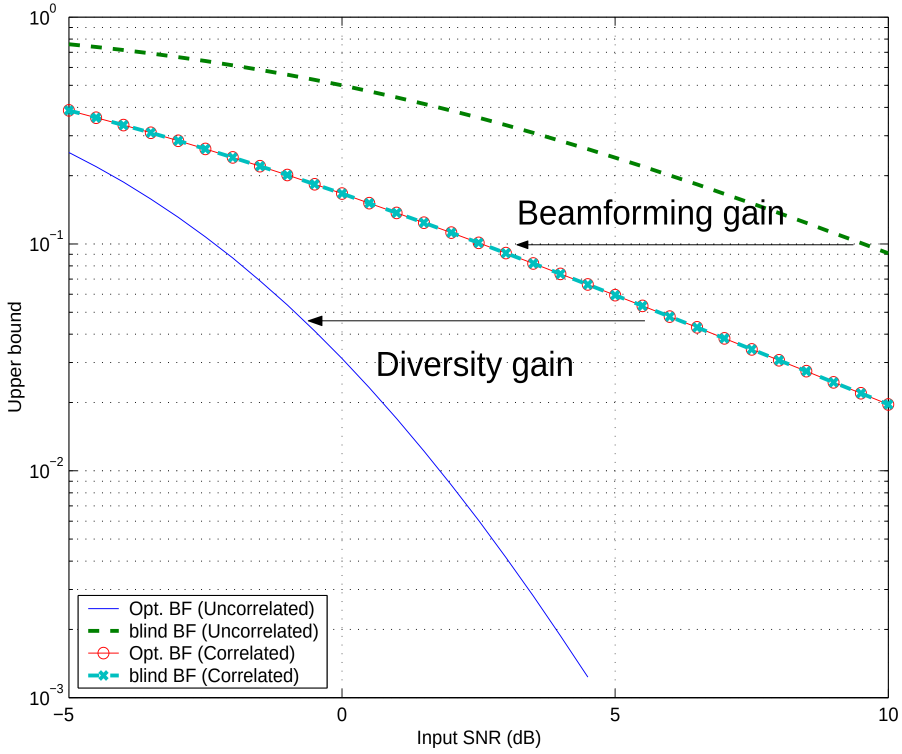
$$\lambda_1 = \lambda_2 = \cdots = \lambda_L = \frac{tr(\mathbf{R_H})}{L} \text{ (perfectly uncorrelated),}$$

while the blind beamforming can provide the best performance when

$$\lambda_1 = tr(\mathbf{R_H}) > \lambda_2 = \cdots = \lambda_L = 0 \text{ (perfectly correlated).}$$

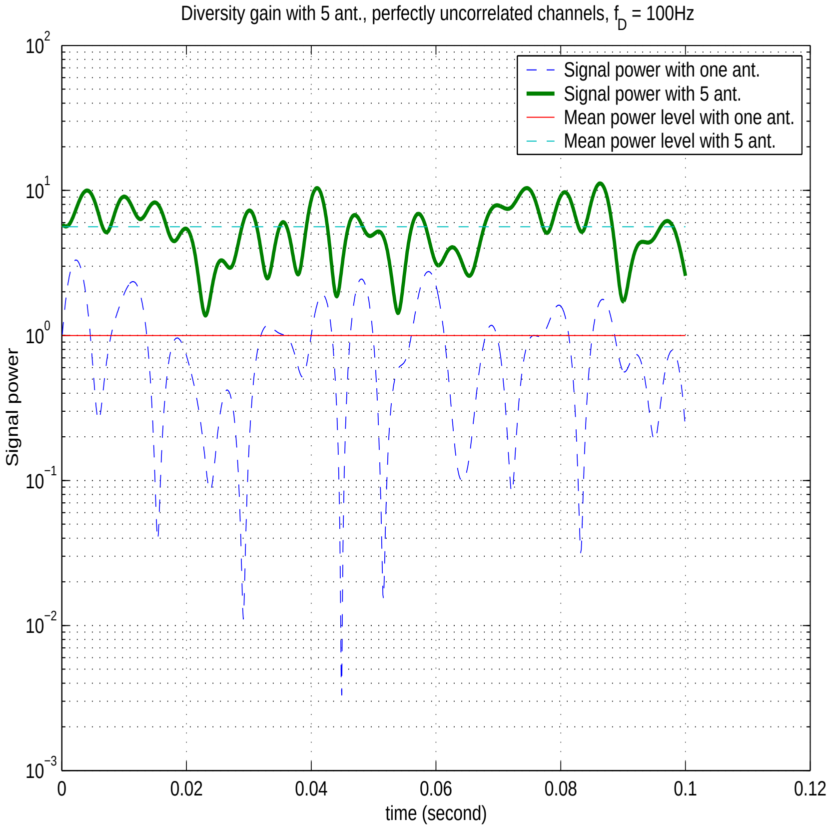
Performance analysis (cont'd)

The blind beamforming cannot perform better than the optimal beamforming.

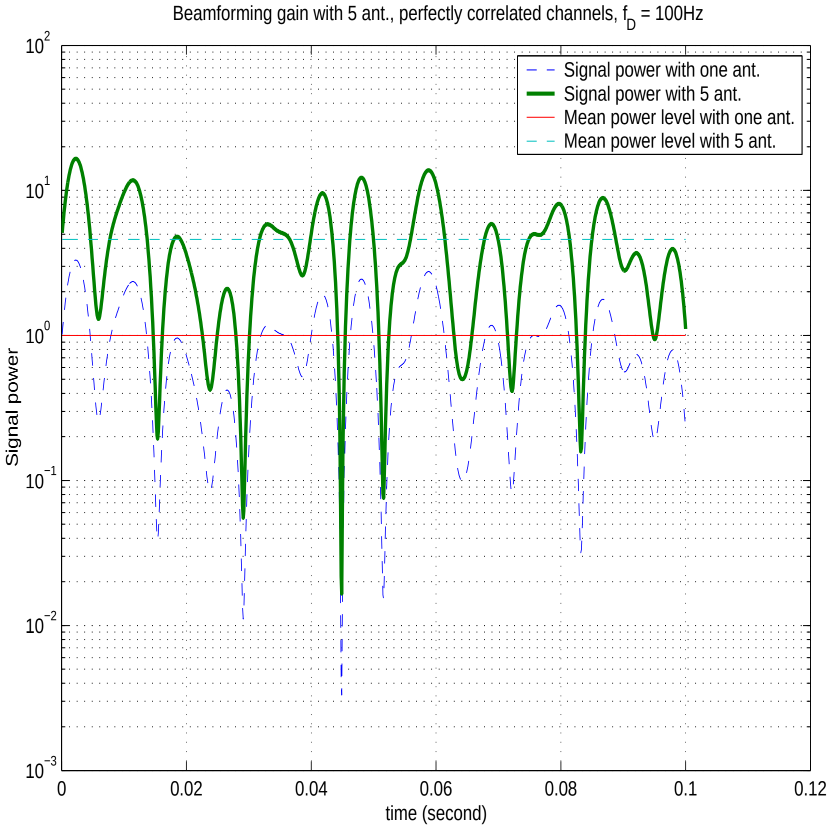


Upper bounds of the BER when $L = 5$ and $M = 1$.

Diversity gain versus beamforming gain

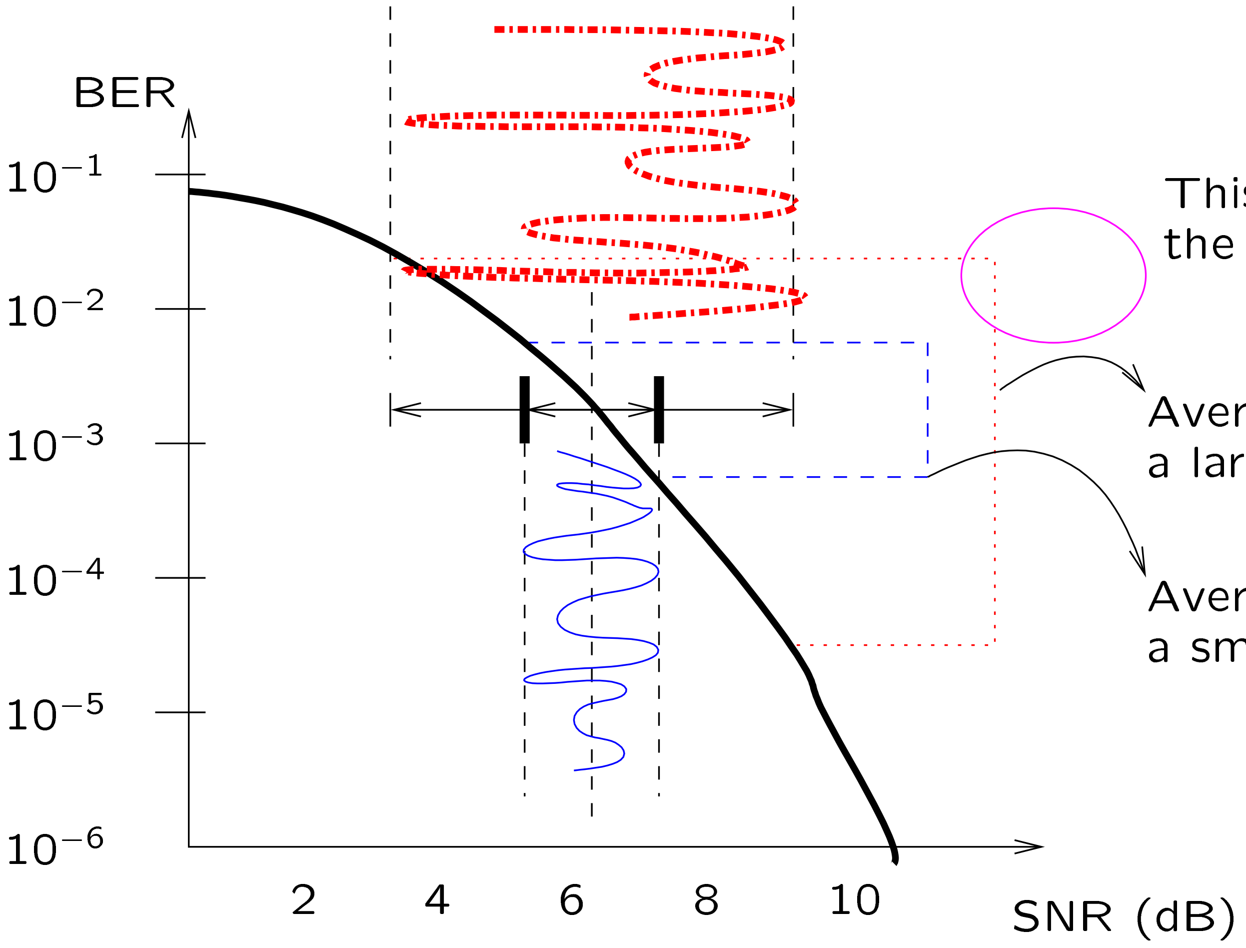


Diversity gain
from uncorrelated channels



Beamforming gain
from correlated channels

Impact of diversity gain beamforming gain on the BER



This higher BER range degrades the average BER much.

Average BER for
a larger SNR variation

∨

Average BER for
a smaller SNR variation

3. Eigenbeamforming

Since the optimal beamforming can provide much better performance due to diversity gain, it is essential to exploit diversity gain to have good performance.

⇒ some feedback for the instantaneous CSI is required to have a diversity gain.

Main idea of the eigenbeamforming:

- Choose N_f best eigenmodes from $\mathbf{R}_{\mathbf{H}}$ (no feedback of the instantaneous CSI)
  ⇒ improve the beamforming gain
- Select one of N_f best eigenmodes that can increase the instantaneous SNR (feedback required)
  ⇒ provide the diversity gain

Eigenbeamforming (cont'd)

In the eigenbeamforming, the beamforming vector is given by

$$\mathbf{w}(t) = \arg \max_{\mathbf{e}_m, m=1,2,\ldots,N_f} ||\mathbf{H}(t)\mathbf{e}_m||^2.$$

- The set of N_f best eigenmodes can be fed back from the mobile station to the base station.
- Alternatively, the base station can obtain this set from uplink measurements by using the channel reciprocity.
- The selection diversity is provided by choosing the best eigenmode that matches with the instantaneous CSI among N_f pre-selected eigenmodes.
- The index of the selected eigenmode is fed back. Hence, $\lceil \log_2 N_f \rceil$ bits are required.

Performance analysis

For simplicity, let $M = 1$ (one receive antenna) and assume that the channel vector is a zero-mean complex Gaussian vector.

We have N_f possible eigenmodes and the selection diversity is applied. Hence, by using order statistics, the cdf of the channel SNR is given by

$$F(x) = \prod_{m=1}^{N_f} \left(1 - e^{-x/\lambda_m}\right) = \sum_{q=0}^{2^{N_f}-1} A_q e^{-B_q x}.$$

Then, the average BER for BPSK or QPSK signaling is given by

$$P_b = \int_0^\infty Q(\sqrt{2\gamma x}) F'(x) dx,$$

where $F'(x) = dF(x)/dx$.

Performance analysis (cont'd)

The average BER becomes

$$P_b = \frac{1}{2}\sum_q A_q \sqrt{\frac{\gamma}{\gamma + B_q}}.$$

Furthermore, an asymptotic expression is available when $\lambda_{N_f} > 0$ as follows:

$$\begin{aligned} P_b &= G\left(\prod_{m=1}^{N_f}\lambda_m\right)^{-1}\gamma^{-N_f} + \underbrace{O(\gamma^{-(N_f+1)})}_{\text{vanishing quickly at high SNR}} \\ &\simeq G\underbrace{\left(\prod_{m=1}^{N_f}\lambda_m\right)^{-1}}_{\text{beamforming gain}} \times \underbrace{\gamma^{-N_f}}_{\text{diversity gain}}, \quad \gamma \gg 0, \end{aligned}$$

where G is a constant.

Performance analysis (cont'd)

Properties of the eigenbeamforming:

- To maximize the beamforming gain, the N_f eigenvectors corresponding to the N_f largest eigenvalues should be chosen.
- The diversity gain is decided by the number of pre-selected eigenmodes, N_f. Hence, as N_f increases, a better diversity gain is expected.

⇒ The eigenbeamforming can effectively obtain **the beamforming gain** as well as **the diversity gain** with a small amount of feedback.

Consequently, for a given number of eigenmodes, N_f, the best performance is achieved when

$$\lambda_1 = \lambda_2 = \cdots = \lambda_{N_f} > \lambda_{N_f+1} = \cdots = \lambda_L = 0.$$

4. Eigenbeamforming for OFDM

The eigenbeamforming can be applied to OFDM systems with N subcarriers.

Since each subcarrier experiences different fading, it should have different beamforming vector.

The signal in the space and frequency domains to be transmitted is as follows:

$$\begin{aligned}\mathbf{S}(t) &= [\mathbf{w}_0(t)s_0(t)\ \mathbf{w}_1(t)s_1(t)\ \cdots\ \mathbf{w}_{N-1}(t)s_{N-1}(t)] \\ &= [\mathbf{w}_0(t)\ \mathbf{w}_1(t)\ \cdots\ \mathbf{w}_{N-1}(t)]\mathbf{D}(t),\end{aligned}$$

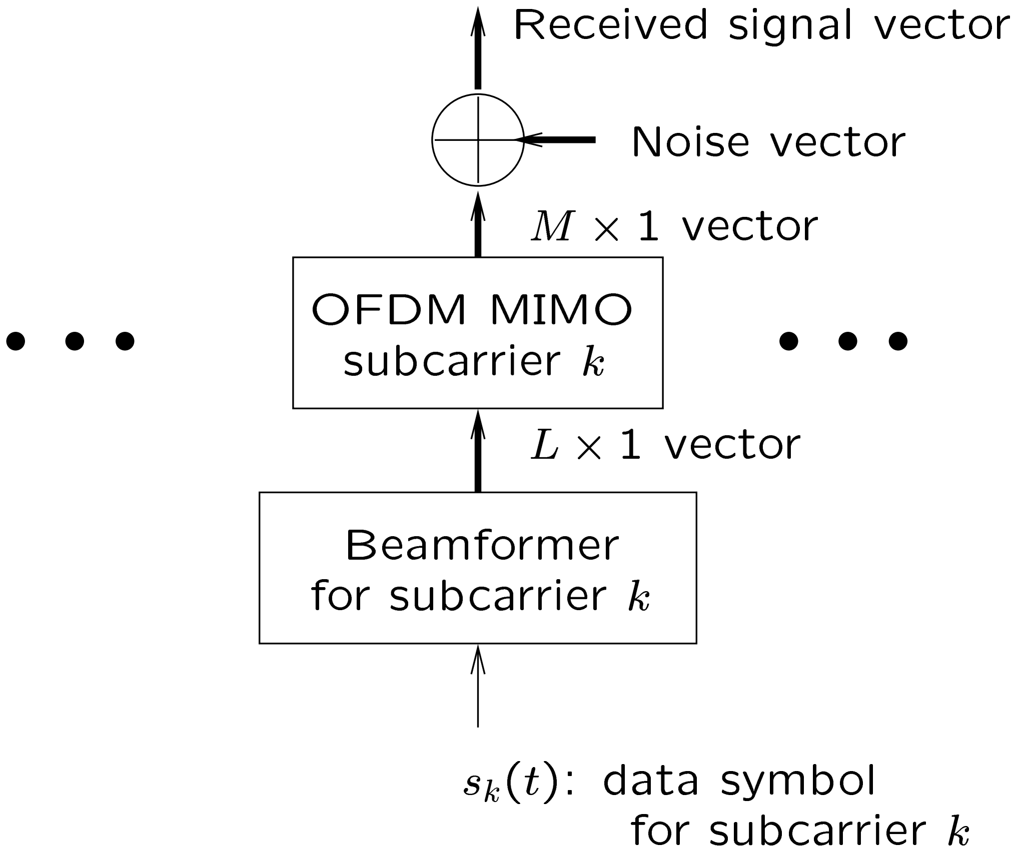
where $\mathbf{D}(t)$ is a diagonal matrix with the data symbols that is written as

$$\mathbf{D}(t) = diag(s_0(t), s_1(t), \ldots, s_{N-1}(t)).$$

After the IDFT of $\mathbf{S}(t)$, the signal is transmitted with cyclic prefix.

Diagram for MIMO OFDM with beamforming

We have N MIMO *flat fading* channels in freq. domain.



Received signal vector

Noise vector

$M \times 1$ vector

OFDM MIMO
subcarrier k

• • •

• • •

$L \times 1$ vector

Beamformer
for subcarrier k

$s_k(t)$: data symbol
for subcarrier k

Received signal

The frequency domain MIMO channel matrices are given by

$$\tilde{h}_{k,m,l}(t) = \sum_{p=0}^{P-1} h_{p,m,l}(t) e^{-j2\pi pk/N},$$

where $\{h_{p,m,l}(t)\}$, $p = 0, 1, \ldots, P-1$, is the channel impulse response (CIR) from transmit antenna l to receive antenna m during the tth symbol interval and P is the length of the CIR.

After the DFT, the received signal matrix is given by

$$\mathbf{R}(t) = [\tilde{\mathbf{H}}_0(t)\mathbf{w}_0(t)\ \tilde{\mathbf{H}}_1(t)\mathbf{w}_1(t)\ \cdots\ \tilde{\mathbf{H}}_{N-1}(t)\mathbf{w}_{N-1}(t)]\mathbf{D}(t) + \mathbf{N}(t).$$

Hence, the received signal vector for the kth subcarrier is given by

$$\mathbf{r}_k(t) = \tilde{\mathbf{H}}_k(t)\mathbf{w}_k(t)s_k(t) + \mathbf{n}_k(t)$$

Beamforming

For each subcarrier, the optimal beamforming vector is given by

$$\mathbf{w}_k(t) = \arg \min_{||\mathbf{w}||^2 \leq 1} \mathbf{w}^H \tilde{\mathbf{H}}_k^H(t) \tilde{\mathbf{H}}_k(t) \mathbf{w}, \ k = 0, 1, \ldots, N-1.$$

Since the amount of feedback would be high, the eigenbeamforming can be applied.

For the eigenbeamforming,

- we need to have N different sets of pre-selected eigenmodes for all the N subcarriers.
  $\Rightarrow$ a large amount of feedback would be required.
- each subcarrier needs the short-term selection of pre-selected eigenmodes.
  $\Rightarrow$ $N \lceil \log_2 N_f \rceil$ bits are required for the feedback

Spatial covariance matrices for subcarriers

To determine the N_f best eigenmodes for each subcarrier, the frequency-domain spatial covariance matrices should be found.

We assume that the CIR's are zero-mean random sequences and

$$E[\mathbf{H}_p^H(t)\mathbf{H}_{p'}(t)] = \sigma_{h,p}^2 \mathbf{R}_{\mathbf{H}_p} \delta_{p,p'},$$

where $[\mathbf{H}_p(t)]_{m,l} = h_{p,m,l}(t)$, $\sigma_{h,p}^2$ is the power delay profile of the CIR, and

$$[\mathbf{R}_{\mathbf{H}_p}]_{s,q} = \frac{1}{\sigma_{h,p}^2} E[h_{p,m,s}^*(t) h_{p,m,q}(t)], \; s,q = 1,2,\ldots,L.$$

It means that the time-domain spatial covariance matrix for each tap is different.

Spatial covariance matrices for subcarriers (cont'd)

The spatial covariance matrix of $\tilde{\mathbf{H}}_k(t)$ is given by

$$\mathbf{R}_{\tilde{\mathbf{H}}_k} = E[\tilde{\mathbf{H}}_k^H(t)\tilde{\mathbf{H}}_k(t)].$$

We can show that

$$\begin{aligned}
[\mathbf{R}_{\tilde{\mathbf{H}}_k}]_{s,q} &= \sum_{m=1}^{M} E[\tilde{h}^*_{k,m,s}(t)\tilde{h}_{k,m,q}(t)] \\
&= \sum_{m=1}^{M} E[(\sum_{p=0}^{P-1} h_{p,m,s}(t)e^{-j2\pi pk/N})^*(\sum_{p'=0}^{P-1} h_{p',m,q}(t)e^{-j2\pi p'k/N})] \\
&= \sum_{p=0}^{P-1} \sigma_{h,p}^2[\mathbf{R}_{\mathbf{H}_p}]_{s,q} = [\mathbf{R}_{\tilde{\mathbf{H}}}]_{s,q} \overset{\triangle}{=} [\sum_{p=0}^{P-1} \sigma_{h,p}^2\mathbf{R}_{\mathbf{H}_p}]_{s,q}.
\end{aligned}$$

It shows that the frequency domain spatial covariance matrices are the same for all the subcarriers.

Eigenbeamforming

- One common set of pre-selected eigenmodes is shared by all the N subcarriers.
  $\Rightarrow$ only a small amount of feedback would be required.
- This makes the eigenbeamforming very attractive for OFDM systems when the set of pre-selected eigenmodes is to be fed back.
- As adjacent subcarriers experience similar fading, a common eigenmode can be selected for $\bar{K}$ adjacent subcarriers.
  $\Rightarrow (N/\bar{K})\lceil \log_2 N_f \rceil$ bits for the short-term selection
- The performance analysis for a flat fading channel is valid for OFDM systems since the signal over each subcarrier experiences a flat fading channel.
  $\Rightarrow$ Beamforming gain + diversity gain

Simulation results

OFDM parameters:

No. of subcarriers	$N = 64$
No. of antennas	$L = 5,\ M = 1, 2$
No. of multipaths	$P = 8$
No. of pre-selected eigenmodes	N_f
No. of subcarriers for short-term selection	$\bar{K}$

A simple spatial covariance matrix is used (ρ is the spatial correlation):

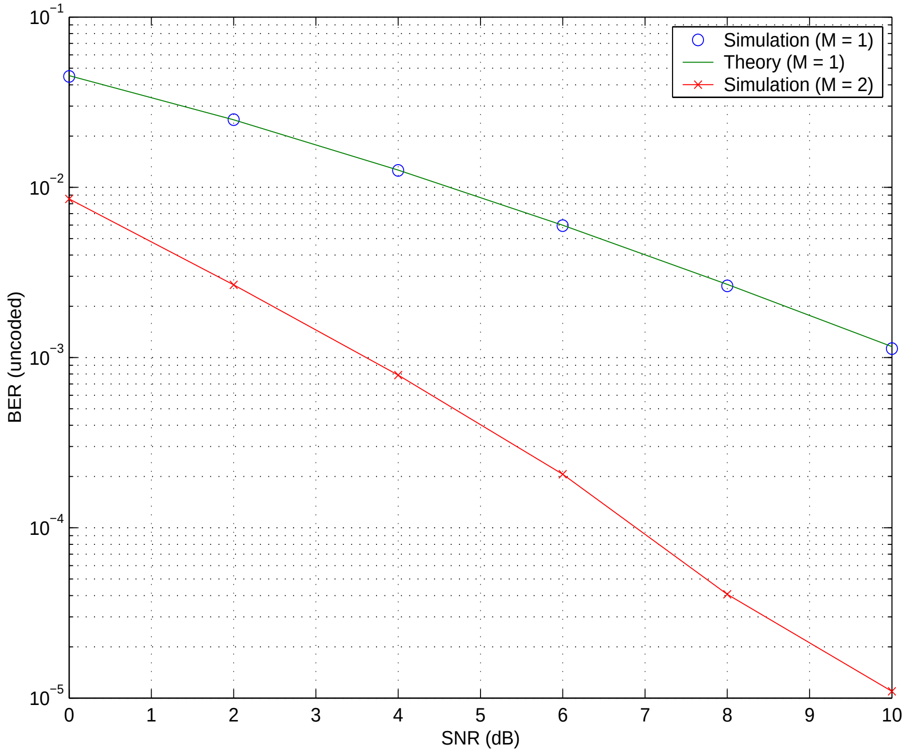
$$\mathbf{R}_{\mathbf{H}_p} = \begin{bmatrix} 1 & \rho & \cdots & \rho^{L-1} \\ \rho & 1 & \cdots & \rho^{L-2} \\ & & \ddots & \\ \rho^{L-1} & \rho^{L-2} & & 1 \end{bmatrix} \text{ for all } p$$

The power delay profile is given by

$$\{\sigma_{h,p}^2\} = \{0.1540, 0.1447, 0.1359, 0.1277, 0.1199, 0.1127, 0.1058, 0.0994\}.$$

Simulation results (cont'd)

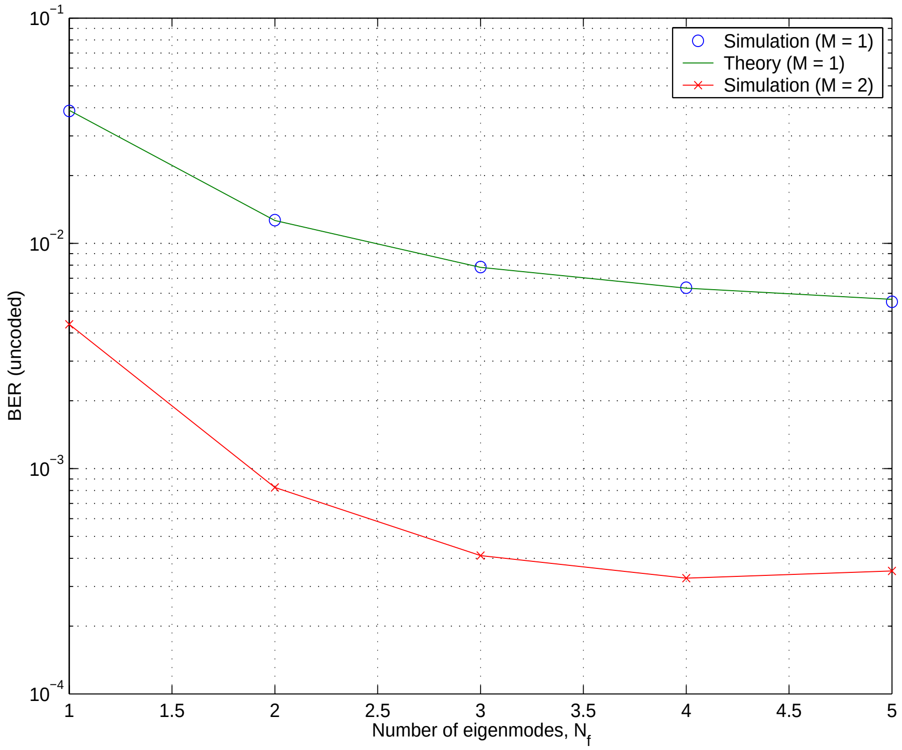
BER versus SNR ($\rho = 0.5, N_f = 2$)



A better diversity gain can be achieved when $M = 2$.

Simulation results (cont'd)

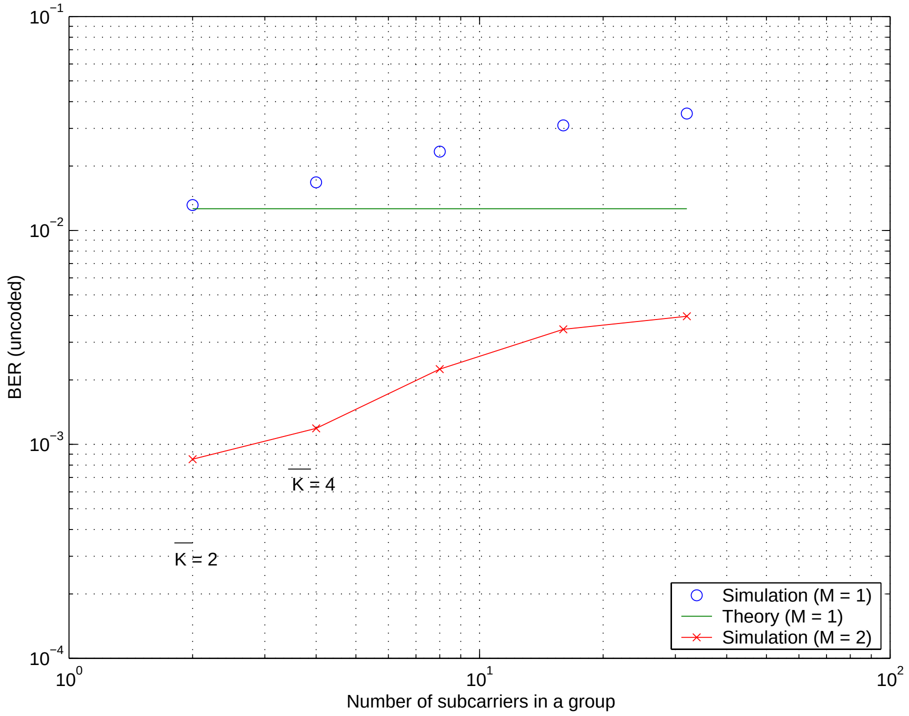
BER versus N_f $(\rho = 0.5, SNR = 4dB)$



Since the channel is spatially correlated, the performance improvement by increasing N_f is saturated.

Simulation results (cont'd)

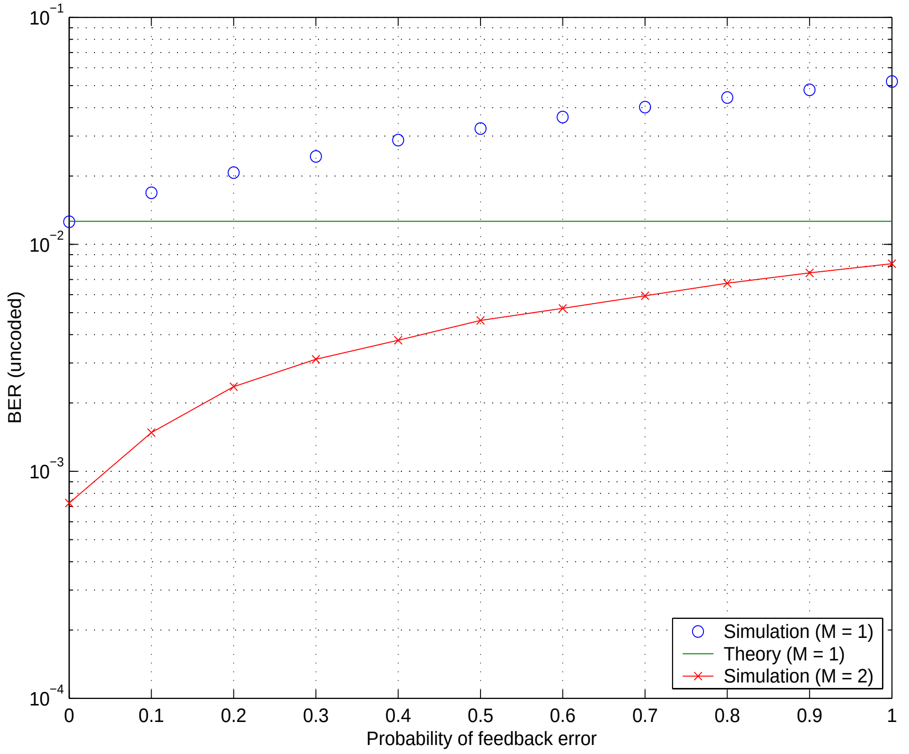
BER versus $\bar{K}$ $(\rho = 0.5, SNR = 4dB)$



The case of $\bar{K} = 2$ introduces an insignificant performance degradation, while the number of bits for the feedback becomes a half.

Simulation results (cont'd)

BER versus feedback error ($\rho = 0.5, SNR = 4dB$)



The eigenbeamforming exhibits a robustness against the feedback error of the short-term selection.

Simulation results (cont'd)

The 3GPP spatial channel model (SCM) for simulations is used.

The channel vector from a transmit antenna array to a receive antenna is given by

$$\mathbf{v}_m(t) = \sum_{c=1}^{C} \sum_{q=1}^{Q} \alpha_{q,c} \mathbf{a}(\theta_c + \tilde{\theta}_{c,q}),$$

where

$\mathbf{a}(\theta)$:	the array response vector of the transmit antenna array
θ_c:	the center angle of departure (AOD) of the cth cluster
$\tilde{\theta}_{c,q}$:	the qth perturbed AOD from the center AOD, θ_c
$\alpha_{q,c}$:	the fading coefficient of the signal departing to the angle of $\theta_c + \tilde{\theta}_{c,q}$

The resulting fading channel matrix becomes

$$\mathbf{H}(t) = [\mathbf{v}_1(t)\ \mathbf{v}_2(t)\ \cdots\ \mathbf{v}_M(t)]^T.$$

Simulation results (cont'd)

In the 3GPP SCM, we assume

- there are two major AOD's at 20° and 50° (i.e., $C = 2$).
- the perturbed AOD's are modeled as Laplace random variables with the angular spreads of 5° and 2° for the center AOD's 20° and 50°, respectively.
- a half-wavelength antenna spacing

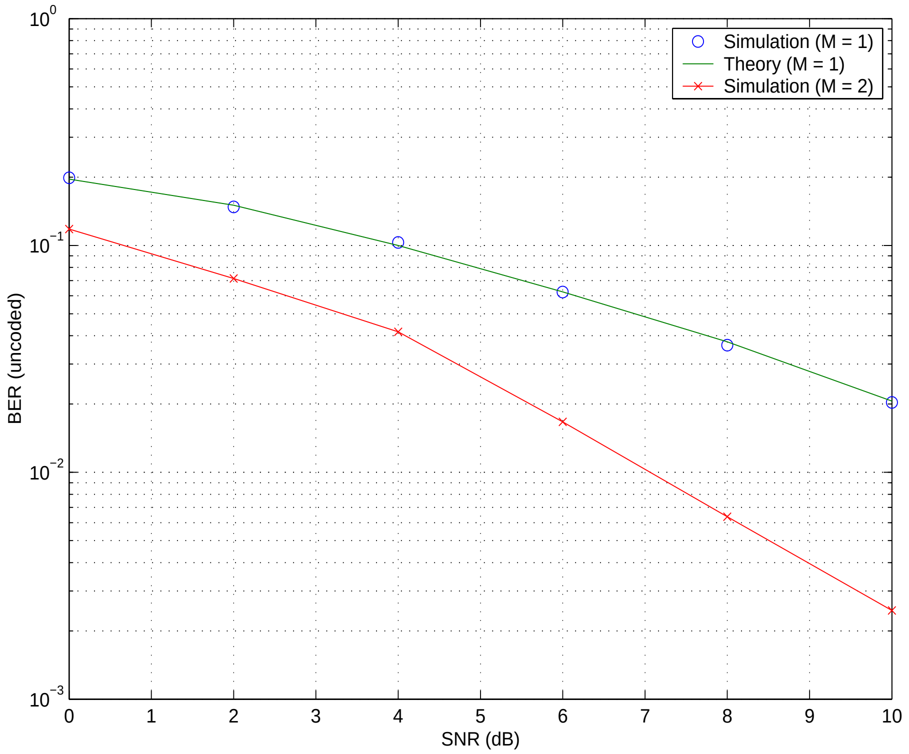
The (empirical) resulting eigenvalues of the spatial covariance matrix are given by

$$\{\lambda_m\} = \{0.6620,\ 0.5738,\ 0.0437,\ 0.0025,\ 0.0001\}.$$

It shows that there are only two dominant eigenmodes.

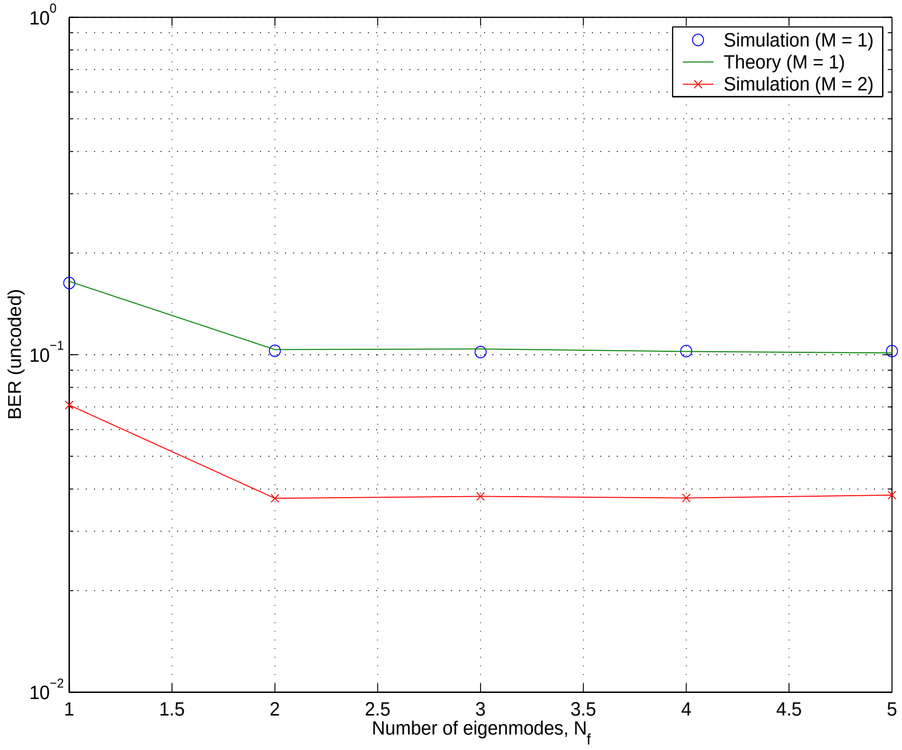
Simulation results (cont'd)

BER versus SNR for SCM ($N_f = 2$)



Simulation results (cont'd)

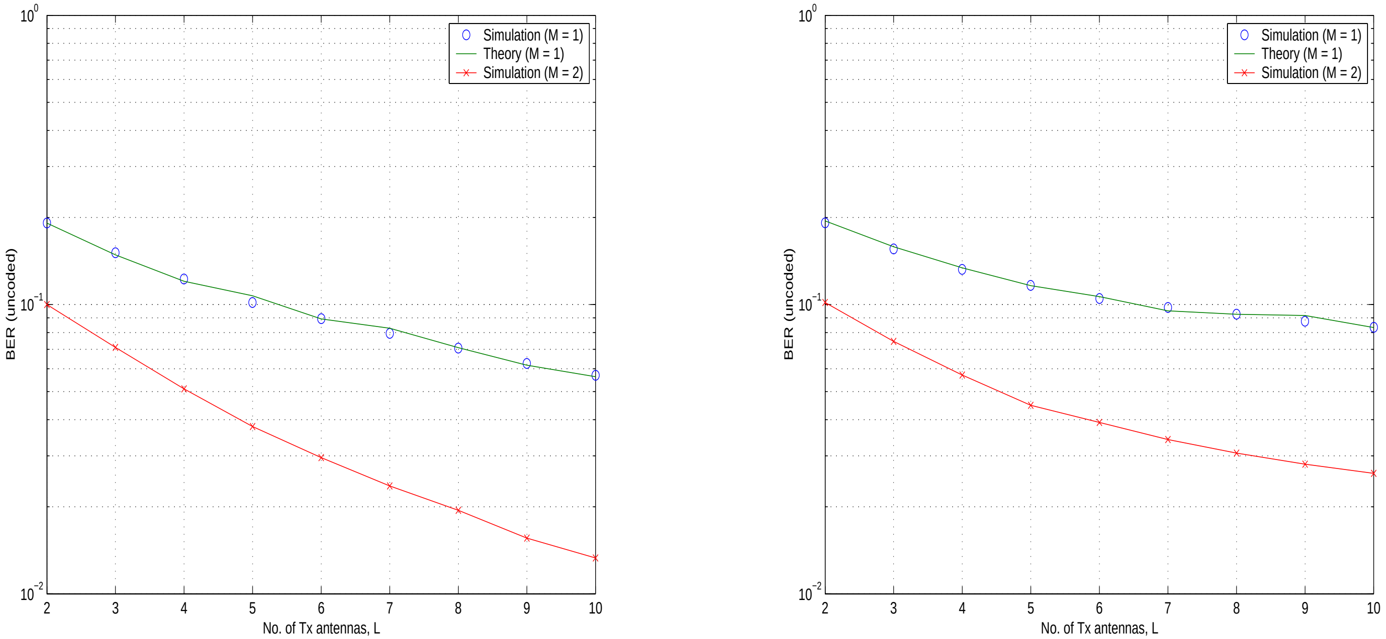
BER versus N_f for SCM $(SNR = 4dB)$



As there are only two dominant eigenmodes, $N_f = 2$ is enough for the SCM.

Simulation results (cont'd)

BER versus L for SCM ($N_f = 2$, $SNR = 4dB$)



A half-wavelength spacing 2 times wavelength spacing

For given two dominant eigenmodes, the beamforming gain should be maximized for a better performance (i.e., a half-wavelength case is better).

5. Conclusions

- Since the downlink channel is spatially correlated due to the absence of local scatters, the beamforming gain should be achieved as much as possible together with the diversity gain when the beamforming is employed for downlink.
- In this sense, the eigenbeamforming is very attractive as it can provide both diversity gain and beamforming gain with a small amount of feedback.
- We can analytically show the beamforming gain and diversity gain of the eigenbeamforming as follows:

$$P_b \simeq G \underbrace{\left(\prod_{m=1}^{N_f} \lambda_m \right)^{-1}}_{\text{beamforming gain}} \times \underbrace{\gamma^{-N_f}}_{\text{diversity gain}}, \quad \gamma \gg 0,$$

- Through simulations, we can observe that the eigenbeamforming is robust against the feedback error of the short-term eigenmode selection.
- It is also shown that the eigenbeamforming is quite suitable for OFDM: Since a common set of pre-selected eigenmodes is shared by all the subcarriers, the amount of feedback for the set of eigenmodes can be greatly reduced.