On Differential Space-Time Modulation across MIMO Radio Channels

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Presentation Outline

- Brief overview of Differential Space-Time Modulation
- Non-coherent/Differential Diagonal Unitary Space-Time Modulation (DUSTM)
  - Improved DUSTM design criteria
  - Design of DUST superset modulation (DUSTSM)
  - Faster Non-Coherent Detection methods
  - Fast fading limitations of DUSTM

Brief review of space-time coding across multiple-input multiple-output (MIMO) Radio Channels

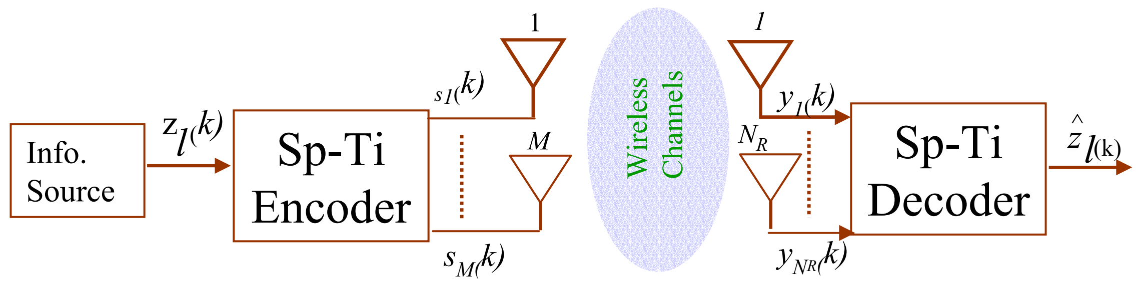


Fig 1.1(a) Block Diagram of typical space-time coded transmission and reception across a MIMO radio channel

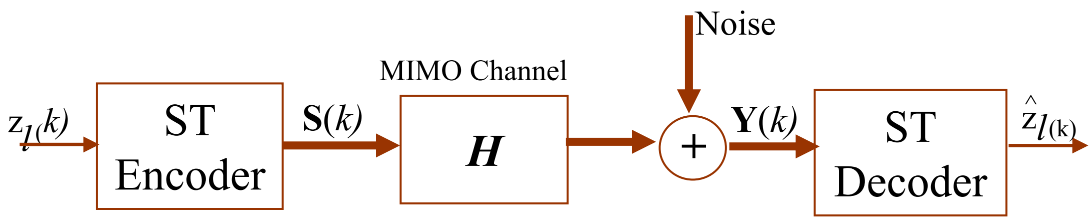


Fig 1.1(b) Conceptual representation of space-time modulation system across a MIMO Channel

Space-Time Codes

- Coherent ST coding
  - Layered Codes [Foschini et al.'96, Foschini '99]
  - Trellis Codes [Tarokh et al.'98, Grimm et al.'98]
  - Block Codes [Alamouti'98, Tarokh et al.'99]
  - Linear Dispersion Codes[Hassibi-Hochwald'00]
  - Constellation Rotation Codes [DaSilva et al'97, Xin-Wang-GG'00]
- Non-Coherent ST Coding => no channel estimation
  - Differential diagonal unitary ST modulation (DUSTM) [Hughes'00,Hochwald et al.'00].
  - Cayley Differential Unitary ST Modulation [Hassibi-Hochwald'02]
  - Trellis-Coded DUSTM [Tao-Cheng'03 etc.]
  - Differential ST-BC[Tarokh-Jafarkhani'00,Shao-Yuan'03]
  - Differential Space-Time Turbo Codes [Schlegel-Grant'03]

More on Various Differential Space-Time Coding Schemes

- Cayley Differential Unitary Space-Time Turbo Codes
  - Claimed efficient encoding & decoding at any R_D – Cayley transform
  - Not inherent simplicity of diagonal DUSTM
  - Claimed at high data rates structure emulates the capacity-achieving input distribution (?) → ie tends to case for equalisation
  - Decoded using sphere decoding, successive nulling and cancelling
- General non-coherent PSK constellation [Tarokh]
  - Simplicity, but capacity achieving of DUSTM at higher R_D.
- Differential Space-Time Block Codes
  - Best? Mixture of block [Alamouti (coherent)], and trellis [Tarokh (coherent) using DUSTM (or PSK codes effectively)
- Differential Space-Time Turbo Codes
  - Serial concatenation of simple error-control codes and differential space-time modulation [pseudo-differential ?].

Non-coherent capacity of MIMO Channels

- Block flat Rayleigh fading channels
  - $h_{ij} \sim \mathcal{CN}(0,1)$: constant over T symbol slots - block
  - h_{ij} independent from block to block
    - if piecewise constant ==>
    - let $\zeta = \min\{M, N_R, \lfloor T/2 \rfloor\}$, then [Zheng-Tse'00]

$$C \approx \zeta\left(1 - \frac{\zeta}{T}\right)\log\rho, \qquad \text{for } \rho >> 1$$

    - not directly applicable to continuous fading model, but a good guideline
- For coherent and non-coherent MIMO channels
    - capacity optimum when $M = N_R$, high SNR

Differential ST modulation

- Coherent space-time modulation channel estimation challenging
  - Wireless Channel may vary rapidly
    - » receiver does not have enough time to learn fading coefficients
  - Excessive Overhead
    - » many channels need to be estimated in MIMO systems
- Differential Unitary Space-Time Modulation
  - Model,

$$\mathbf{R}(k) = \sqrt{\rho}\,\mathbf{S}(k)\mathbf{H} + \mathrm{AWGN}(k)$$

Differential Diagonal Unitary Space-Time Modulation (DUSTM)

- Signals, S_τ transmitted grouped in time blocks
  - size M, τ used to index the time blocks

    $$R_\tau = \sqrt{\rho} S_\tau H_\tau + V_\tau \quad \text{for } \tau = 0, 1, \ldots \tau_k$$

  - H contains i.i.d. fading coefficients
  - V_τ, additive independent receiver noise
  - integer data sequence, $z_1 \ldots z_\tau \in \{0, 1, \ldots, L-1\}$

    $$S_\tau = G_{z_\tau} S_{\tau-1}, \tau = 1, 2 \ldots, \text{with } S_0 = I_M$$

  - L signals in constellation $\mathcal{G}$, $L = 2^\wedge(R_D M)$
  - Code matrices G_l diagonal, zeros off diagonal, commutative cyclic group construction

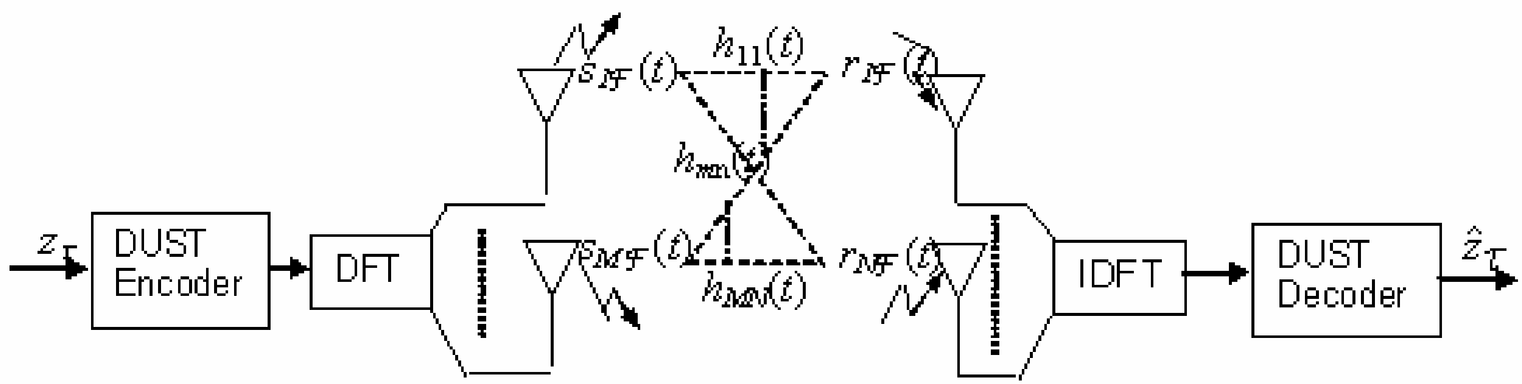
    $$G_l = G_1{}^l, \text{where } G_1 = \text{diag}\left[e^{j2\pi u_1/L}, e^{j2\pi u_2/L} \cdots e^{j2\pi u_M/L}\right], 0 \le l < L$$

  - Full transmitter diversity
    - » because $\mathcal{G}$ forms a group, every S_τ belongs to $\mathcal{G}$

→ Only one antenna transmits a phase-shift keying (PSK) symbol at any time

Differential diagonal unitary space-time modulation scheme across Rayleigh flat fading MIMO wireless channel, incorporating DFT to spread PSK symbols across multiple antennas (reducing power requirements)

Possible due to phase invariance when applying a unitary transform (DFT) at transmitter and correspondingly at the receiver



Improved DUSTM Design Criteria

- u_m in G_l code matrix are integers between 1 and L-1 and the constellation is thus entirely defined by $[u_1,\ldots,u_M]$.
- Quality of the constellation $\mathcal{G}$ is defined by its diversity product.

$$\zeta = \frac{1}{2}\min_{0\le l<l'<L}\left|\det\left(G_l - G_{l'}\right)\right|^{\frac{1}{M}}$$

$$\Longrightarrow \quad \zeta = \min_{l\in\{1,\ldots,L-1\}}\left|\prod_{m=1}^{M}\sin\left(\pi u_m l/L\right)\right|^{1/M}$$

- Stated DUSTM design criteria, optimal $[u_1,\ldots,u_M]$ must contain 1
  - Incorrect, in fact a set of optimal $[u_1,\ldots,u_M]$ for a given data rate, R_D, and number of transmit antennas, M, in which every relative prime to L is listed at each once for (1,….,L/2-1), i.e. every odd number, for DUSTM constellations
    - Number of optimal cyclic group codes, $N_{\mathcal{G}} \ge \left\lceil \frac{2^{R_D M}}{4M} \right\rceil$
      - » Eg for Data Rate 2, M = 6, $N_{\mathcal{G}} \ge 171$

Improved DUSTM Design

- Non-exhaustive set, specified by $\{u_1, u_2, u_3, u_4\}$ of optimal constellations for $R_D = 2$ bits/channel use and $M = 4$ transmit antennas, corresponding to $L = 256$; where every relative prime to L, from 1,…, $L/2$-1 is listed at least once, the corresponding maximized diversity product, ζ, for each constellation is 0.2208.

$[u_1, u_2, u_3, u_4]$				$[u_1, u_2, u_3, u_4]$			
1	35	41	119	17	59	109	115
1	67	99	117	21	33	61	93
1	71	75	95	21	39	45	53
3	29	31	43	23	37	79	81
3	35	65	75	23	63	73	99
3	101	105	123	25	113	115	121
5	37	99	119	27	59	73	93
5	51	81	83	29	55	65	105
7	11	31	65	33	39	63	85
7	13	15	103	33	53	57	127
7	19	81	89	33	73	87	125
9	29	91	123	35	67	95	107
11	29	61	127	39	49	57	77
13	45	49	101	45	47	77	123
15	19	37	51	47	49	91	105
15	41	101	111	49	55	111	123
15	69	81	119	75	83	89	107
17	27	87	113	85	97	99	125

DUSTSM (DUST Superset Modulation)

- Is the diversity product, ζ, a sufficient measure of the quality of DUSTM
  - Can effective "multicyclic" constructions be used.
- DUSTSM constellation spectral efficiency achieving supersets
  - $M_{\text{sup}} = M + M_s$, $L_{sup} = 2\text{^}(R_D M_{sup}) = L \times 2\text{^}(R_D M_s)$
  - Use $\mathcal{G}$ size L, and effectively create a "multicylic" construction.
    - Multiplying $\mathcal{G}$ by $2\text{^}(R_D M_s)$code vectors corresponding to group size to create $\mathcal{G}_{\mathbf{sup}}$
    - Zero diversity product with good BER performance across a continuous Rayleigh flat fading MIMO channel ?

DUSTSM Transmission

- Consider simplest superset $M_s = M + 1$
  - For reduced spectral efficiency, where R_D is scaled by $M/(M+1)$

$$\Longrightarrow S_\tau^{\text{sup}} = \text{diag}\left[\mathbf{d}\left(S_\tau\right), s_{m,m}\right]$$

  - $\mathbf{U} = [U_1,\ldots, U_T]$ be the set of all optimal $U_t = [u_{1,t},\ldots,u_{M,t}]$, choose $u_{m'}$ from $[u_{1,1}, \ldots u_{M,1}]$, which when retransmitted maximises

$$\Longrightarrow \zeta = \min_{l \in \{1,\ldots,L-1\}} \left| \prod_{m=1}^{M} \sin\left(\pi u_m l/L\right) \right|^{1/M}$$

  - For each $k = 1, 2\ldots,2^{R_D}$ $u_k \in U_t$, where $U_t \in \mathbf{U}$, to reuse and form an effective group U_{supk} of the form $[u_{1,t},\ldots,u_{M,t}, u_{m',t}]$

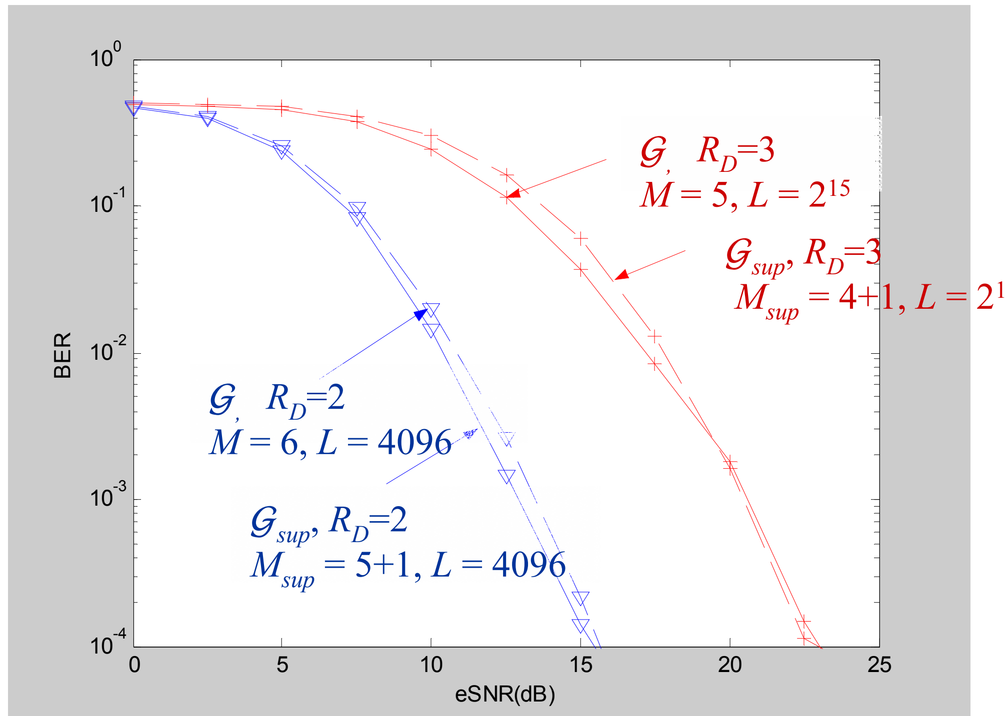
$$G_{l_{sup}} = \mathbf{P}_K\left(G_k'\right).G_1^{\,l}, \ \left\{\mathbf{P}_1\left(G_k'\right).G_1^{\,l}, \mathbf{P}_2\left(G_k'\right).G_1^{\,l} \ldots \mathbf{P}_{2^{R_D}}\left(G_k'\right).G_1^{\,l}\right\} \in \mathcal{G}_{sup},$$

$$G_1 = \text{diag}\left(\left[e^{j2\pi u_{1,1}/L},\ldots,e^{j2\pi u_{M,1}/L},e^{j2\pi u_{m',1}/L}\right]\right), \quad 0 \le l < L$$

$$\text{and } G_k' = \text{diag}\left(\left[e^{j2\pi u_{1,t}/L},e^{j2\pi u_{2,t}/L}\ldots e^{j2\pi u_{M,t}/L},e^{j2\pi u_{k,t}/L}\right]\right), \quad 1 \le k \le 2^{R_D}$$

Results of DUSTSM Implementation

- BER performance comparison for DUSTSM, constellations represented by $\mathcal{G}_{sup}$, with DUSTM, $\mathcal{G}$, using ML decoding, for $R_D = 2, 3$ bits/channel use in a Rayleigh flat fading MIMO channel with a Doppler fading parameter $f_D T_s = 0.0025$. $N_R = 2$, eSNR is the expected signal-to-noise ratio at the receiver and the BER was derived using Gray labelling for the constellations. The dashed lines represent DUSTM performance.



Fast Differential Decoding of MIMO Channels

- Fast Lattice Decoding
  - "LLL" algorithm [Lenstra et al.'82] a polynomial-time approximation, using basis reduction, to find the approximate decoder answer
    - Successfully decreased the level of approximation when using multiple receive antennas from [Clarkson et al. '01].
- Fast Block pseudo-DPSK (FBp-DPSK) decoding
  - No basis reduction required, using mathematical algorithm derived from lattice technique
    - Can check a subset of points using ML decoding around decoder answer for more efficiency
- Optimal subset of transmit antennas can be used

Fast Lattice Decoding across MIMO Radio Channels

- Improving Decoding Efficiency
  - Decoder more approximate as number of dimensions increases
    - Able to utilise the cyclic group nature of the constellations and the block matrix representation of the basis vectors
      - Smaller Lattice s.t. basis vectors for M' and N_R', where $\max(M') = \max(N_R') = 3$.
        - » *Faster and more accurate*
      - Still can achieve diversity gains by optimising choice of transmit and receive antennas
  - Improve decoder estimate by checking subset (cyclotomic cosets ~ an approximate partition) of points around $\hat{z}$ to find, typically $\sim L/20 \rightarrow L/16$
      - Use ML decoding to find $\hat{z}^{\text{dec}}$

Fast Block pseudo-DPSK decoding

- Derivation from basis representation for lattice procedure

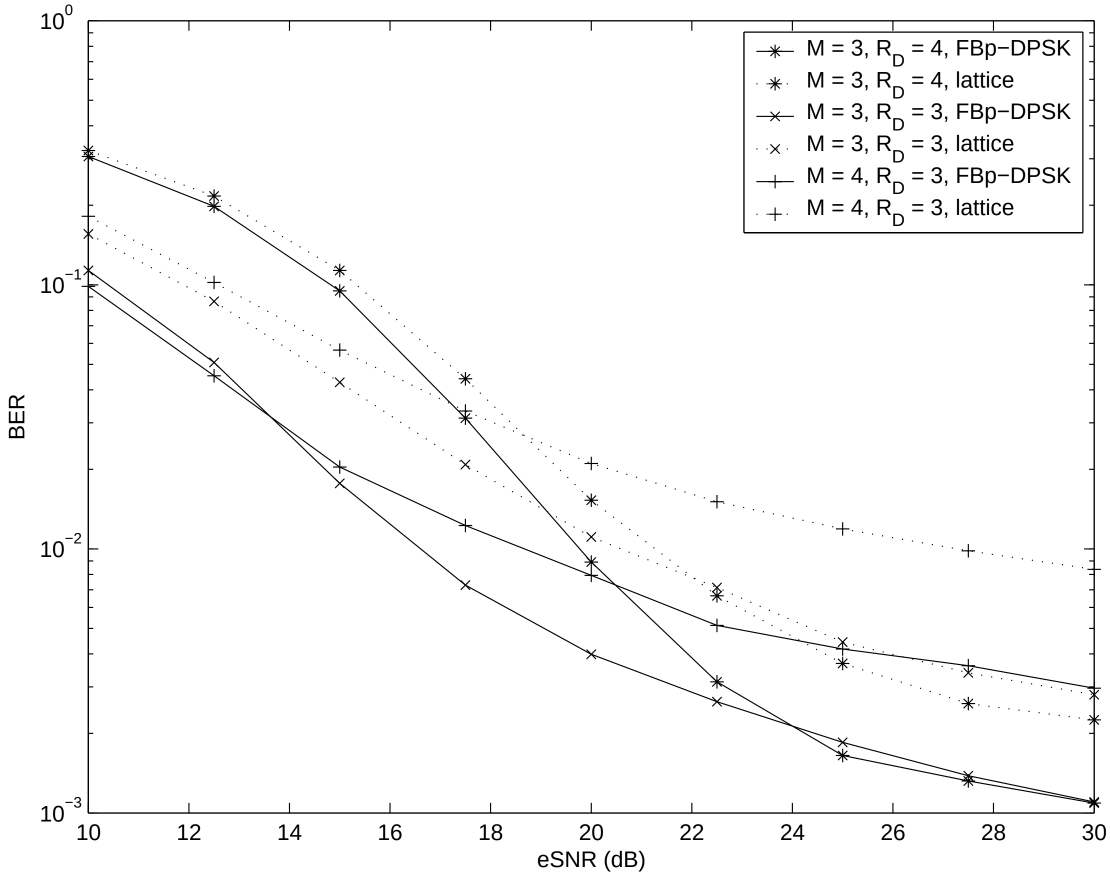
$$\hat{z}^{\mathrm{fb}} = \left\lceil \sum_{n_r=1}^{N_R} \left(\varphi_{1,n_r} \Big/ N_R - \sum_{m=2}^{M} A_{m,n_r} \varphi_{m,n_r} \Big/ A_{1,n_r} L \right) \mathrm{mod}^* (L / N_R) \right\rfloor \mathrm{mod}\, L$$

- Can use another estimate also (based on block before last)

$$\hat{x}_p^{\mathrm{fb}} = \left\lceil \sum_{n_r=1}^{N_R} \left(\varphi_{p;1,n_r} \Big/ N_R - \sum_{m=2}^{M} A_{p;m,n_r} \varphi_{p;m,n_r} \Big/ L \right) \mathrm{mod}^* (L / N_R) \right\rfloor$$

$$\hat{z}_2^{\mathrm{fb}} = \left(\hat{x}_p^{\mathrm{fb}} - \hat{z}_p^{\mathrm{dec}} \right) \mathrm{mod}\, L$$

FBp-DPSK decoding and lattice decoding comparison



MIMO radio channel of varying size. The MIMO channel was simulated for continuous flat fading for ULAs at the BS, with $d_{sp} = 10\,\lambda$, $f_D Ts = 0.01$ and $N_R = 3$. The dotted lines show the results for fast lattice decoding.

Performance limitations of DUSTM over rapidly flat fading wireless channels

- Trend of performance degradation due to time selectivity in a Rayleigh fast flat fading wireless scenario
- Characterised by

$$R_{c_m c_m}(\tau)' = -J_1(z), \quad \text{where} \quad z = \mathbf{2}\pi f_{\boldsymbol{max}} T_{\boldsymbol{s}} \tau$$

$$R_{c_m c_m}(\tau)'' = J_1(z)/z - J_0(z),$$ Bessel functions of the first kind

- Bounds for degradation set for DUSTM can be set at the length of two concurrent blocks of received signals R_τ of length $2M$, where $\tau = 2M$ received time samples, or symbols

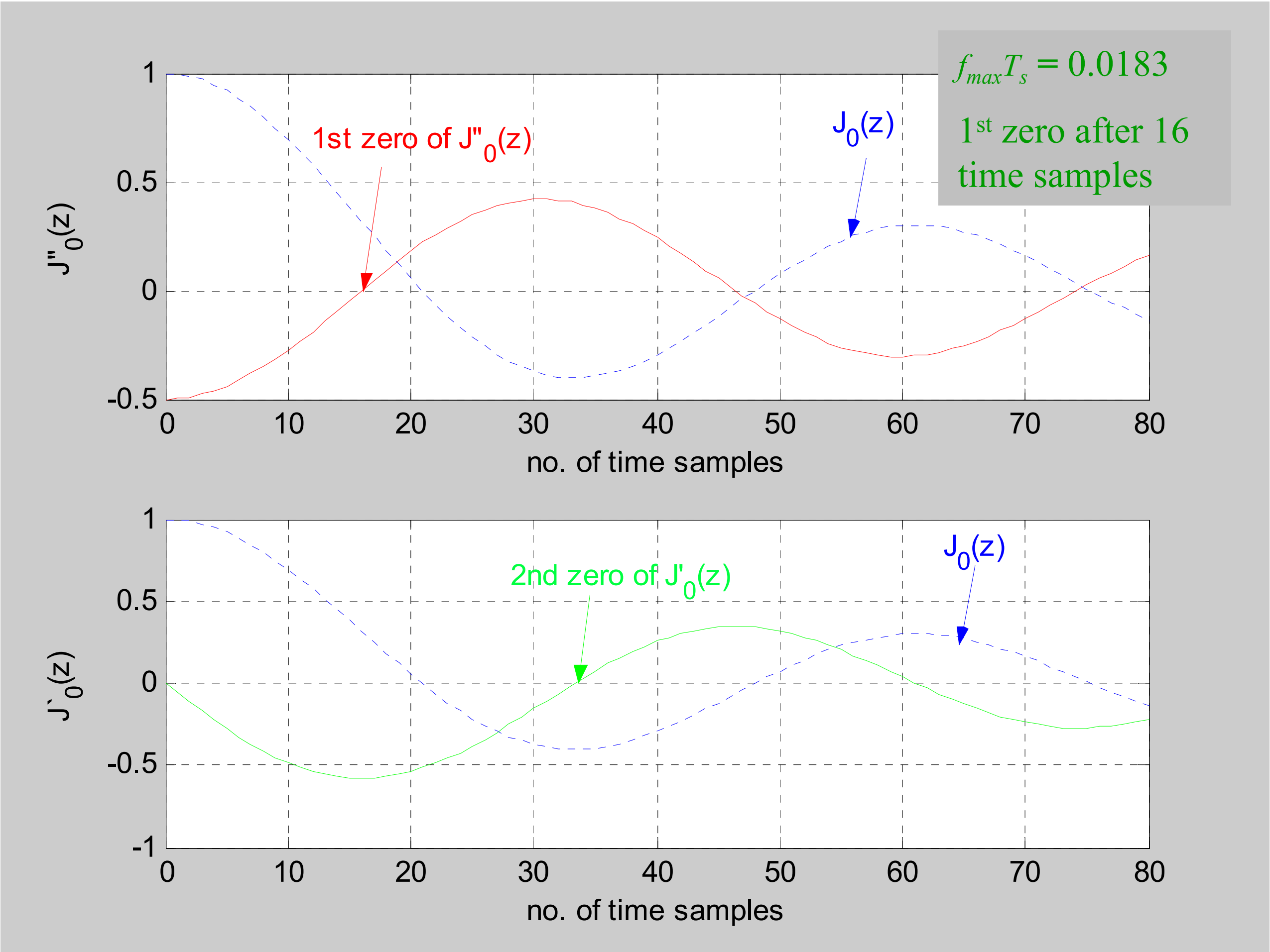
MIMO Radio Channel Model

- Derivation of Space-Time Cross Correlation function for an (M, N_R) channel
  - Starting from generalized flat fading channel distortion equation model on previous slide
    - » $\left|R_{c_m c_{m'}}\left(\tau, d_{sp}\right)\right|, \ \arg\left(R_{c_m c_{m'}}\left(\tau, d_{sp}\right)\right)$
  - Requires approximation of transmit antenna configuration centre-point
- From $R_{c_m c_{m'}}\left(\tau, d_{sp}\right)$ obtain transmission correlation function matrix and hence channel transfer matrix

$$\hat{R}_M\left(\tau, d_{sp}\right) = \begin{bmatrix} J_0(2\pi f_D \tau) & R_{c_1 c_2}\left(\tau, d_{sp}\right) & \cdots & R_{c_1 c_M} \\ R_{c_2 c_1}\left(\tau, d_{sp}\right) & \ddots & \ddots & \vdots \\ \vdots & \ddots & J_0(2\pi f_D \tau) & R_{c_{M-1} c_M} \\ R_{c_M c_1} & \cdots & R_{c_M c_{M-1}} & J_0(2\pi f_D \tau) \end{bmatrix} \rightarrow$$

$$\mathbf{H}_{M,N_R}(\tau) = \frac{\hat{R}_M\left(\tau, d_{sp}\right)^{1/2}}{\sigma} A_{M,N_R}$$

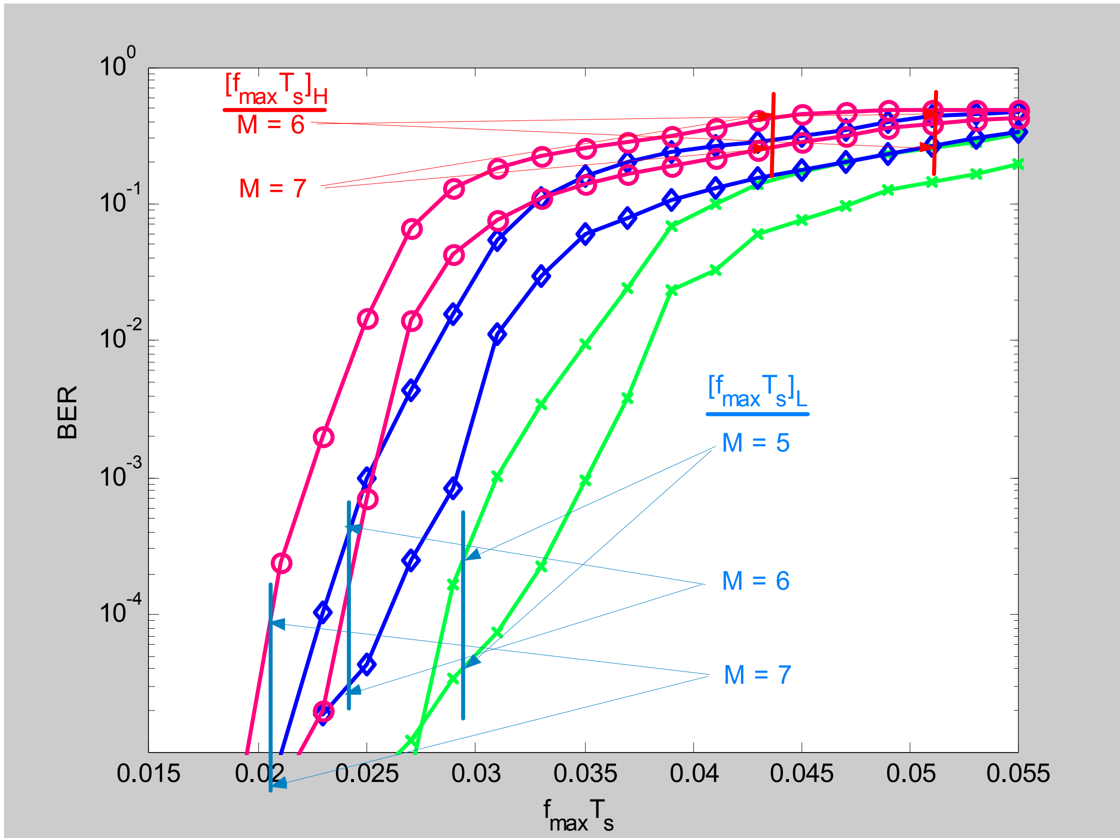
First and second order differential of Jakes autocorrelation function, $J_0(z)$, $z = 2\pi f_{max} T_s \tau$.



Performance Degradation of DUSTM for $M = 3 \ldots 8$ transmit antennas

M - Number of Transmit Antennas	$[f_{max}T_s]_L$ - Lower BER Performance Degradation Bound	$[f_{max}T_s]_H$ - Upper Degradation Bound
3	0.0488	
4	0.0366	
5	0.0292	
6	0.0244	0.0508
7	0.0209	0.0436
8	0.0183	0.0381

The upper and lower bounds of performance degradation of DUSTM for $M = 3 \ldots 8$ transmit antennas, for any data rate, R_D.



Simulated trend of degradation related to increasing $f_{max}T_s$, $N_R = M$, of DUSTM; at $R_D = 1$, eSNR = 10 dB (dashed lines) and $R_D = 2$, eSNR = 20 dB respectively.

Related Miscellaneous investigation

- Methods for faster derivation of DUSTM codes for higher data rates, R_D, and larger numbers of transmit antennas, M.
  - Making derivation of larger constellations, $L = 2^{(R_D M)}$ feasible.
- Investigation of subset selection at the receiver when using DUSTM, based on simple statistical selection techniques.

Concluding Remarks

- New DUSTM Design Criteria
  - Possible solution space for DUSTM modulation
- DUST superset modulation
  - Find a generic framework for development of DUSTSM
  - General performance criteria for DUSTSM
- Fast fading limitations of DUSTM
  - Proof behind fading degradation related to autocorrelation function
- Fast decoding methods (FBp-DPSK or lattice decoding) of DUSTM
  - Methods for choosing optimal subsets of received data
- Use of G.A. optimisation on DUSTM using macroscopic MIMO radio channel model

Relevant Publications

- D. B. Smith, “An Application of a Generalised Jakes Model for MIMO Channels”, *Proc. WARS’02, http://www.ips.gov.au/IPSHosted /NCRS/wars2002/proceedings/comm-c*, NSW, Australia, Feb. 2002, pp. 1-6.
- D. B. Smith and T. A. Aubrey, “Selection of MIMO Antenna Subsets and Supersets using Differential Unitary Space-Time Modulation”, in *Proc. Australian Communications Theory Workshop, 2003*, *AusCTW’03*, Melbourne, Australia, Feb. 2003, pp. 104-107.
- D. B. Smith, “Fast Differential Unitary Space-Time Modulation Decoding for a MIMO Radio Channel,” *Wireless Personal Communications,* vol. 25, no. 4, pp. 343-349, July 2003
- D. B. Smith and T. A. Aubrey, “Performance Degradation of Differential Space-Time Modulation in Fast Frequency Flat Fading Rayleigh MIMO channels,” *Electronics Letters,* vol. 39, no. 17, pp. 1278-1280, Aug. 2003
- *D. B. Smith and T. A. Aubrey, “Differential Unitary Space-Time Superset Modulation”, *Submitted to European Transactions on Telecommunications*
- *D. B. Smith and T. A. Aubrey, “Fast Block pseudo-Differential Phase Shift Keying (FBp-DPSK) Decoding of Differential Space-Time Modulation Across Multiple Antennas”, *Submitted to Wireless Personal Communications.*

*Pending publication