On Differential Space-Time Modulation across MIMO Radio Channels

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Presentation Outline

• Brief overview of Differential Space-Time Modulation

• Non-coherent/Differential Diagonal Unitary Space-Time Modulation (DUSTM)
  – Improved DUSTM design criteria
  – Design of DUST superset modulation (DUSTSM)
  – Faster Non-Coherent Detection methods
  – Fast fading limitations of DUSTM
Brief review of space-time coding across multiple-input multiple-output (MIMO) Radio Channels

Fig 1.1(a) Block Diagram of typical space-time coded transmission and reception across a MIMO radio channel

Fig 1.1(b) Conceptual representation of space-time modulation system across a MIMO Channel
Space-Time Codes

• Coherent ST coding
  • Layered Codes [Foschini et al.’96, Foschini ’99]
  • Trellis Codes [Tarokh et al.’98, Grimm et al.’98]
  • Block Codes [Alamouti’98, Tarokh et al.’99]
  • Linear Dispersion Codes[Hassibi-Hochwald’00]
  • Constellation Rotation Codes [DaSilva et al’97, Xin-Wang-GG’00]

• Non-Coherent ST Coding => no channel estimation
  • Differential diagonal unitary ST modulation (DUSTM) [Hughes’00,Hochwald et al.’00].
  • Cayley Differential Unitary ST Modulation [Hassibi-Hochwald’02]
  • Trellis-Coded DUSTM [Tao-Cheng’03 etc.]
  • Differential ST-BC[Tarokh-Jafarkhani’00,Shao-Yuan’03]
  • Differential Space-Time Turbo Codes [Schlegel-Grant’03]
More on Various Differential Space-Time Coding Schemes

• Cayley Differential Unitary Space-Time Turbo Codes
  – Claimed efficient encoding & decoding at any $R_D$ – Cayley transform
  – Not inherent simplicity of diagonal DUSTM
  – Claimed at high data rates structure emulates the capacity-achieving input distribution (?) → ie tends to case for equalisation
  – Decoded using sphere decoding, successive nulling and cancelling

• General non-coherent PSK constellation [Tarokh]
  – Simplicity, but capacity achieving of DUSTM at higher $R_D$.

• Differential Space-Time Block Codes
  – Best? Mixture of block [Alamouti (coherent)], and trellis [Tarokh (coherent) using DUSTM (or PSK codes effectively)]

• Differential Space-Time Turbo Codes
  – Serial concatenation of simple error-control codes and differential space-time modulation [pseudo-differential ?].
Non-coherent capacity of MIMO Channels

- Block flat Rayleigh fading channels
  - $h_{ij} \sim \mathcal{CN}(0,1)$: constant over $T$ symbol slots - block
  - $h_{ij}$ independent from block to block
    - if piecewise constant $\Rightarrow$
    - let $\zeta = \min\{M,N_R,\lfloor T/2 \rfloor\}$, then [Zheng-Tse’00]
      \[
      C \approx \zeta \left(1 - \frac{\zeta}{T}\right) \log \rho, \quad \text{for } \rho \gg 1
      \]
    - not directly applicable to continuous fading model, but a good guideline

- For coherent and non-coherent MIMO channels
  - capacity optimum when $M = N_R$, high SNR
Differential ST modulation

• Coherent space-time modulation channel estimation challenging
  – Wireless Channel may vary rapidly
    » receiver does not have enough time to learn fading coefficients
  – Excessive Overhead
    » many channels need to be estimated in MIMO systems

• Differential Unitary Space-Time Modulation
  • Model,

\[
R(k) = \sqrt{\rho}S(k)H + \text{AWGN}(k)
\]
Differential Diagonal Unitary Space-Time Modulation (DUSTM)

- Signals, $S_\tau$ transmitted grouped in time blocks
  - size $M$, $\tau$ used to index the time blocks
    \[ R_\tau = \sqrt{\rho} S_\tau H_\tau + V_\tau \text{ for } \tau = 0, 1, \ldots, \tau_k \]
  - $H$ contains i.i.d. fading coefficients
  - $V_\tau$, additive independent receiver noise

- integer data sequence, $z_1, \ldots, z_\tau \in \{0, 1, \ldots, L-1\}$
  \[ S_\tau = G z_\tau S_{\tau-1}, \tau = 1, 2, \ldots, \text{ with } S_0 = I_M \]

- $L$ signals in constellation $G$, $L = 2^{(R_D M)}$

- Code matrices $G_l$ diagonal, zeros off diagonal, commutative cyclic group construction
  \[ G_l = G_1^l, \text{ where } G_1 = \text{diag}\left[e^{j2\pi l_1/L}, e^{j2\pi l_2/L}, \ldots, e^{j2\pi l_M/L}\right], 0 \leq l < L \]

- Full transmitter diversity
  » because $G$ forms a group, every $S_\tau$ belongs to $G$

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Only one antenna transmits a phase-shift keying (PSK) symbol at any time
Differential diagonal unitary space-time modulation scheme across Rayleigh flat fading MIMO wireless channel, incorporating DFT to spread PSK symbols across multiple antennas (reducing power requirements)
Possible due to phase invariance when applying a unitary transform (DFT) at transmitter and correspondingly at the receiver
Improved DUSTM Design Criteria

• \( u_m \) in \( G_l \) code matrix are integers between 1 and \( L-1 \) and the constellation is thus entirely defined by \([u_1, \ldots, u_M]\).

• Quality of the constellation \( \mathcal{G} \) is defined by its diversity product.

\[
\zeta = \frac{1}{2} \min_{0 \leq l < l' < L} \left| \det (G_l - G_{l'}) \right|^{1/M}
\]

\[
\zeta = \min_{l \in \{1, \ldots, L-1\}} \left| \prod_{m=1}^{M} \sin \left( \frac{\pi u_m l}{L} \right) \right|^{1/M}
\]

• Stated DUSTM design criteria, optimal \([u_1, \ldots, u_M]\) must contain 1
  – Incorrect, in fact a set of optimal \([u_1, \ldots, u_M]\) for a given data rate, \( R_D \), and number of transmit antennas, \( M \), in which every relative prime to \( L \) is listed at each once for \((1, \ldots, L/2-1)\), i.e. every odd number, for DUSTM constellations
  – Number of optimal cyclic group codes, \( N_{\mathcal{G}} \geq \left\lceil \frac{2^{R_D M}}{4M} \right\rceil \)

  » Eg for Data Rate 2, \( M = 6 \)

\[ N_{\mathcal{G}} \geq 171 \]
Improved DUSTM Design

- Non-exhaustive set, specified by \{u_1, u_2, u_3, u_4\} of optimal constellations for \(R_D = 2\) bits/channel use and \(M = 4\) transmit antennas, corresponding to \(L = 256\); where every relative prime to \(L\), from 1,…, \(L/2-1\) is listed at least once, the corresponding maximized diversity product, \(\zeta\), for each constellation is 0.2208.

| \(|u_1|, u_2, u_3, u_4\) | \(|u_1|, u_2, u_3, u_4\) |
|---|---|
| 1  | 35 | 41 | 119 | 17 | 59 | 109 | 115 |
| 1  | 67 | 99 | 117 | 21 | 33 | 61 | 93 |
| 1  | 71 | 75 | 95 | 21 | 39 | 45 | 53 |
| 3  | 29 | 31 | 43 | 23 | 37 | 79 | 81 |
| 3  | 35 | 65 | 75 | 23 | 63 | 73 | 99 |
| 3  | 101 | 105 | 123 | 25 | 113 | 115 | 121 |
| 5  | 37 | 99 | 119 | 27 | 59 | 73 | 93 |
| 5  | 51 | 81 | 83 | 29 | 55 | 65 | 105 |
| 7  | 11 | 31 | 65 | 33 | 39 | 63 | 85 |
| 7  | 13 | 15 | 103 | 33 | 53 | 57 | 127 |
| 7  | 19 | 81 | 89 | 33 | 73 | 87 | 125 |
| 9  | 29 | 91 | 123 | 35 | 67 | 95 | 107 |
| 11 | 29 | 61 | 127 | 39 | 49 | 57 | 77 |
| 13 | 45 | 49 | 101 | 45 | 47 | 77 | 123 |
| 15 | 19 | 37 | 51 | 47 | 49 | 91 | 105 |
| 15 | 41 | 101 | 111 | 49 | 55 | 111 | 123 |
| 15 | 69 | 81 | 119 | 75 | 83 | 89 | 107 |
| 17 | 27 | 87 | 113 | 85 | 97 | 99 | 125 |
DUSTSM (DUST Superset Modulation)

• Is the diversity product, $\zeta$, a sufficient measure of the quality of DUSTM
  – Can effective “multicyclic” constructions be used.

• DUSTSM constellation spectral efficiency achieving supersets
  - $M_{sup} = M + M_s$, $L_{sup} = 2^{(R_D M_{sup})} = L \times 2^{(R_D M_s)}$
  - Use $G$ size $L$, and effectively create a “multicylic” construction.
    - Multiplying $G$ by $2^{(R_D M_s)}$ code vectors corresponding to group size to create $G_{sup}$
    - Zero diversity product with good BER performance across a continuous Rayleigh flat fading MIMO channel?
DUSTSM Transmission

• Consider simplest superset $M_s = M + 1$

  • For reduced spectral efficiency, where $R_D$ is scaled by $M/(M+1)$
    \[ S_{\text{sup}} = \text{diag}\left[ d\left(S_t\right), s_{m,m}\right] \]

  • $U = [U_1, ..., U_T]$ be the set of all optimal $U_t = [u_{1,t}, ..., u_{M,t}]$, choose $u_{m'}$ from $[u_{1,1}, ..., u_{M,1}]$, which when retransmitted maximises
    \[ \zeta = \min_{l \in \{1, ..., L-1\}} \prod_{m=1}^{M} \sin(\pi u_m l/L) \]

  • For each $k = 1, 2, ..., 2^{R_0}$ $u_k \in U_t$, where $U_t \in U$, to reuse and form an effective group $U_{\text{supk}}$ of the form $[u_{1,t}, ..., u_{M,t}, u_{m',t}]$
    \[ G_{l,\text{sup}} = P_k \left(G'_k\right) G_1^l, \left\{ P_1 \left(G'_1\right) G_1^l, P_2 \left(G'_2\right) G_1^l, ..., P_{2^{R_0}} \left(G'_k\right) G_1^l \right\} \in G_{\text{sup}}, \]

    \[ G_1 = \text{diag}\left[ e^{j2\pi u_{1,1}/L}, ..., e^{j2\pi u_{M,1}/L}, e^{j2\pi u_{m,1}/L} \right], \quad 0 \leq l < L \]

    \[ G_k' = \text{diag}\left[ e^{j2\pi u_{1,1}/L}, e^{j2\pi u_{2,1}/L}, ..., e^{j2\pi u_{M,1}/L}, e^{j2\pi u_{m',1}/L} \right], \quad 1 \leq k \leq 2^{R_0} \]
Results of DUSTSM Implementation

- BER performance comparison for DUSTSM, constellations represented by $G_{sup}$, with DUSTM, $G$, using ML decoding, for $R_D = 2, 3$ bits/channel use in a Rayleigh flat fading MIMO channel with a Doppler fading parameter $f_D T_s = 0.0025$. $N_R = 2$, eSNR is the expected signal-to-noise ratio at the receiver and the BER was derived using Gray labelling for the constellations. The dashed lines represent DUSTM performance.
Fast Differential Decoding of MIMO Channels

• Fast Lattice Decoding
  – “LLL” algorithm [Lenstra et al.’82] a polynomial-time approximation, using basis reduction, to find the approximate decoder answer
    • Successfully decreased the level of approximation when using multiple receive antennas from [Clarkson et al. ’01].

• Fast Block pseudo-DPSK (FBp-DPSK) decoding
  – No basis reduction required, using mathematical algorithm derived from lattice technique
    • Can check a subset of points using ML decoding around decoder answer for more efficiency

• Optimal subset of transmit antennas can be used
Fast Lattice Decoding across MIMO Radio Channels

• Improving Decoding Efficiency
  – Decoder more approximate as number of dimensions increases
  • Able to utilise the cyclic group nature of the constellations and the block matrix representation of the basis vectors
    – Smaller Lattice s.t. basis vectors for $M'$ and $N_R'$, where $\max(M') = \max(N_R') = 3$.
      » Faster and more accurate
    – Still can achieve diversity gains by optimising choice of transmit and receive antennas
      – Improve decoder estimate by checking subset (cyclotomic cosets ~ an approximate partition) of points around $\hat{z}$ to find, typically $\sim L/20 \rightarrow L/16$
        – Use ML decoding to find $\hat{z}^{\text{dec}}$
Fast Block pseudo-DPSK decoding

- Derivation from basis representation for lattice procedure

\[
\hat{z}_{fb} = \left[ \sum_{n_r=1}^{N_R} \phi_{1,n_r} \mod* \left( \frac{L}{N_R} \right) \right] \mod L
\]

- Can use another estimate also (based on block before last)

\[
\hat{x}_{p}^{fb} = \left[ \sum_{n_r=1}^{N_R} \phi_{p;1,n_r} \mod* \left( \frac{L}{N_R} \right) \right] \mod L
\]

\[
\hat{z}_{fb} = \left( \hat{x}_{p}^{fb} - \hat{z}_{p}^{dec} \right) \mod L
\]
FBp-DPSK decoding and lattice decoding comparison

MIMO radio channel of varying size. The MIMO channel was simulated for continuous flat fading for ULAs at the BS, with

\[ d_{sp} = 10 \lambda , \]
\[ f_D T_S = 0.01 \] and
\[ N_R = 3. \] The dotted lines show the results for fast lattice decoding.
Performance limitations of DUSTM over rapidly flat fading wireless channels

• Trend of performance degradation due to time selectivity in a Rayleigh fast flat fading wireless scenario

• Characterised by

\[ R_{c_m c_m} (\tau) = -J_1(z), \quad \text{where} \quad z = 2\pi f_{\text{max}} T_s \tau \]

\[ R_{c_m c_m} (\tau)'' = \frac{J_1(z)}{z} - J_0(z), \quad \text{Bessel functions of the first kind} \]

• Bounds for degradation set for DUSTM can be set at the length of two concurrent blocks of received signals \( R_\tau \) of length \( 2M \), where \( \tau = 2M \) received time samples, or symbols

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MIMO Radio Channel Model

- Derivation of Space-Time Cross Correlation function for an \((M, N_R)\) channel
  - Starting from generalized flat fading channel distortion equation model on previous slide
    \[
    |R_{c_m c_{m'}}(\tau, d_{sp})|, \arg(R_{c_m c_{m'}}(\tau, d_{sp}))
    \]
  - Requires approximation of transmit antenna configuration centre-point
- From \(R_{c_m c_{m'}}(\tau, d_{sp})\) obtain transmission correlation function matrix and hence channel transfer matrix

\[
\hat{R}_M(\tau, d_{sp}) = \begin{bmatrix}
J_0(2\pi f_D \tau) & R_{c_1 c_2}(\tau, d_{sp}) & \cdots & R_{c_{M-1} c_M}
\end{bmatrix}
\]

\[
H_{M,N_R}(\tau) = \frac{\hat{R}_M(\tau, d_{sp})^{1/2}}{\sigma} A_{M,N_R}
\]
First and second order differential of Jakes autocorrelation function, $J_0(z)$, $z = 2\pi f_{\text{max}} T_s \tau$. 

\[ f_{\text{max}} T_s = 0.0183 \]

1st zero after 16 time samples
Performance Degradation of DUSTM for $M = 3\ldots8$ transmit antennas

<table>
<thead>
<tr>
<th>$M$ - Number of Transmit Antennas</th>
<th>$[f_{\text{max}} T_s]_L$ - Lower BER Performance Degradation Bound</th>
<th>$[f_{\text{max}} T_s]_H$ - Upper Degradation Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.0488</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.0366</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.0292</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.0244</td>
<td>0.0508</td>
</tr>
<tr>
<td>7</td>
<td>0.0209</td>
<td>0.0436</td>
</tr>
<tr>
<td>8</td>
<td>0.0183</td>
<td>0.0381</td>
</tr>
</tbody>
</table>

The upper and lower bounds of performance degradation of DUSTM for $M = 3\ldots8$ transmit antennas, for any data rate, $R_D$.

Simulated trend of degradation related to increasing $f_{\text{max}} T_s$, $N_R = M$, of DUSTM; at $R_D = 1$, eSNR = 10 dB (dashed lines) and $R_D = 2$, eSNR = 20 dB respectively.

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Related Miscellaneous investigation

• Methods for faster derivation of DUSTM codes for higher data rates, $R_D$, and larger numbers of transmit antennas, $M$.
  – Making derivation of larger constellations, $L = 2^{(R_D M)}$ feasible.

• Investigation of subset selection at the receiver when using DUSTM, based on simple statistical selection techniques.
Concluding Remarks

• New DUSTM Design Criteria
  – Possible solution space for DUSTM modulation

• DUST superset modulation
  – Find a generic framework for development of DUSTSM
  – General performance criteria for DUSTSM

• Fast fading limitations of DUSTM
  – Proof behind fading degradation related to autocorrelation function

• Fast decoding methods (FBp-DPSK or lattice decoding) of DUSTM
  – Methods for choosing optimal subsets of received data

• Use of G.A. optimisation on DUSTM using macroscopic MIMO radio channel model
Relevant Publications


• D. B. Smith, “Fast Differential Unitary Space-Time Modulation Decoding for a MIMO Radio Channel,” *Wireless Personal Communications*, vol. 25, no. 4, pp. 343-349, July 2003


• *D. B. Smith and T. A. Aubrey, “Differential Unitary Space-Time Superset Modulation”, Submitted to European Transactions on Telecommunications*


*Pending publication*