Iterative Detection in Code-Division Multiple-Access with Error Control Coding

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Abstract

A code-division multiple-access system with channel coding may be viewed as a serially-concatenated coded system. In this paper we propose a low complexity method for decoding the resulting inner code (the spreading sequence), which allows iterative (turbo) decoding of the serially-concatenated code pair. The per-bit complexity of the proposed decoder increases only linearly with the number of users. Performance within a fraction of a dB of the single user bound for heavily loaded asynchronous CDMA is shown both by simulation and analytically.

1 Introduction

It is well known that channel coding can be used to combat mobile radio channel effects such as noise and fading. For multiple-access channels, information theory promises an increase in capacity if joint decoding of the users is employed [1]. The design of good (easily decodeable) multiple-access codes is difficult, and we do not address that problem here. However, information theory also tells us that under the assumption of joint decoding, judicious use of pseudo-random direct-sequence spreading incurs only a minimal loss in capacity [2]. With these motivations, we consider the joint decoding of a code-division multiple-access (CDMA) system in which each user, in addition to spreading, employs a single-user channel code. As we shall show, the concatenation of direct-sequence spreading with the asynchronous multiple-access channel may be viewed as a special form of a convolutional code. We shall consider this convolutional code as the inner code of a serially-concatenated system, with the single-user channel codes forming the outer code. With the serially-concatenated system so defined, we propose an iterative decoding technique inspired by [3]. Our main contribution is a low-complexity decoder for the inner (CDMA) code suitable for use in such an iterative scheme. This contrasts with the view of Giallorenzi *et al.* [4] where the generator polynomials pertaining to the inner and outer code are combined into one and decoding in one trellis at the receiver. The resulting number of states in that case is proportional to the exponential of the product of the number of users and the memory in the convolution codes employed for FEC. Such a system is intractable for realistic system parameters.

The optimal joint decoder for the inner code has a number of states that increases exponentially with the number of users. As a result, many suboptimal, linear complexity solutions have been proposed, such as the matched filter [5], the decorrelator [6, 7] and the projection receiver [8]. None of these techniques were designed with the idea of iteration in mind. Several authors have however proposed iterative decoders. In [9], the synchronous CDMA channel is considered as a block code and an iterative decoder is developed from this basis. A multistage decoder, which resembles the iterative one proposed here is described in [10]. The main difference being our exploitation of the maximum-a-posteriori (MAP) decoding technique rather than soft output Viterbi algorithm. Other relevant prior work includes [11, 12, 13, 14] where various alternative inner decoders are suggested. In addition to proposing the receiver structure we also provide performance analysis using techniques not found in the literature as yet.

The paper is arranged in the following way. In Section 2 we define the interference model incorporating the random spreading sequences and multipath propagation. Furthermore, we make the observation that the resulting CDMA channel is nothing more than a convolutional code. We proceed to define the iterative receiver structure in Section 3. In Section 4 we derive analytical expressions to evaluate the performance as a function of the number of iterations for each user, who may have a different encoding polynomials and multipath channels. Performance examples are given in Section 5, where simulation and analysis are compared. We summarise our results in Section 6.

Throughout the paper vectors and matrices are indicated as underlined lower case and underlined upper case respectively. If $\underline{x}$ is a vector, then the notation x_i refers to element i of $\underline{x}$. The superscript * is the conjugate-transpose operator. If $\underline{M}$ is a matrix, the vector $\underline{m}_i$ is its ith column. The vector $\underline{0}_n$ is the length n all-zero vector, and the matrix $\underline{I}_n$ is the $n \times n$ identity matrix.

2 Model

With reference to Figure 1, let each of the K users in the multiple access system encode their binary information sequences using a (possibly different) rate k/n binary convolutional code. Without loss of generality, we shall consider the transmission of L code symbols per user, corresponding to a total of KL/n information bits. Each user independently interleaves their encoded sequence. Interleaving is conventionally employed for time diversity, but here we shall require it for the implementation of the receiver. Denote the sequence output from the interleaver of user $k = 1, 2, \ldots, K$ as $x_k[i]$, where $i = 1, 2, \ldots, L$ is the symbol index. At each time symbol time i, each user transmits $\underline{s}_k[i]x_k[i]$, which is the real multiplication of $x_k[i]$ with the N-chip spreading sequence, $\underline{s}_k[i] \in \{-1, +1\}^N$. In practice, the spreading sequences for each symbol interval are chosen pseudo-randomly from symbol to symbol, and are known to the receiver.

We shall assume multipath propagation with maximum delay spread MT_c where M is some integer and T_c is the chip duration. The entire received signal after down converting $\underline{e} \in \mathbb{C}^{(L+1)N+M}$ may be written as follows

$$\underline{e} = \underline{A}\,\underline{d} + \underline{n},$$

where we make the following definitions. The matrix $\underline{A} \in \mathbb{C}^{(L+1)N+M, LK}$ has as column $j = 1, 2, \ldots, LK$ the vector $\underline{a}_j$, given by

$$\underline{a}_j = \begin{pmatrix} \underline{0}_{N(i-1)} \\ \underline{h}_k[i] \star \underline{s}_k[i] \\ \underline{0}_{N(L-i)} \end{pmatrix} \in \mathbb{C}^{(L+1)N+M}$$

where $\underline{h}_k[i] \star \underline{s}_k[i]$ represents the discrete-time convolution of $\underline{h}_k[i] \in \mathbb{C}^N + M$, the complex chip-rate-sampled channel impulse response for user k at symbol i with the corresponding spreading sequence. The channel vector $\underline{h}_k[i]$ accounts for any asynchronism by placing

$\tau_k < N$ zeros at the start of the vector corresponding to the delay $\tau_k T_c$. The received power for symbol j (corresponding to some particular user and symbol interval) is $w_j = \underline{a}_j^* \underline{a}_j$. The vector $\underline{d}$ has elements $d_j = \underline{x}_k[i]$. The index variables i,k and j are uniquely related by $j = (i-1)K + k$. The vector $\underline{n} \in \mathbb{C}^{(L+1)N+M}$ contains independent and identically distributed complex Gaussian noise samples, $\mathrm{E}\{\underline{n}\,\underline{n}^*\} = \sigma_n^2 \underline{I}_{(L+1)N+M}$.

The output of the RAKE receiver front end (which assumes neither chip, nor symbol synchronism between users) can now be written as

$$\underline{y} = \hat{\underline{A}}^* \underline{e} = \hat{\underline{A}}^* \underline{A}\,\underline{d} + \underline{z}. \tag{1}$$

The matrix $\hat{\underline{A}}$ is the receiver's estimate of $\underline{A}$. Channel estimation for such a system using random spreading sequences is non-trivial, but may be accomplished using the techniques described in [15]. Assuming perfect channel knowledge, let $\underline{R} = \underline{A}^* \underline{A}$. From (1) we have

$$\underline{y} = \underline{R}\,\underline{d} + \underline{z}, \tag{2}$$

which defines $\underline{y}$ to be nothing more than a convolutional encoding of the sequence $\underline{d}$ in coloured Gaussian noise with $\mathrm{E}\{\underline{z}\,\underline{z}^*\} = \underline{R}\sigma_n^2$. This realisation comes about due to the band diagonal nature of the $\underline{R}$ (encoding) matrix. The bandwidth of $\underline{R}$ is $2\nu - 1 = 2(K+M) - 1$. Our conclusion from this realisation is that we can employ convolutional code decoding techniques, such as Viterbi or MAP algorithms, to determine $\underline{d}$ given $\underline{y}$ and $\underline{R}$. Note that $\underline{R}$ defines a time-varying convolutional code over the complex field.

For the purpose of describing the decoder for this code we write $\underline{y}$ as a perturbed version of $\underline{d}$

$$\underline{y} = \underline{W}\,\underline{d} + \underline{z} + \underline{M}\,\underline{d}, \tag{3}$$

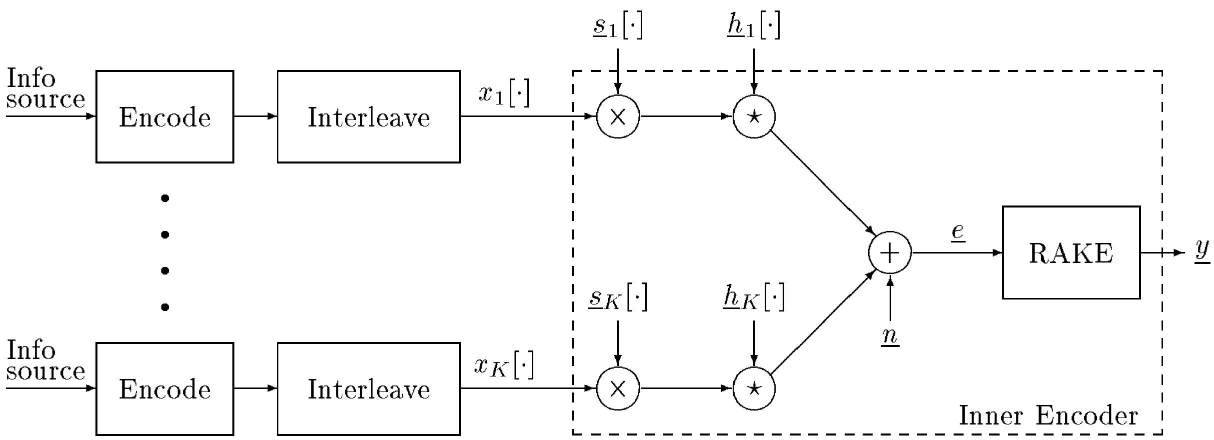

Figure 1: Code-Division Multiple-Access Channel

where

$$\underline{W} = \text{diag}\,(\underline{R})$$
$$\underline{M} = \underline{R} - \underline{W}.$$

or for a particular element of $\underline{y}$, corresponding to one output of a particular RAKE

$$y_j = w_j d_j + z_j + \underline{m}_j \underline{d} \tag{4}$$

where $\underline{m}_j$ is row j of $\underline{M}$. Using our view that the CDMA channel is a convolutional code,

- y_j is the (noise perturbed) j^{th} codeword emitted by the encoder, and
- $\underline{m}_j$ behaves like the convolutional encoder shift register since the non-zero portion of $\underline{m}_j$, which selects the subsequence of $\underline{d}$, acts like a liding window on $\underline{d}$ due to the band diagonal nature of $\underline{R}$.

3 Iterative Decoder

Due to the serially-concatenated convolutional code structure of the transmitter we propose an iterative decoding principle [3], in which the inner code creates reliability information for the outer code, which in turn creates reliability for the inner code. The iterative process can then be continued until further iteration yields minimal improvement.

The receiver block diagram is shown in Figure 2. The joint CDMA MAP decoder block ideally[1] performs joint symbol-wise maximum-a-posteriori decoding [16] of the inner CDMA code, based on the encoding matrix $\underline{R}$. The trellis over which the MAP algorithm for the inner CDMA code would operate has 2^{ν} states, which is intractable for large K. It is this system block that we shall sub-optimise. The De-interleaver simply reverses the interleaving performed by each user at the transmitter. This interleaving is used to spread burst errors that may arise in the single user decoders and more generally to "whiten" the errors at the input of the MAP decoders. The K single user (SU) MAP decoder block performs single-user symbol-wise MAP decoding of the outer channel codes, according to [16]. Both the CDMA MAP and the K SU MAP decoders produce soft information in the form of the likelihoods for $\{d_j : j = 1, \cdots, LK\}$. It is this information which is passed iteratively around the decoder.

We now describe our sub-optimisation of the joint CDMA MAP decoder. Our aim is to construct prior distributions of the symbols $\{d_j : j = 1, \cdots, LK\}$ for the single user MAP decoders. These priors will be formed subject to the CDMA channel code constraint, ignoring the single user code constraints. The approximation may be arrived at using an approach similar to the Expectation-Maximisation algorithm [17, 18]. In the language of the EM algorithm we have$\{\underline{y}, \underline{d}_{\mathcal{I}_j}\}$ as the complete data ($\underline{y}$ incomplete, $\underline{d}_{\mathcal{I}_j}$ missing) and

[1]To be more precise we would like to do codeword MAP decoding.

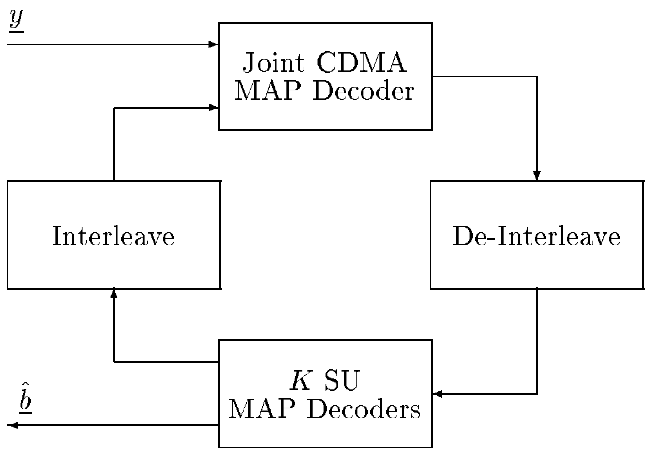

Figure 2: Iterative Receiver Structure

d_j as the parameter. We now get

$$Q(d_j; d_j^m) = \mathrm{E}\left\{\log p\left(\underline{d}_{\mathcal{I}_j}, \underline{y}|d_j\right)|\underline{y}, \underline{d}_j^{m-1}\right\}$$

as an approximation to the likelihood for d_j. This is the expectation step of the EM algorithm. We now employ two assumptions, which apart from the EM algorithm itself account for the sub-optimality of our technique

1. We use only the most "relevant" element of $\underline{y}$ so that

$$p\left(\underline{d}_{\mathcal{I}_j}, \underline{y}|d_j\right) \approx p\left(\underline{d}_{\mathcal{I}_j}, y_j|d_j\right)$$

   and,

2. there are sufficiently many users so that $\underline{m}_j\underline{\epsilon}$ can be considered as a realisation of a Gaussian distributed random variable.

The second assumption allows simple computation of the required likelihood as follows.

We have from (4), that

$$p(y_j|\underline{d}) = f_{y_j|\underline{d}}(\underline{d}) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left[-\frac{1}{2\sigma_n^2}\left|y_j - \underline{m}_j \underline{d}_{\mathcal{I}_j} - w_j d_j\right|^2\right], \tag{5}$$

Taking logs and then the conditional expectation we get

$$\begin{aligned}
\lambda(d_j) &= \mathrm{E}_{\underline{d}_{\mathcal{I}_j}}\{\log p(y_j|\underline{d})\} \\
&= C_1 - \frac{1}{2\sigma_n^2}\left|y_j - \underline{m}_j \underline{d}_{\mathcal{I}_j} - w_j d_j\right|^2 \\
&= C_1 - \frac{1}{2\sigma_n^2}\mathrm{E}_{\underline{d}_{\mathcal{I}_j}}\left\{\left(|y_j|^2 + w_j^2 - 2w_j d_j(y_j - \underline{m}_j \underline{d}_{\mathcal{I}_j})\right)\right\} \\
&= C_1 - \frac{1}{2\sigma_n^2}\left(|y_j|^2 + w_j^2 - 2w_j d_j(y_j - \underline{m}_j \mathrm{E}\left\{\underline{d}_{\mathcal{I}_j}\right\})\right)
\end{aligned}$$

We can ignore additive terms that are independent of the hypothesis d_j, since doing so will not change the likelihood ratio. We may now form

$$\lambda(d_j) = f_{y_j|\underline{d}}(d_j, \underline{d}_{\mathcal{I}_j} = \mathrm{E}\left\{\underline{d}_{\mathcal{I}_j}\right\})$$

Since, in general, $\mathrm{E}\left\{\underline{d}_{\mathcal{I}_j}\right\} \neq \underline{d}_{\mathcal{I}_j}$ we set

$$\mathrm{E}\left\{\underline{d}_{\mathcal{I}_j}\right\} = \underline{d}_{\mathcal{I}_j} + \underline{z}_{MU} \tag{6}$$

It then follows that

$$\begin{aligned}
x_j &= y_j - \underline{m}_j \mathrm{E}\left\{\underline{d}_{\mathcal{I}_j}\right\} \qquad (7) \\
&= y_j - \underline{m}_j \underline{d}_{\mathcal{I}_j} - \underline{m}_j \underline{z} \\
&= w_j d_j - \underline{m}_j \underline{z}_{MU} + n_j
\end{aligned}$$

Note that x_j may be thought of as an interference canceller output and accounts for the constraints imposed by the CDMA channel code. Using assumption 2 above we have that $\underline{m}_j \underline{z}$ is Gaussian distributed with zero mean and variance σ_{MU}^2 so

$$\begin{aligned}
\lambda(d_j) &= f_{y_j|\underline{d}}(d_j, \underline{d}_{\mathcal{I}_j} = \mathrm{E}\left\{\underline{d}_{\mathcal{I}_j}\right\}) \\
&= f_{x_j|d_j}(x_j, d_j) \\
&= \mathcal{N}(d_j, \sigma^2)
\end{aligned}$$

where

$$\sigma^2 = \sigma^2_{MU} + \sigma^2_n$$

and $\mathcal{N}(\mu, \sigma^2)$ is the Gaussian pdf with mean μ and variance σ^2. The noise variance σ^2 can be computed in practice as

$$\sigma^2 = \mathrm{var}\left(\underline{x} - \underline{W}\mathrm{E}\{\underline{d}\}\right)$$

Note that when the estimate $\mathrm{E}\{\underline{d}\}$ is equal to $\underline{d}$ we have $\underline{z}_{MU} = 0$ and single user performance would be achieved.

The entire complexity of our sub-optimal CDMA MAP decoder is approximately equal to a single branch metric computation of the full complexity MAP decoder.

4 Analysis

To begin our analysis, we write the output of both the sub-optimal CDMA MAP, which we shall now refer to as the interference canceller (IC), and the single user (SU) MAP decoders as perturbed versions of the original coded sequence $\underline{d}$. Specifically, for the IC we have

$$\underline{d}_{IC} = \underline{d} + \underline{n}_{IC},$$

and the output of the SU MAP block is

$$\underline{d}_{CC} = \underline{d} + \underline{n}_{CC}.$$

It remains to find the distributions of the noise terms $\underline{n}_{IC}$ and $\underline{n}_{CC}$. It can be seen from (7) that

$$\begin{aligned}\underline{n}_{IC} &= \underline{M}(\underline{d} - \underline{d}_{IC}) + \underline{n} \\ &= \underline{M}\underline{\epsilon} + \underline{n},\end{aligned}$$

where $\underline{\epsilon} = \underline{d} - \underline{d}_{IC}$ is the "error sequence". When the number of users is large each element of $\underline{n}_{IC}$ will therefore be Gaussian. Any correlations that exist between the elements of $\underline{n}_{IC}$ are assumed to be destroyed by the de-interleaving operation. The variance of element j of the noise in the output of the IC is

$$\begin{aligned}\sigma^2_{IC}(j) &\triangleq \mathrm{E}\left\{(\underline{n}_{IC})^2_j\right\} \\ &= \mathrm{E}\left\{(\underline{m}_j\underline{\epsilon} + n_j)^2\right\}.\end{aligned}$$

Further assuming the independence of the error sequence $\underline{\epsilon}$ and the AWGN term n_j, and additionally, uniform inter-user asynchronism, we can apply the results of [19] for pseudo-random spreading sequences and no multipath to find

$$\begin{aligned}\sigma^2_{IC}(j) &= \mathrm{E}\left\{(\underline{m}_j\underline{\epsilon})^2\right\} + \sigma^2_n \\ &= \sigma^2_n + \frac{w_j\sigma^2_{CC}}{KN}\sum_{i=j-K+1}^{j+K-1}|j-i| \\ &= \sigma^2_n + w_j\sigma^2_{CC}\frac{(K-1)}{N}\end{aligned} \tag{8}$$

where w_j is the received power for symbol d_j and we define σ^2_{CC} to be the average power per bit in the error sequence $\underline{\epsilon}$. We have also employed the result that $\sum_{i=1}^{n} = \frac{1}{2}n(n+1)$. The noise corrupting the SU MAP decoder outputs is

$$\sigma^2_{CC} \triangleq \mathrm{var}\left(d_i - \mathrm{E}\left\{d_i|\underline{y}\right\}\right), \tag{9}$$

which is assumed to be independent of i. We make no further claims on the joint distribution of the error sequence. When the received powers are all equal to a constant, w, we can drop the dependence on j and simply write

$$\sigma^2_{IC} \quad = \quad w\sigma^2_{CC}\frac{K-1}{N} + \sigma^2_n. \tag{10}$$

Observe that the attenuation of the noise power through the IC is dependent on the load of the system K/N. The lighter the load, the more attenuation. The SU MAP decoders operate in an AWGN environment with noise variance described by (8) or (10) if the received powers are equal. We can see from the above analysis that we require the noise variance output by the SU MAP decoders. There does not seem to be a simple way to calculate this quantity, and we therefore resort to simulation. In particular the quantity σ^2_{CC}, by (9), is estimated by simulation over a range of input variances, σ^2_{IC}. Similarly the probability of information bit error, P_e, for a particular input variance can be computed by simulation.

We have constructed functions which yield the noise variance output given the noise variance of the input for both the inner CDMA code decoder, and SU MAP decoders. By linking these functions in a recursive manner we can track the noise variance of the iterated decoder as a function of the iteration number. Observe that the minimum value of the variance out of the CDMA channel decoder is σ^2_n which corresponds to single user performance.

The initial condition of the analysis system is determined from the initial condition of the convolutional code output. Specifically, we initialise the output of the convolutional codes to be

$$P\left(d_j = -1\right) = P\left(d_j = 1\right) = \frac{1}{2}. \tag{11}$$

This leads to $\mathrm{E}\{d_i|\underline{y}\} = 0$ and therefore

$$\begin{aligned}\sigma^2_{CC,0} &= \mathrm{var}\left(d_i - \mathrm{E}\{d_i|\underline{y}\}\right)\\ &= 1.\end{aligned}$$

Adding an additional subscript denoting the iteration number to σ^2_{CC} and σ^2_{IC} we have

$$\sigma^2_{IC,m} = w\sigma^2_{CC,m-1}\frac{K-1}{N} + \sigma^2_n,$$

which describes the IC operation and

$$\sigma^2_{CC,m} = f_{CC}\left(\sigma^2_{IC,m}\right),$$

where f_{CC} is derived by simulation for the particular convolutional codes employed by the users. We need only simulate each different convolutional code once, even though several users may employ this code for FEC.

5 Performance Discussion

We now present performance results for the proposed system. Both simulation and analytical results are given. The ratio of the number of users K to the spreading code length N (or processing gain) is kept high for good spectral efficiency (and according to the information theoretic arguments of [2]). Actually unlike conventional results we shall "overload" the channel by selecting $K > N$. The convolutional codes employed by the users were the maximal free distance rate 1/2, 4 state convolutional codes with generators $5, 7$. If a more powerful code was employed the single user performance would improve and the multiuser receiver proposed here would follow this improvement [14]. We do not terminate the single user convolutional code trellis and use information sequences of length 100 resulting in length $L = 200$ interleaving.

5.1 Symbol Error Performance

We simulate the $K = 30$, $N = 14$ system which prohibits execution of the full complexity inner CDMA code MAP algorithm. With $K = 30$ the CDMA channel trellis has 2^{29} states in the multipath free case. The average bit error rate over users and iterations of the receiver are shown in Figure 3 versus E_b/N_0. Simulated and analytical curves are shown and it can be seen that there is some small discrepancy between the results. This discrepancy may be due to the small correlations that exist in the noise at the input to the SU MAP decoders. Another possible source of error may arise since $\underline{m}_{j\epsilon}$ is only Gaussian if $\nu \to \infty$.

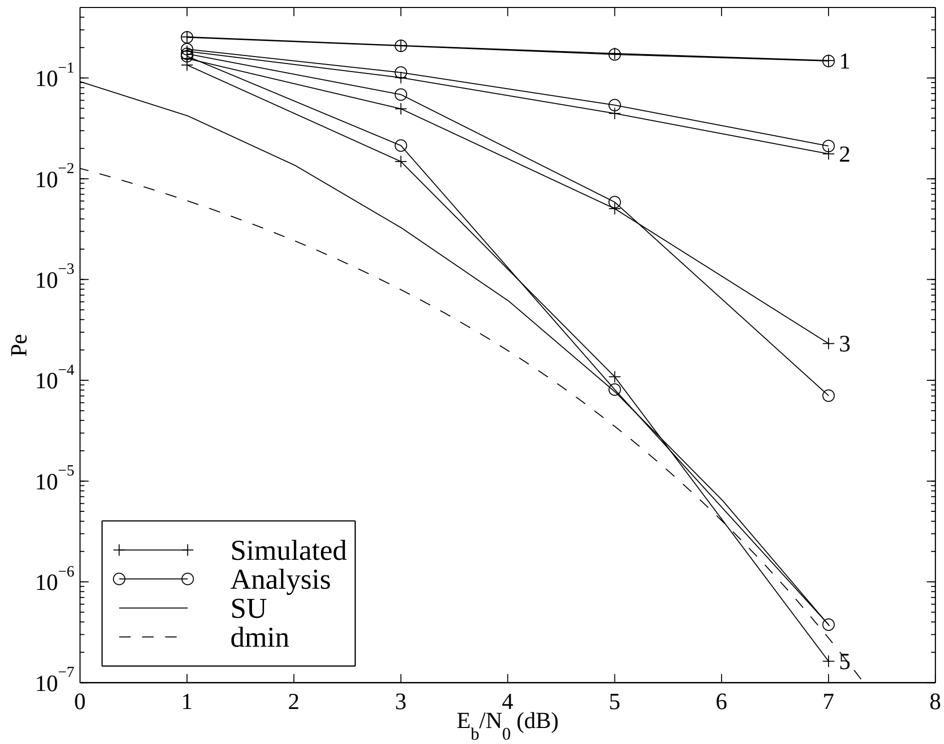

Figure 3: Simulated System Performance

The performance for the case where only a single user is present is also included (SU). We see that after 5 iterations that single user performance is achieved for all 30 users using the receiver defined here.

The large gap between single user performance the final iteration of the proposed receiver at low E_b/N_0 is explained by our analysis. In Figure 4 we have shown how we compute the variance of the noise into and out of both of the blocks in the iterative receiver. The vertical line corresponds to the target variance which is dependent on the E_b/N_0, at the receiver, of each of the users. In this case we have set this E_b/N_0 to be low (-1 dB) in order to illuminate the reason for the large gap between single user, and obtained performance at low E_b/N_0 as shown in Figure 3. The performance of the iterative receiver is limited by the point at which the "CC" and "IC" lines cross. We see, in this case, the intersection is at $\sigma^2_{IC} = 1.95$ which is higher than the ideal value of $\sigma^2_n = 1.26$.

Figure 4 yields further insight into the operation of the iterative receiver. We see that as the underlying E_b/N_0 is improved the point at which the IC line crosses the horizontal axis will shift left. This has the effect of moving the intersection of the "IC" and "CC" lines closer to the ideal operating point. Alternatively, reducing the ratio K/N (K/N = 30/14 in Fig. 4) has the effect of increasing the the slope of the IC line. This would also have the effect of moving the intersection point closer to the ideal. The other mechanism available for performance improvement is the convolutional code characteristic. If the variance in versus variance out profile was improved, perhaps by using a stronger code, the CC curve would move down and again the intersection point would move left, closer to the ideal.

Previous proposals such as the conventional system [5] and the maximum likelihood linear preprocessing technique known as the projection receiver [8] are several dB away from the single user bound. In fact, the result after the first iteration of the proposed system is equivalent to a conventional system. The projection receiver is bounded away from the single user curve by approximately $10\log_{10}(N/(N-K)) = 8dB$. In addition, The projection receiver requires the complicated computation of $\underline{R}^{-1}$.

5.2 Near-Far Performance

Let us now study the near-far performance of the proposed system. We shall have 13 users received at 1.5 dB and 14 users received at 4.5 dB, resulting in a 3dB difference. There are a total of 27 users in the system with spreading codes of length $N = 31$ with the other parameters of the simulation as before. The average performance of the 1.5 dB

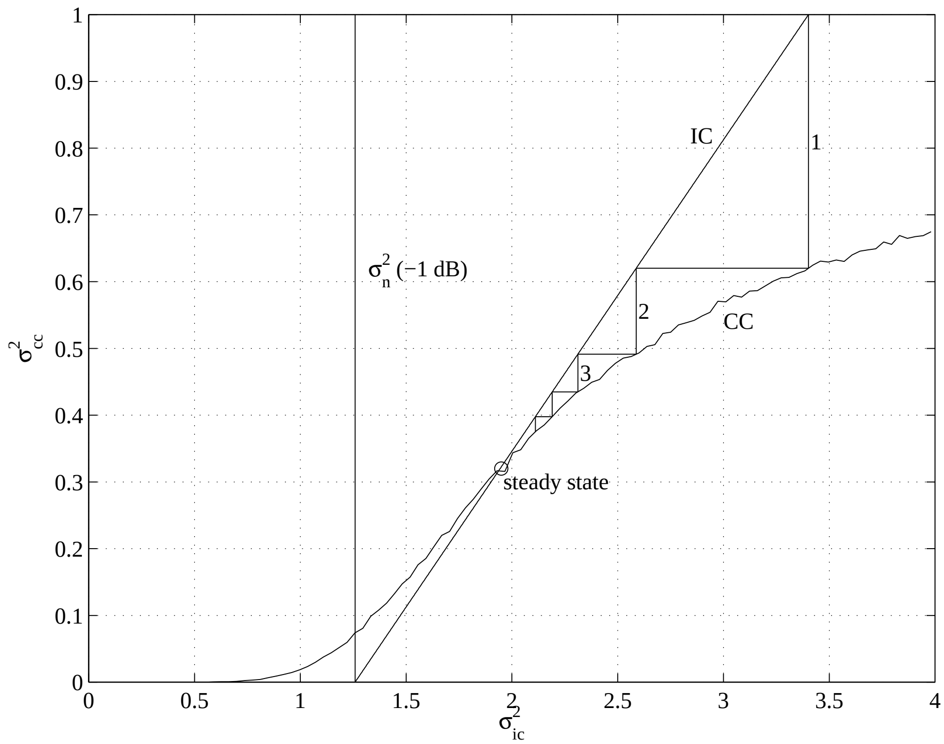

Figure 4: Noise Variance Trace

set of users along with the average performance of the 4.5 dB set of users is shown in Figure 5 over iterations. The case where all 27 users have the same receiver power is also included. The ideal scenario is that performance of the two sets of users are decoupled, with the performance being dependent only on the received power of the user. We see that for the final iteration the strong users are degraded by less than 0.5 dB relative to the case when all users are at 4.5 dB. We also see that the weak users are improved by more than 0.5 dB. A point of interest is the first iteration. Here we see that the strong users are doing better in the presence of 13 weak users instead of 13 users at 4.5 dB. This follows since, as can be seen from (11), at the first iteration the other users are treated as noise. Interestingly as the iterations proceed the roles are reversed. The near far effect is not severe in this receiver.

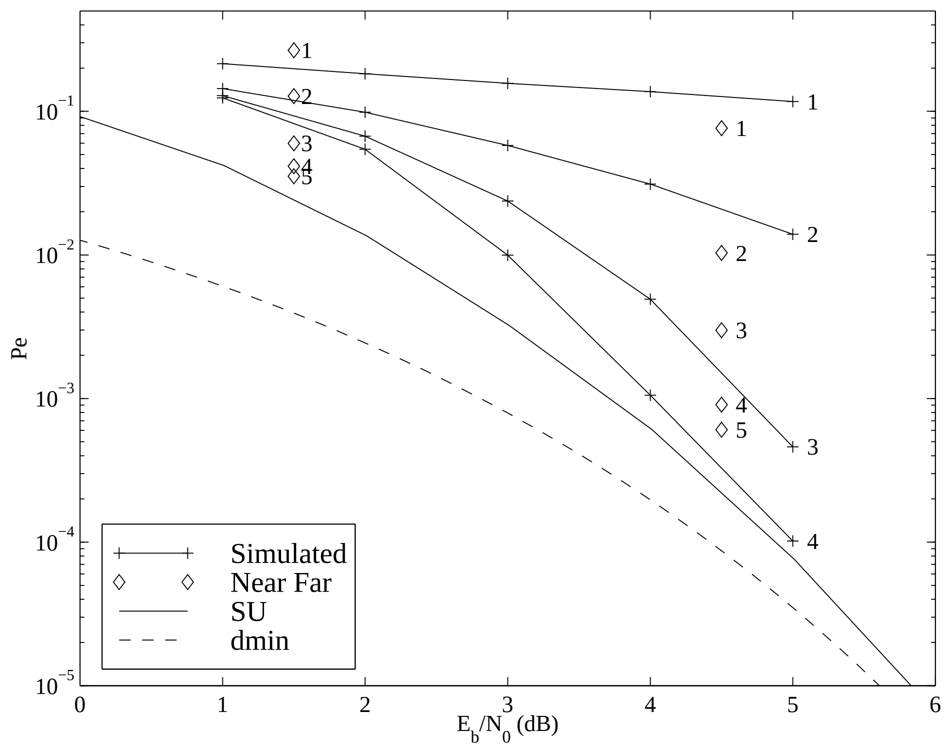

Figure 5: Simulated System Performance in Near Far Scenario

5.3 Channel Estimation Errors

Finally let us study the effect of channel estimation errors at the receiver. In particular $\underline{\hat{a}}_j = \alpha_k \underline{a}_j$ where the distribution of the α_k is Gaussian with hard limits. We limit the estimated power to be within ± 3 dB of the actual power. The multiplicative factors α_k are distributed normal with unit mean and variance 0.02 (2%). The same erroneous estimate is used for the entire signaling interval of length L. The estimate errors are statistically independent across users and the actual powers of all users are identical. The number of users in this test was $K = 13$ with $N = 15$ chips per spreading code. These values follow the theme of this work which is to highly load the available bandwidth.

Figure 6 shows the performance of the receiver for the erroneous amplitude estimates and the exact estimates. We see that the fourth iteration at high E_b/N_0 seems bound away from single user performance. This is justified since we used a multiplicative method for introducing channel estimate errors. If we had employed an additive error then an error floor would have been evident. Assuming that a 2% error on the estimated amplitude is realistic, then acceptable error rates for continuous data applications are realisable.

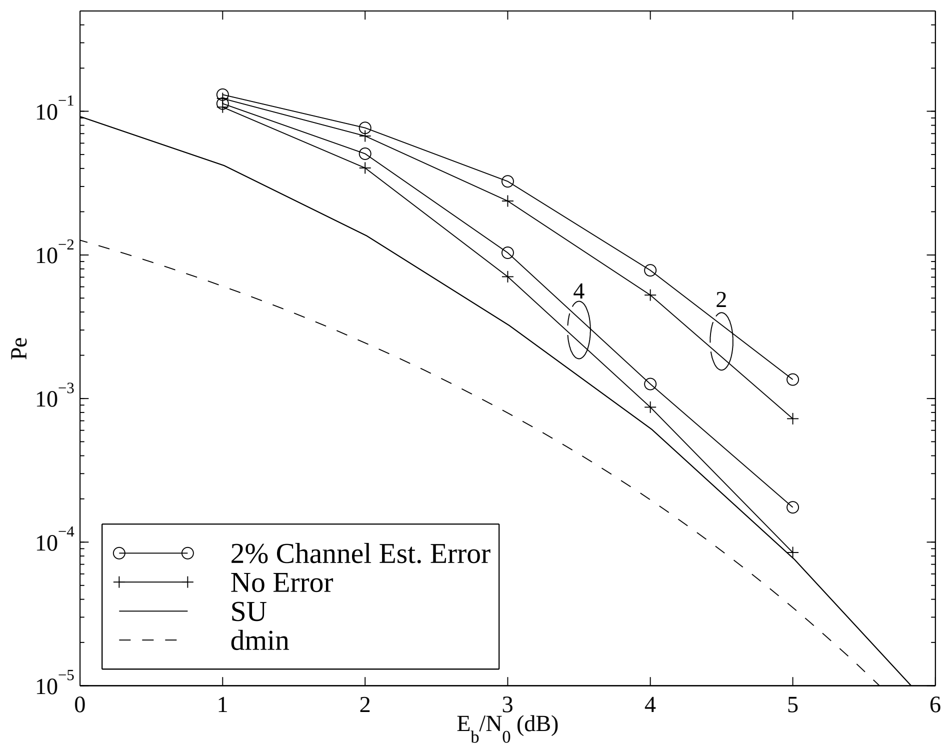

Figure 6: Simulated System Performance with Channel Estimation Errors

6 Summary

We have proposed a low-complexity iterative decoder for the uplink in a CDMA communications system. The receiver provides unusually good performance for the complexity spent. Although the system must iterate several times, the complexity of one iteration is extremely low. The complexity of the CDMA MAP decoder is approximately equal to a single branch metric computation for a corresponding full complexity MAP decoder. The performance of the receiver is computed using both simulation and analytical tools, and good agreement is observed. System performance after a only a few iterations is as if there were only one user in the system, i.e. the interference has been almost completely eliminated. Although this has been done in the past with small numbers of users or orthogonal codes we drop both restrictions and allow the codes to be random and the number of users to exceed the spreading code length. Multipath propagation is easily incorporated in the receiver design, even taking into account errors in channel estimation. Future studies will study the possibility of integrated channel estimation procedures. The receiver is also shown to have excellent near-far resistance.

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