ITERATIVE MULTI-USER DETECTION FOR DS-CDMA WITH FEC

Mark C. Reed, Paul Alexander, John Asenstorfer
Mobile Communications Research Centre
University of South Australia
SPRI Building, Warrendi Road
The Levels SA 5095, Australia
FAX: +61 8 8302 3873

Christian B. Schlegel
Department of Electrical Engineering
3262 Merrill Engineering Building
University of Utah
Salt Lake City UT 84112, U.S.A
FAX: +1 801 581 5281

Abstract — This paper discusses an iterative multi-user receiver for direct-spread code-division multiple-access (DS-CDMA) using random spreading codes with forward error control (FEC) coding. The maximum a-posteriori (MAP) criterion is used to derive the receiver for the joint received signal, with the decoding performed by single user MAP decoders. Iterations of the receiver/decoder set are used as a mechanism for reducing multiple access interference. Using this technique single user performance is achieved.

1. INTRODUCTION

Direct sequence code-division multiple-access (DS-CDMA) systems in mobile communications, are now becoming available on the market. The performance of these systems is limited by their inherent multiple access interference. For this reason we study ways of improving the implementation of multi-user (MU) systems. In this paper we describe an iterative DS-CDMA multi-user receiver, primarily intended for the uplink.

In some research proposals synchronous systems are proposed for the uplink, allowing the use of orthogonal spreading codes. This synchronisation is difficult to maintain due to limitations from timing control and multi-path propagation. We propose to use random codes and demonstrate our system as a synchronous one, although the extension to an asynchronous system is possible [1]. The performance of an asynchronous system using random codes is on average the same as a synchronous system using random codes.

A number of papers motivate us to search for spectrally efficient DS-CDMA systems using random codes. Work done by Grant et al. [2] has shown that the capacity penalty vanishes, for a large number of users, using randomly selected spreading codes, as the ratio between the number of users and the spreading length becomes large. Also Jana et al. [3] have shown a slightly different result; they showed that the upper bound of the normalised minimum distance for a trellis coded multi-user DS-CDMA system with non-orthogonal spreading is identical to that of the single-user case. This means that asymptotically, using non-orthogonal codes, or random codes, single user performance should be possible.

Recently a coding technique called “Turbo Codes” was described by Berrou et al. [4]. This technique achieves near Shannon capacity performance, by utilising the concepts of iterations, statistically independent codes (by interleaving), soft-in/soft-out decoding, and parallel concatenated convolutional codes (PCCC). Further to this paper, work has been done by Benedetto et al. [5] on serial concatenated convolutional codes (SCCC). Several authors have proposed using turbo codes for DS-CDMA systems [6, 7]. These papers discuss system implementations but show no performance results. In this paper we adapt these turbo coding techniques to the multiuser receiver.

Throughout this paper scalars are lower case, vectors are underlined lower case, and matrices are underlined upper case. The symbol $(·)^T$ is the matrix transposition operator. Variables have subscripts that refer to the time increment and superscripts that refer to the user, except if stated otherwise.

2. SYSTEM MODEL

We model the uplink of a DS/CDMA communication system, as a coded, discrete-time system. The channel adds zero-mean white Gaussian noise with variance $\sigma^2 = N_0/2$, where $N_0$ is the single sided noise power spectral density. The channel model is chip and symbol
synchronous. $K$ users each transmit $L$ coded symbols $d_t^{(k)} \in \{+1, -1\}$, where $k \in \{1, \ldots, K\}$ is the user number, and $t \in \{0, \ldots, L-1\}$ identify the symbol interval. The spreading code employed by user $k$ at symbol interval $t$ consists of $N$ chips and is denoted $\mathbf{s}_t^{(k)} \in \{-1/\sqrt{N}, \ldots, +1/\sqrt{N}\}^N$. The matched filter output $y_t$ at time $t$ can therefore be expressed as

$$y_t = H_t \mathbf{d}_t + \mathbf{n}_t$$  

(1)

where $\mathbf{d}_t$ is the data vector, $H_t$ is the cross correlation matrix of the spreading sequences, and $\mathbf{n}_t$ is the noise vector. For a synchronous system

$$y_t^T = (y_t, \ldots, y_{t-N})$$

and $H_t = A_t^T A_t \in \mathbb{R}^{K \times K}$ where

$$A_t = (s_1^t, \ldots, s_K^t)$$

is the bank of spreading-code matched filters. The noise samples $\mathbf{n}_t$ are Gaussian distributed and have variance $\mathbb{E}\{\mathbf{n}_t \mathbf{n}_t^T\} = \mathbf{H}_t \sigma^2$. The coding method we use is limited to trellis codes and FEC is provided by convolutional codes.

3. THE ITERATIVE MULTI-USER RECEIVER

The problem faced when designing a multi-user receiver with a turbo code structure is that of generating the correct probability information for the soft-in/soft-out decoders and of supplying appropriate a-priori information to the multi-user receiver on each iteration. Fortunately both these problems can be solved simultaneously.

Figure 1 shows the iterative multi-user receiver system. The receiver takes the matched filter channel output as described in (1) and generates the conditional channel probabilities $p(y_t|d_t)$, which is the well known multivariate Gaussian conditional probability [8]. The metric generator then calculates the likelihood $p(y_t|d_t^{(k)})$ for each user $k$. This is the maximum-a-posteriori (MAP) result as discussed below. Single user soft-in/soft-out decoders generate the a-posteriori symbol probabilities $\Pr\{d_t^{(k)} = d|y_t^{(k)}\}$ (where $d \in \{+1, -1\}$) which are then used as a-priori information for the metric generator on the next iteration. When the required number of iterations have been completed a decision $(\hat{d}_t)$ is output by the single user decoders.

In this work we constrain the system to use $K$ single user decoders. These decoders calculate a-posteriori probabilities (APP) according to Bahl et al. [9]. They typically take as input the metric $p(y_t|d_t^{(k)})$, and output

$$\Pr\{d_t^{(k)} = d|y_t\} = \frac{p(y_t|d_t^{(k)})}{p(y_t)}$$

$$= C_p \Pr\{d_t^{(k)}, y_t\}$$

(3)

where $C_p$ is a constant. This operation is symbol based and does not take into account the FEC code. The above MAP criteria (3) was also used in [10] to compute sub-optimal MAP metrics for single user decoders in multi-user CDMA.

4. ITERATING THE RECEIVER

Multiuser systems describe receivers where users share information. If this is done correctly a joint detection process results with improved performance over systems without joint
In Turbo Code decoding [4], for example, the output probability of the first MAP decoder, \( \Pr\{b_t = b|y\} \), is used as a-priori information for a second MAP decoder. In a similar fashion we assign the single-user MAP decoder output probabilities from iteration \( i \) to the a-priori input probabilities to the metric generator for iteration \( i + 1 \) in (2), i.e., we set

\[
\Pr\{d_t^{(k)} = d\} = \Pr\{d_t^{(k)} = d|y^{(k)}\}.
\]

As in [4] this is justified due to the weak correlation between the single user convolutional codes and the spreading codes. On the first iteration the multiuser receiver’s a-priori information is \( \Pr\{d_t^{(k)} = d\} = 1/2 \), i.e., all symbol sequences are assumed to be equi-probable.

Because the FEC codes are not systematic the a-priori information cannot be factorised from the output of the metric generator as is done in [4] where the extrinsic information is calculated. If however systematic convolutional coding is used then the systematic information could be removed from the systematic bit probabilities to produce the extrinsic output.

### 5. SIMULATION RESULTS

The simulation result in Fig. 2 shows the performance of the system for 1, 2, 3 and 4 iterations. “single user” is single user performance. The simulation was for a synchronous chip and symbol channel with random spreading codes, spreading length \( N = 7 \), and users \( K = 5 \). All of the possible \( 2^K = 32 \) likelihood values \( (p(y_t|d_t)) \) were calculated. A convolutional code of rate \( R = 1/2 \), constraint length \( \nu = 3 \) with 4 states and \( d_{acc} = 5 \) was used as the error control code. “ML-linear” is the best performance from an equivalently coded linear system without iterations [11]. Simulation results show that the iterative system has performance very close to single user performance. Most of this improvement occurs on the second iteration, when the a-priori information is used for the first time.

### 6. CONCLUSIONS

In this paper we discussed the DS-CDMA channel before deriving the iterative multiuser receiver in terms of the MAP criteria. We also discussed the metric generation and single user decoding in terms of the iterative solution. Finally simulation results show that the iterative multi-user receiver has performance very close to single user performance. Most of this improvement occurs on the second iteration, when the a-priori information is used for the first time.

### REFERENCES


Figure 2: Iterative multi-user receiver performance.


