Iterative Multiuser Interference Reduction: Turbo CDMA

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***Abstract*— We view the asynchronous random code-division multiple-access (CDMA) channel as a time-varying convolutional code. We study the case where the users encode their data, and, therefore, the single user transmitters and the CDMA channel appear as the concatenation of two coding systems. At the receiver we employ serial Turbo decoding strategies. Unlike conventional Turbo codes where both the inner and outer code may be selected, in our case, the inner code is due to the CDMA channel which we assume to be random. Nevertheless, the decoding system resembles the decoder of a serial Turbo code and single-user performance is obtained even for numbers of users approaching the spreading code length.**

***Index Terms*—Code-division multiple access, iterative methods, random codes, Turbo codes.**

I. INTRODUCTION

THE OUTPUT of a code-division multiple-access (CDMA) channel is a linear transformation of the input. In the case where the channel output noise is whitened, using the noise whitening-matched filter [1], [2], the transformed signal has some special properties. These properties enable us to view the CDMA channel as a convolutional code with the number of states being exponential in the number of users. Due to the complexity of this code, decoding schemes such as the decorrelator [3] or matched filter receivers [4] are often employed.

When the users first encode their information sequences via single-user convolutional codes, the resulting system can be viewed as a concatenated coding system. Proposals that have a linear preprocessing filter to decode the inner CDMA code have been presented [5], [6]. The performance of these systems for the case where the number of users approaches the spreading code length is several decibels away from the single-user case. This loss is due to the linear decoding of the CDMA channel, which is a convolutional code in its own right. Convolutional codes are typically decoded using nonlinear techniques such as the Viterbi algorithm [7]. When the number of users is small compared to the spreading code length, a linear method is an appropriate decoder and overall system performance is satisfactory [4]–[6]. Other low complexity schemes such as a decision-feedback canceler have also been employed [8].

The maximum-likelihood solution for the asynchronous CDMA channel where the users encode their data has been formalized recently by Giallorenzi and Wilson [9]. They were able to achieve near single-user performance for the two-user case and fixed spreading codes. The decoder consisted of a Viterbi algorithm running over a trellis with a number of states that was exponential in the product of the number of users and the constraint length of the convolutional codes. Due to this complexity problem, they proposed a suboptimal technique [10]. In this paper, for large numbers of users, they were able to get within 2 dB of single-user performance. The system employed single-user decoders and there were multiple passes through each of these decoders. As will be shown, this structure is similar to the system in this paper. Recent work by Alexander *et al.* [11] provided a performance analysis for this system which shows the receiver achieves single-user performance for moderate E_b/N_o.

For the synchronous CDMA case, Hagenauer [12] realized that the CDMA channel could be viewed as a block code. He proposed a suboptimal scheme for decoding the CDMA channel that provided soft information in an iterative decoding scheme. A performance comparison with asynchronous schemes was not possible since, in the synchronous case, spreading codes may be selected according to their correlation properties. In the asynchronous case, where any possible delay profile is admitted, such explicit code design is not feasible.

It is not practical to decode the CDMA channel code using full-complexity decoding techniques due to the large number of states. In this paper, we propose a suboptimal maximum *a posteriori* (MAP) probability technique for decoding the CDMA channel code, and the single-user codes are decoded using full-complexity MAP probability algorithms, as described originally by Bahl *et al.* [13].

We make the realization that the concatenation of two convolutional codes may be decoded in a Serial Turbo Code [14] fashion at the receiver. A feature of our system is that the convolutional code corresponding to the CDMA channel has random generator polynomials. We do, however, have control over the high-level statistics of the polynomials via the spreading code length and the level of asynchronism in the channel. The exploitation of this control is not studied in this paper.

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We are encouraged by the result of Jana and Wei [15] that states that the minimum distance of the single-user codes are not compromised by the process of spreading and despreading in a multiuser environment. Additionally, Douillard *et al.* [16] realized that the intersymbol interference (ISI) channel may be viewed as a convolutional code and, therefore, were able to view a system incorporating FEC as a concatenated convolutional code system. We make the same realization here, but with the CDMA channel substituted for the ISI channel. Indeed, the model of a CDMA channel is equivalent to the model of an ISI channel where the taps of the ISI channel model are time varying.

Random spreading codes are explicitly considered since they allow for interferer diversity, which is not the case for fixed spreading codes. Asynchronous transmission is permitted to move complexity from the mobile terminal into the base station. We assume a random-access system where the amount of coordination between base station and user is minimized.

This paper is organized as follows: in Section II, the convolutional code model for the CDMA channel is described where a noise-whitening matched filter is required. The decoder for the serially concatenated convolutional transmitter model is discussed in Section III and simulated in Section IV. Conclusions are given in Section V.

Throughout this paper, scalars are lowercase, vectors are underlined lowercase, and matrixes are underlined uppercase. Subscripting is dropped when no ambiguities will arise. The symbols $(\cdot)^\top$ and $(\cdot)^{-1}$ are the transposition and inversion operators, respectively. The delimiter $\{\cdot\}^y$ defines a space of dimension y. Vector subscripting can be of the form $[\underline{\boldsymbol{x}}]_j$, denoting the jth component of vector $\underline{\boldsymbol{x}}$ When $\underline{\boldsymbol{x}}$ is some constant, then the subscript implies vector size, i.e., $\underline{0}_m$ is a column vector of zeros of size $\boldsymbol{m}$. We shall use a notation for subvectors, whereby the vector consisting of elements $\boldsymbol{i}$ through $\boldsymbol{j}$ inclusive of vector $\underline{\boldsymbol{x}}$, i.e, $(\boldsymbol{x}_i, \boldsymbol{x}_{i+1}, \cdots, \boldsymbol{x}_j)^\top$, shall be denoted $\underline{\boldsymbol{x}}_i^j$.

II. Channel Model

A well-known model for the L-symbol, K-user symbol-asynchronous CDMA channel employing spreading codes of length N is described presently. In particular, the output of a filter matched to the common chip waveform employed by all users is

$$\underline{\boldsymbol{e}} = \underline{\boldsymbol{A}}\,\underline{\boldsymbol{W}}\,\underline{\boldsymbol{d}} + \underline{\boldsymbol{n}} \tag{1}$$

where $\underline{\boldsymbol{A}}$ describes the CDMA channel over which each of the users transmits and is defined in terms of the spreading codes employed by each user and the relative delays of the users' transmissions measured at the receiver.

$$\begin{aligned}
\underline{\boldsymbol{A}} &= (\underline{\boldsymbol{a}}_1, \cdots, \underline{\boldsymbol{a}}_{LK}) \in \{\mathcal{C}, 0\}^{(L+1)N,\, LK} \\
\underline{\boldsymbol{a}}_j &= \left(\underline{0}^\top_{iN+\tau_k}, \underline{\boldsymbol{s}}^\top_{k,\,i}, 0, \cdots, 0\right)^\top \in \{\mathcal{C}, 0\}^{(L+1)N} \\
\tau_k &\in \{0, 1, \cdots, N-1\}
\end{aligned}$$

where $\mathcal{C} = \{-(1/\sqrt{N}), +(1/\sqrt{N})\}$ is the set of possible chip amplitudes. The channel symbol interval $i \in \{1, \cdots, L\}$ and user index $k \in \{1, \cdots, K\}$, are derived from the transmission number $j \in \{1, \cdots, LK\}$ as follows:

$$\begin{aligned}
i &= \lceil (j-1)/K \rceil \\
k &= ((j-1) \bmod K) + 1
\end{aligned}$$

where $\lceil \boldsymbol{x} \rceil$ is the smallest integer not less than $\boldsymbol{x}$. The spreading code $\underline{\boldsymbol{s}}_{k,\,i}$, is of length N chips and is employed by user k to transmitted bit i. We have restricted consideration to the chip synchronous case. The diagonal matrix $\underline{\boldsymbol{W}}$ contains, on the diagonal, the received amplitudes of each user's waveform. The noise vector $\underline{\boldsymbol{n}}$ represents the sampled AWGN in the system. It has the property $E\{\underline{\boldsymbol{n}}\,\underline{\boldsymbol{n}}^\top\} = \sigma^2 \underline{\boldsymbol{I}}$, where σ^2 is the variance of the noise process.

The binary code words of length n generated by each of the K users' encoders appear in the data vector $\underline{\boldsymbol{d}}$ as follows:

$$\underline{\boldsymbol{d}} = (d_1, \cdots, d_{LK}) \in \{-1, +1\}^{LK}. \tag{2}$$

Considering the fact that the elements of $\underline{\boldsymbol{d}}$ result from K single-user encoders of rate 1/n, we have that bit $l \in (1, n)$ of symbol $i_b \in (1, L_b)$ transmitted by user $k \in (1, K)$ is element j of $\underline{\boldsymbol{d}}$. The transmission index j may be formed as where

$$\begin{aligned}
j &= (i-1)K + k \\
&= ((i_b - 1)n + l - 1)K + k
\end{aligned}$$

and $L = L_b n$ is the collective number of channel bits transmitted by the users. We shall place the information symbols of all users into the vector $\underline{\boldsymbol{b}}$. The objective of the receiver is to determine an estimate for this vector.

When the noise whitening matched filter is applied to $\underline{\boldsymbol{e}}$, the resulting CDMA channel may be viewed as a real time-varying convolutional code. The encoder does not employ modulo-2 additions as is conventionally the case, but still possesses properties such as distance spectra and generating polynomials. The filter is the orthogonal basis resulting from the Gram–Schmidt orthogonalization of $\underline{\boldsymbol{A}}$, i.e.,

$$\underline{\boldsymbol{A}} = \underline{\boldsymbol{Q}}\,\underline{\boldsymbol{F}}$$

where $\underline{\boldsymbol{F}} \in \mathbb{R}^{LK,\, LK}$ is lower triangular with bandwidth K, and $\underline{\boldsymbol{Q}}$ has the same dimensions as $\underline{\boldsymbol{A}}$. The number of memory elements required in the convolutional encoder is one less than the bandwidth of the encoding matrix [17].

Note that an identical model results for the multipath fading channel case [18]. The output of the filter is

$$\begin{aligned}
\underline{\boldsymbol{y}} &= \underline{\boldsymbol{Q}}^\top \underline{\boldsymbol{e}} \\
&= \underline{\boldsymbol{F}}\,\underline{\boldsymbol{W}}\,\underline{\boldsymbol{d}} + \underline{\boldsymbol{z}} \qquad (3) \\
&= \underline{\boldsymbol{G}}\,\underline{\boldsymbol{d}} + \underline{\boldsymbol{z}}. \qquad (4)
\end{aligned}$$

The statistics of the noise are preserved due to the orthogonal nature of the projection $\underline{\boldsymbol{Q}}^\top$. Specifically, z_j are independently identically, distributed (i.i.d.) Gaussian with variance σ^2, i.e., $E\{\underline{\boldsymbol{z}}\,\underline{\boldsymbol{z}}^\top\} = \sigma^2 \underline{\boldsymbol{I}}$. We may think of the columns of $\underline{\boldsymbol{Q}}$ as modifications of the conventional coded matched filters $\underline{\boldsymbol{A}}$. In fact, $\underline{\boldsymbol{Q}} = \underline{\boldsymbol{A}}\,\underline{\boldsymbol{F}}^{-1}$, which states that the columns of $\underline{\boldsymbol{Q}}$ are linear combinations of the original spreading codes. Since $\underline{\boldsymbol{F}}$

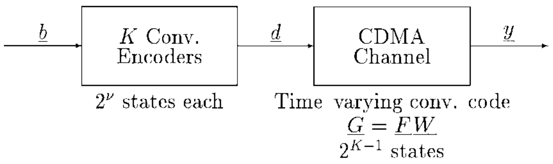

Fig. 1. Signal at receiver model.

is invertible for linearly independent columns of $\underline{\boldsymbol{A}}$, $\underline{\boldsymbol{y}}$ is a sufficient statistic. The two key features of $\underline{\boldsymbol{y}}$ that distinguish it from the conventional matched filter statistic are:

1) the noise component in $\underline{\boldsymbol{y}}$ is white;
2) the interference results from only $K-1$ other transmissions rather than $2K-1$ in the conventional case.

We can view the $LK \times LK$ matrix $\underline{\boldsymbol{G}} = \underline{\boldsymbol{F}}\,\underline{\boldsymbol{W}}$ as an encoding matrix. The convolutional code described by $\underline{\boldsymbol{G}}$ produces one of 2^K codewords for each input bit. Therefore, the code is rate $1/K$ and has $K-1$ memory elements. In real time, the input and output symbol rates are identical, only the cardinality of the input and output alphabets are different. The generator polynomials, set by the CDMA channel, are, in general, time-varying due to the use of random spreading codes. The sequence observed by the receiver is modeled as shown in Fig. 1 where we have assumed each of the single-user encoders employ convolutional codes with ν memory elements each.

Observe the concept of an “inner code” and an “outer code” in the model. The outer code pertains to the single-user encoders and the inner code is the CDMA channel. We have not specified the details of the “outer code” since there are several possibilities and the details are not important to our decoding system. The total number of memory elements in the “outer code” is $K\nu$. One can place the information symbols from each of the users into a vector $\underline{\boldsymbol{b}}$ and construct a generator matrix $\underline{\boldsymbol{G}}_{\text{SU}}$ corresponding to a code with $K\nu$ memory elements where $\underline{\boldsymbol{d}} = \underline{\boldsymbol{G}}_{\text{SU}}\underline{\boldsymbol{b}}$. The critical realization is that it is correct to model the output of the “outer code” as a multiplexing of outputs of the single-user encoders according to (2). In summary, a valid abstraction of an asynchronous CDMA system with independent single-user encoding is a concatenated code where the constituent codes are convolutional codes with generators $\underline{\boldsymbol{G}}_{\text{SU}}$ and $\underline{\boldsymbol{G}}$. With this realization inhand, new techniques for decoding serially concatenated codes may be employed.

Note that there are other filters that allow the CDMA channel to be viewed as a convolutional code, i.e., the raw $\underline{\boldsymbol{A}}$ channel of (1) is a convolutional code with varying-rate and binary coefficients. Although the overall rate of the code is still $1/K$, the number of bits consumed by the decoder varies according to the number spreading sequences that overlap in the channel matrix $\underline{\boldsymbol{A}}$. The more columns of $\underline{\boldsymbol{A}}$ that overlap, the more bits consumed by the encoder. The rate for the $\underline{\boldsymbol{F}}$ channel realization is constant at $1/K$. We employ the $\underline{\boldsymbol{F}}$ channel for reasons that allow effective complexity reduction in the decoder. Note that the receiver must derive $\underline{\boldsymbol{F}}$ which requires knowledge of $\underline{\boldsymbol{A}}$. The matrix $\underline{\boldsymbol{A}}$ is uniquely defined by the set of delays $\{\tau_1, \cdots, \tau_K\}$ and the random spreading codes employed by the users.

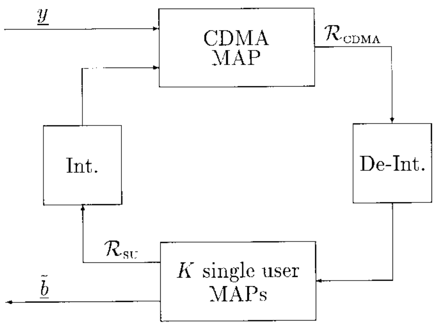

Fig. 2. Receiver structure.

A joint decoder for this serially concatenated code would require a trellis with $2^{K(\nu+1)-1}$ states. Note that this number of states satisfies the following boundary conditions:

1) single-user: $K=1$, $2^{K(\nu+1)-1} = 2^{\nu}$;
2) no coding: $\nu = 0$, $2^{K(\nu+1)-1} = 2^{K-1}$.

We shall not attempt to find a suboptimal solution for the prohibitively large joint trellis. Rather, we focus on the serially concatenated nature of the encoder. We shall assume that the single-user encoders interleave their output sequence at the code word bit level to aid in the decoding process.

III. DECODER

Due to the serially concatenated convolutional code structure of the transmitter, we propose a Turbo code style decoding principle where the inner code creates reliability information for the outer code, which in turn creates reliability for the inner code. The iterative process continues until further iterations yield minimal improvement. The *ad hoc* nature of the system is rooted in the *ad hoc* nature of Turbo codes. No criteria has been given for the stability of what amounts to a nonlinear recursive filter. Turbo codes have acquired notoriety, and we explore their application to our abstraction of the CDMA communications system.

The receiver system diagram is shown in Fig. 2 where the presence of interleaving blocks requires that, in the transmitter, the users interleave their encoder output streams before transmission across the CDMA channel. The interleaving is employed to spread burst errors that arise in the single-user decoders. In Fig. 2 we have shown the interleaving blocks as one device, although they consist of K parallel devices. The decoder operates on the sequence $\underline{\boldsymbol{d}}$. The soft output of the CDMA MAP decoder is

$$\mathcal{R}_{\text{CDMA}} = \{p(\underline{\boldsymbol{y}}|d_j)\colon j = 1, \cdots, LK\}$$

and is produced as follows:

$$p(\underline{\boldsymbol{y}}|d_j) = p(\underline{\boldsymbol{y}}, d_j)/P(d_j) \tag{5}$$

where

$$p(\underline{\boldsymbol{y}}, d_j) = \sum_{\underline{\boldsymbol{d}}_{j-K+2}^{j-1} \in \{-1,+1\}^{K-2}} p\Big(\underline{\boldsymbol{y}}, \underline{\boldsymbol{d}}_{j-K+2}^{j}\Big) \tag{6}$$

and

$$p\Big(\underline{\boldsymbol{y}}, \underline{\boldsymbol{d}}_{j-K+2}^{j}\Big) = p\Big(\underline{\boldsymbol{y}}_1^{j}, \underline{\boldsymbol{d}}_{j-K+2}^{j}\Big)p\Big(\underline{\boldsymbol{y}}_{j+1}^{LK}|\underline{\boldsymbol{d}}_{j-K+2}^{j}\Big). \tag{7}$$

We may think of the summation as averaging out the other users' contributions, thus forming a probability for the user of interest. Defining the state of the CDMA encoder at symbol interval $j-1$ to be $\underline{\boldsymbol{d}}_{j-K+1}^{j-1} = (d_{j-K+1}, \cdots, d_{j-1})$, we see that the summation in (6) is over all states on the CDMA trellis at symbol interval $j-1$. Note that since the bandwidth of $\underline{\boldsymbol{G}}$ is $K-1$, the state has $K-1$ binary elements and therefore there are 2^{K-1} states. The state probability can be computed using Bahl's method of forward and backward recursions [13] as follows:

$$p\Big(\underline{\boldsymbol{y}}_1^j, \underline{\boldsymbol{d}}_{j-K+2}^j\Big) = \sum_{d_{j-K+1}} p\Big(y_j | \underline{\boldsymbol{d}}_{j-K+1}^j\Big) \times P\Big(\underline{\boldsymbol{y}}_1^{j-1}, \underline{\boldsymbol{d}}_{j-K+1}^{j-1}\Big) P(d_j) \tag{8}$$

$$p\Big(\underline{\boldsymbol{y}}_{j+1}^{LK} | \underline{\boldsymbol{d}}_{j-K+2}^j\Big) = \sum_{d_{j+1}} p\Big(\underline{\boldsymbol{y}}_{j+2}^{LK} | \underline{\boldsymbol{d}}_{j-K+2}^{j+1}\Big) \times p\Big(y_{j+1} | \underline{\boldsymbol{d}}_{j-K+2}^{j+1}\Big) P(d_{j+1}). \tag{9}$$

The probability $P(d_j)$ in (5) is the *a priori* information about the symbol d_j and at the first iteration is set to be 0.5 for both $d_j = +1$ and $d_j = -1$. On subsequent executions of the CDMA MAP decoder, this probability will be furnished by the single-user MAP decoders. The likelihood $p(y_j | \underline{\boldsymbol{d}}_{j-K+1}^j)$ is, from (4), Gaussian with variance σ^2

$$p\Big(y_j | \underline{\boldsymbol{d}}_{j-K+1}^j\Big) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-1}{2\sigma^2}\left(y_j - \underline{\boldsymbol{g}}_j \underline{\boldsymbol{d}}_{j-K+1}^j\right)^2\right) \tag{10}$$

where $\underline{\boldsymbol{g}}_j$ is the nonzero part of row j of $\underline{\boldsymbol{G}}$, i.e.,

$$\underline{\boldsymbol{g}}_j = (G_{j,\, j-K+1}, \cdots, G_{j,\, j}).$$

The deinterleaver, with knowledge of the K single-user deinterleavers, implements one deinterleaving operation on the soft output of the CDMA MAP decoder. Each user's MAP decoder produces both a soft output pertaining to its part of $\underline{\boldsymbol{d}}$ and an estimate of its information sequence (the original input to its convolutional encoder). For user k, the soft output information is

$$\mathcal{R}_k = \{P(d_j | \underline{\boldsymbol{y}}) \colon j = iK + k,\, i = 1, \cdots, L\}. \tag{11}$$

The single-user MAP decoders operate identically to the method proposed by Benedetto and Montorsi [14] for second-stage decoders in a serial Turbo decoder. This method is equivalent to the MAP algorithm of Bahl *et al.* [13] that generates the *a posteriori* probabilities of the states and state transitions at each time interval for a finite-state machine based on noisy observations.

Here, we take the perspective that the input to the MAP decoder is a sequence of *a priori* likelihoods, say λ_j, for the symbols on the trellis transitions d_j. In this way, the transition probabilities of Bahl *et al.* are

$$\gamma_{i_b}(m', m) \frac{1}{2} \prod_{j \in \mathcal{T}_{k,\, i_b}} \lambda_j \tag{12}$$

where $\mathcal{T}_{k,\, i_b}$ is the set of integers specifying the channel symbols generated by the transition from state m to state m' in symbol interval i_b of user k's encoder

$$\mathcal{T}_{k,\, i_b} = \{(i-1)K + k \colon \quad i = (i_b - 1)n + 1, \cdots, i_b n - 1\}.$$

In our development we provide the *a priori* likelihoods for the symbols on the trellis transitions as

$$\lambda_j = p(\underline{\boldsymbol{y}} | d_j). \tag{13}$$

The algorithm of Bahl *et al.* is used to compute the state probabilities at each trellis interval. These are then employed in the construction of information symbol and channel symbol *a posteriori* likelihoods. The MAX operation of the MAP algorithm is then applied to make a decision on the information symbols by selecting the symbol with the highest *a posteriori* likelihood. Central to the execution of our iterative algorithm is the collection of the *a posteriori* channel symbol likelihoods into the reliability information $\mathcal{R}_k$. These are then collected across users as follows:

$$\mathcal{R}_{\mathrm{SU}} = \{\mathcal{R}_k \colon k = 1, \cdots, K\}.$$

This information is interleaved for use in the next pass through the CDMA MAP decoder. The interleaving amounts to a permutation, according to the interleaver of user k, of the i index in (11). The reliability information $\mathcal{R}_{\mathrm{SU}}$ is employed by the following assignment:

$$P(d_j) = P(d_j | \underline{\boldsymbol{y}})$$

which is *a priori* information in (5). This process is identical to the operation of a decoder for serial Turbo codes [14].

A. Approximate Solution

The trellis over which the CDMA MAP decoder operates has 2^{K-1} states which is too large for even moderate numbers of users. We shall employ a method proposed by Mehlan and Meyr for the ISI channel [19] that was adapted to the CDMA case[1] by Alexander and Rasmussen [20]. This previous work neglected the option of iterating that has been realized in this paper. Two approximations result in an approximate solution that is $O(K)$, instead of $O(2^{K-1})$ as for the full MAP decoder for the CDMA channel.

The first approximation is justified by an observation made by Wei and Schlegel [21]. Their results imply that an M algorithm applied to the forward recursion in (8) will return, at time j, M of the largest $p(\underline{\boldsymbol{y}}_1^j, \underline{\boldsymbol{d}}_{j-K+2}^j)$ with very high probability. The application of the M algorithm to the $\boldsymbol{F}$ channel of (3) is pivotal. The sum of (6) is over all 2^{K-1} states in the CDMA trellis. We shall replace this with a sum over states that are surviving paths in the M algorithm for the forward recursion and assume the other states have a negligible contribution.

$$p(\underline{\boldsymbol{y}}, d_j) \approx \sum_{\underline{\boldsymbol{d}}_{j-K+1}^{j-1} \in \mathcal{M}_j} p\Big(\underline{\boldsymbol{y}}, \underline{\boldsymbol{d}}_{j-K+1}^j\Big) \tag{14}$$

where $\mathcal{M}_j \subset \{-1, +1\}^{K-1}$ is the set of M states surviving at time j. This approximation impacts on the backward recursion.

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[1] The CDMA channel is a time-varying ISI channel.

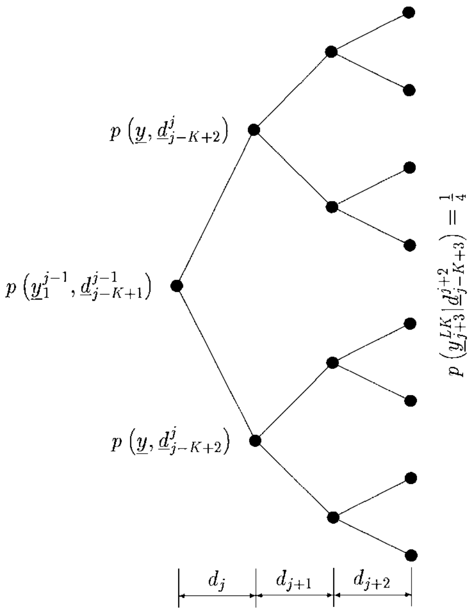

Fig. 3. Approximate MU CDMA MAP decoder step.

Specifically, we only require the $p(\underline{\boldsymbol{y}}_{j+1}^{LK} | \underline{\boldsymbol{d}}_{j-K+2}^{j})$ that grow from the states in $\mathcal{M}_j$.

The second approximation involves the backward recursion of (9). Again, following the development in [19] and [20], we cut the length LK recursion into LK recursions of depth two. There is little point reducing the complexity of the forward recursion if we still require a full computation of the backward recursion in order to compute the state probabilities. We could allow the forward recursion to execute then compute the required probabilities from the backward recursion. Given that we would like to use state probabilities to steer the forward recursion, we require results from both backward and forward recursions. Specifically we set

$$P\Big(\underline{\boldsymbol{y}}_{j+1}^{LK} | \underline{\boldsymbol{d}}_{j-K+2}^{j}\Big) \approx \sum_{d_{j+1}} P\Big(y_{j+1} | \underline{\boldsymbol{d}}_{j-K+2}^{j+1}\Big) P(d_{j+1}) \times \sum_{d_{j+2}} \frac{1}{4} P\Big(y_{j+2} | \underline{\boldsymbol{d}}_{j-K+3}^{j+2}\Big) P(d_{j+2}) \tag{15}$$

where it is apparent that we have set the initial condition $P(\underline{\boldsymbol{y}}_{j+3}^{LK} | \underline{\boldsymbol{d}}_{j-K+3}^{j+2})$ for each of the four recursions to 1/4. The bits d_{j+1} and d_{j+2} are set by the summations and bits d_{j-K+2} through d_j ($\underline{\boldsymbol{d}}_{j-K+2}^{j}$) are set by the surviving state from the forward recursion, i.e., an element of $\mathcal{M}_j$. This approximation is equivalent to assuming that the distant future values of $p(\underline{\boldsymbol{y}}_{j+1}^{LK} | \underline{\boldsymbol{d}}_{j-K+2}^{j})$ have minimal effect on the current value even though they are involved in the recursion of (8). Specifically, we have assigned a uniform distribution to $P(\underline{\boldsymbol{y}}_{j+3}^{LK} | \underline{\boldsymbol{d}}_{j-K+3}^{j+2})$ of which there are four candidates due to the possible values of the 2-tuple (d_{j+1}, d_{j+2}).

A graphical view of a step in the CDMA trellis is shown in Fig. 3. The binary tree of depth three is grown from each of the surviving paths in the forward recursion at time $j-1$. The new transitions pertain to channel symbols j, $j+1$, and $j+2$. The extension to symbol j creates $2M$ nodes. For each of these $2M$ nodes, the approximate backward recursion is computed using symbol intervals $j+1$ and $j+2$. The joint likelihood $p(\underline{\boldsymbol{y}}, \underline{\boldsymbol{d}}_{j-K+1}^{j})$ is then computed for each of the $2M$ nodes using (7). Equations (5) and (6) are then implemented for $d_j = 1$ and $d_j = -1$ to produce the required likelihoods $p(\underline{\boldsymbol{y}} | d_j)$. The M of $2M$ node selection process retains the nodes with largest $p(\underline{\boldsymbol{y}}, \underline{\boldsymbol{d}}_{j-K+1}^{j})$.

In the original work of Mehlan and Meyr [19], the depth of the backward approximation was one. With this depth look-ahead, a high level of noise in the look-ahead bit interval can steer the M algorithm off course. In our case we chose the look-ahead depth to be two in order to reduce the likelihood of such occurrences. Although higher depth look-aheads may be employed, the complexity of the system increases exponentially with the depth. As the complexity of the algorithm increases exponentially with the look-ahead depth, we do not consider depth three or more.

B. Complexity

As can be seen from Fig. 3 there are 14 conditional probabilities, of the form shown in (10), to be computed for the growth of each of the M surviving nodes. Using the number of such probability computations as the measure of complexity, we can show that the full complexity MU MAP requires 2^K per symbol interval, whereas the proposed scheme requires only $14M$. As will be shown, it is reasonable to set $M = K$ in order to achieve similar performance and, hence, our algorithm has complexity $O(K)$.

IV. SIMULATION

In this section, we present the performance of the proposed system. The ratio of the number of users K to the spreading code length N (or processing gain) is kept high for good spectral efficiency. The error control codes employed by the users were simple rate 1/2, four-state convolutional codes with generators $G = 5_8, 7_8$. If a more powerful code was employed, the single-user performance would improve and it is our expectation that the multiuser receiver proposed here would follow this improvement. We do not terminate the convolutional code trellis.

Since the system is aimed at packet radio systems, the users encode blocks of length $L_u = 100$. The chips $[s_{k,i}]_j$ constituting each user's binary spreading code are chosen randomly and independently across user (k), chip number (j) and symbol interval (i).

We first simulate the $K = 5$, $N = 7$ system which allows execution of the full-complexity MU MAP algorithm. With $K = 5$ the MU trellis has 16 states. The performance for full-complexity forward $(M = 16)$ recursion and full-complexity backward recursion is shown in Fig. 4 (FC 5/7) along with the approximate reduced complexity method with $M = 3$ (SO 5/7). With $K = 5$ there are 16 states in the full trellis. In both cases, results are shown for $I = 3$ iterations since no perceivable performance improvement was obtained by subsequent iteration.

Ideally, the performance of a multiuser receiver should be independent of the number of users so that quality-of-service can be guaranteed on a per user basis. This is not true as can

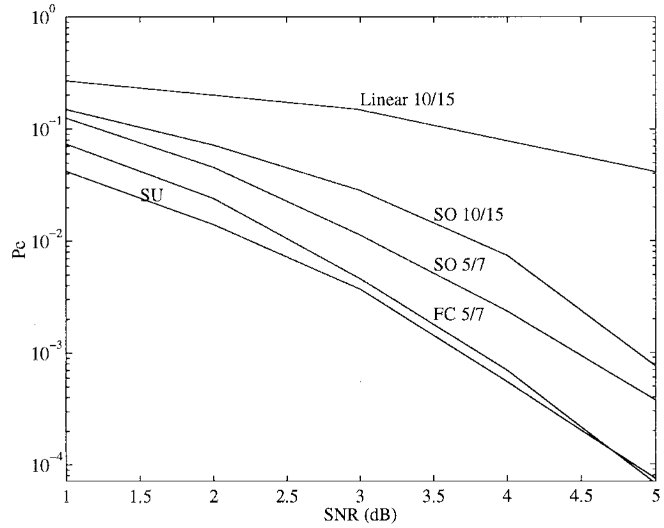

Fig. 4. Simulated system performance for $(K = 5, N = 7)$ and $(K = 10, N = 15)$.

be seen in Fig. 4 where plots for both $K = 1$ (SU) and $K = 5$ are shown. The effect of increasing the number of users from 1 to 5 is equivalent to reducing the power of each user by 0.5 dB. This loss is small compared with all other known linear systems that employ random codes or, equivalently, allow symbol asynchronism.

The full complexity MU MAP could not be executed for $K = 10$ since the trellis has 512 states. As can be seen in Fig. 4 (SO 10/15), the approximate solution of the MU MAP with $M = 3$ provides performance within 1 dB of the performance of the system in the absence of multiuser interference. The performance improvement of our system over the case where the MU CDMA trellis is decoded using a linear method [6] (Linear 10/15) is several decibels.

V. Conclusion

In this paper, we have proposed a multiuser receiver for the highly loaded asynchronous CDMA channel where each user encodes their data before transmission. The operation of the receiver is equivalent to a serial Turbo decoder in a single-user scenario. One of the decoders is for the CDMA channel which we view as a convolutional code and the other is for the single-user encoders. A suboptimal method was proposed that achieves performance remarkably close to single-user performance with complexity that is linear in the number of users. The number of users is set to be a large fraction of the processing gain in order to achieve good spectral efficiency. The results obtained in this work also indicate that there is no need to decode all of the users jointly in one large trellis. A reduced complexity solution for the multiuser trellis that outperforms the current proposal remains to be found.

It is important to question the impact of spectrally efficient CDMA, where K approaches N, on intercell interference in a cellular CDMA communications network. For the IS-95 [4] case, where no attempt is made to remove the impact of intercell users, the question had been addressed [22]. In [23], Newson *et al.* show that for propagation loss exponent four and shadowing variance 8 dB, half of the interference power at the output of a user's correlator comes from intracell users, and the other half comes from all other cells in the network. The proposed scheme is still valid when the channels are multipath fading, provided the multipath channels can be tracked to sufficient accuracy. Any error in the channel estimates will manifest as a noise floor in the error rate performance of the system. In this paper, we have described a receiver system that removes the intracell half of the interference power. This amounts to a 3-dB improvement, in terms of power control setpoint, with respect to conventional CDMA. Equivalently, twice as many users can now be supported as compared to conventional CDMA.

Further improvement can be obtained if the users at the fringe of the nearby cells are incorporated in the detection process in the base station of interest. In this way the power of the interfering users is removed from the intercell interference power. Note that since these users would be in a soft-handover state for IS-95, the infrastructure required is already in place.

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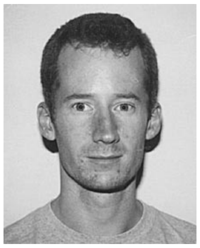

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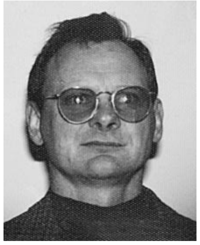

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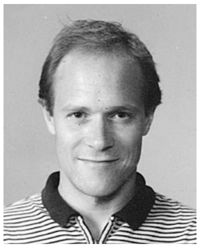

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