Approximated Maximum Likelihood Estimation of Carrier Frequency Offset in Practical OFDM Systems

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***Abstract*— This paper proposes a high-performance and relatively low-complexity approximated maximum likelihood estimator (MLE) for carrier frequency offset (CFO) in practical orthogonal frequency multiplexing (OFDM) systems, where the preamble has repetition structure in time domain, null subcarriers are allocated at two ends of the frequency spectrum, and the channel impulse response is assumably not longer than the cyclic prefix. We show the optimality of conventional MLEs do not hold in such systems, and derive a universal MLE and its modified Cramer-Rao lower bound in the general case. To reduce the complexity, we propose to cascade it with a low-complexity rough CFO estimator so that only a moderate number of maxima searches are needed to achieve satisfactory performance. Simulation with the IEEE 802.16 preamble symbol confirms the excellent performance of the proposed method.**

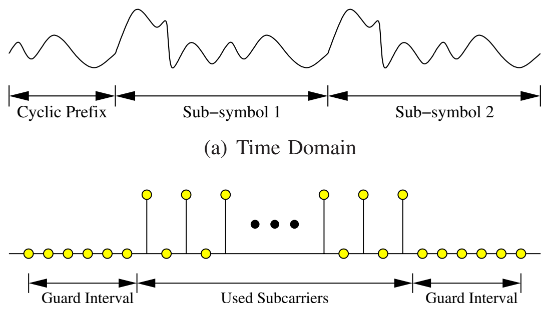


Fig. 1. Diagram of one preamble symbol in practical OFDM systems

I. Introduction

Carrier frequency offset (CFO) introduces inter-carrier interference to OFDM systems, which may seriously degrade the system performance [1].

Many CFO estimation methods have been proposed in the literature using the knowledge of preamble symbol. When the preamble consists of two repeating sub-symbols in time domain, Moose suggested a maximum likelihood estimator (MLE) in [1]. Later, a more general MLE for any number of repeating sub-symbols was proposed in [2]. Schmidl [3] showed the null subcarriers in the preamble can be utilized to expand the estimation range of Moose's method, while [4] suggested a blind CFO estimator that only requires the null subcarrier knowledge to work. Recently, it was reported in [5] that when there are neither time-domain repetition structure nor null subcarriers in the preamble, CFO estimation is still possible if the length of channel impulse response is known and shorter than one OFDM symbol period.

A diagram of the preamble symbol in practical OFDM systems like IEEE 802.16 [6] is shown in Figure 1. Usually the preamble symbol has a repetition structure in time domain, and several null subcarriers known as guard bands at two ends of the spectrum in frequency domain. The cyclic prefix that precedes every OFDM symbol is chosen to be longer than the channel impulse response so that the inter-symbol interference (ISI) can be eliminated. In such OFDM systems, we can show the aforementioned techniques are no longer optimal or efficient because they attempt to minimize the *a priori* information required by the estimators rather than jointly exploit all the information available about the preamble and channel.

This paper extends the work of [7] to practical OFDM systems where rough fractional CFO can be estimated with low complexity. We first derive a universal maximum likelihood CFO estimator in general OFDM systems, and then propose to cascade a low-complexity rough CFO estimator with the MLE so that only a few search steps are required to achieve virtually MLE performance.

II. System Model

We consider packet based OFDM systems where a known preamble symbol is transmitted at the start of every packet to provide initial channel and frequency offset estimation. The time-domain signal of the preamble symbol reads

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{L_X-1} X_n e^{j2\pi(f_0+\Delta_f \alpha_n)t} \tag{1}$$

where N is the number of subcarriers; f_0 is the central frequency; Δ_f is the subcarrier spacing; $\mathbf{X}$ is a known phase-shift keying (PSK) modulated sequence that satisfies $\mathbf{X}^H\mathbf{X} = \mathbf{I}_{L_X}$, where $\mathbf{I}_{L_X}$ is the identity matrix with order L_X; α_n is the index of the subcarrier that carries the n^{th} element of $\mathbf{X}$.

Each OFDM symbol is preceded by a cyclic prefix (CP) that is chosen to be longer than the channel impulse response to eliminate ISI. With perfect timing, after cyclic prefix removal, the OFDM preamble symbol at the receiver side is given by

$$\mathbf{y} = \sqrt{N}\, e^{j\phi_0}\mathrm{diag}(\mathbf{F}(\epsilon))\, \mathbf{W}^H\mathrm{diag}(\mathbf{X})\, \mathbf{V}\mathbf{h} + \mathbf{w} \tag{2}$$

where ϕ_0 is a constant phase difference between the transmitter and receiver; $\mathbf{w}$ is a vector of independent additive Gaussian noise samples whose variances are σ^2; $\mathbf{h}$ is the channel impulse response, which is assumed to be L_h long and invariant for one symbol period; $\mathbf{W}$ is part of the discrete Fourier transform (DFT) matrix whose entries are $W_{n,k} \triangleq \frac{1}{\sqrt{N}}e^{-j2\pi\alpha_n\frac{k}{N}}$, and the first L_h columns of $\mathbf{W}$ made up another $(L_X \times L_h)$ truncated DFT matrix $\mathbf{V}$. ϵ denotes the carrier frequency offset normalized by subcarrier spacing Δ_f, and the vector $\mathbf{F}(\epsilon) \triangleq [1, e^{j2\pi\epsilon/N}, \cdots, e^{j2\pi\epsilon(N-1)/N}]^T$ describes the phase rotating effect caused by frequency offset on each time domain samples at the reciver.

III. Universal Maximum Likelihood Estimator

In this section we derive a universal maximum likelihood CFO estimator in the general case, and provide some insights into its behavior. The modified Cramer-Rao lower bound (MCRLB) [8] is computed for performance analysis. In comparison, we show the conventional methods are not optimal for practical OFDM systems as described earlier in Section I.

A. Derivation

Define $\mathbf{A} \triangleq \sqrt{N}\, e^{j\phi_0}\mathbf{W}^H\mathrm{diag}(\mathbf{X})\, \mathbf{V}$. From (2), we can write the probability distribution of $\mathbf{y}$ for given $\mathbf{h}$ and ϵ as

$$p(\mathbf{y}|\mathbf{h},\epsilon) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}}\exp\left(-\frac{1}{2\sigma^2}\|\mathbf{y} - \mathrm{diag}(\mathbf{F}(\epsilon))\mathbf{A}\mathbf{h}\|^2\right). \tag{3}$$

The joint maximum likelihood estimator then maximizes

$$\Lambda(\mathbf{h},\epsilon) = -\|\mathbf{y} - \mathrm{diag}(\mathbf{F}(\epsilon))\mathbf{A}\mathbf{h}\|^2. \tag{4}$$

For any given ϵ, when $(\mathbf{A}^H\mathbf{A})$ is not singular, or equivalently, when the number of used subcarriers L_X is not less than the number of channel taps L_h, the maximum likelihood estimate of $\mathbf{h}$ exists [9], and can be written as a function of ϵ:

$$\hat{\mathbf{h}}(\epsilon) = \left(\mathbf{A^H A}\right)^{-1}\mathbf{A}\,\mathrm{diag}(\mathbf{F}(-\epsilon))\mathbf{y}. \tag{5}$$

Substituting (5) into (4), we can define a new objective function only about ϵ, and simplify it as

$$\begin{aligned}\tilde{\Lambda}(\epsilon) &\triangleq \Lambda(\hat{\mathbf{h}}(\epsilon),\epsilon) + \mathbf{y}^H\mathbf{y}\\ &= \mathbf{y}^H\mathrm{diag}(\mathbf{F}(\epsilon))\,\mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\,\mathrm{diag}(\mathbf{F}(-\epsilon))\mathbf{y}\\ &= \sum_{n=0}^{N-1}\sum_{k=0}^{N-1} y_n^*\, y_k\, D_{n,k}\, e^{j2\pi\epsilon(n-k)/N} \end{aligned}\tag{6}$$

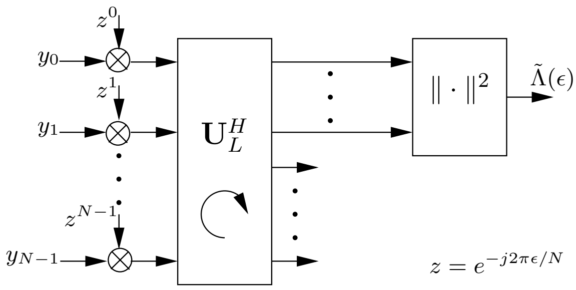


Fig. 2. An implementation of the objective function $\tilde{\Lambda}(\epsilon)$

where $D_{n,k}$ are elements of the matrix defined as

$$\mathbf{D} \triangleq \mathbf{W}^H\mathbf{X}\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H\mathbf{X}^H\mathbf{W}. \tag{7}$$

The universal CFO MLE can then be given by

$$\hat{\epsilon} = \arg\max_{\epsilon}\ \tilde{\Lambda}(\epsilon). \tag{8}$$

Although the exact channel length is unknown at the receiver, one still can take CP length as L_h, and calculate matrix $\mathbf{D}$ beforehand. So, there is no need to compute (7) in real time. However, solving (8) is a global maximum search problem that usually requires prohibitive complexity. In Section IV, an approximation to the global maximum search is proposed to reduce the complexity at a negligible loss on performance.

B. Explanation

Denote matrix $\mathbf{A}$ with its singular value decomposition as

$$\mathbf{A} = \mathbf{U}_L\,\mathbf{\Sigma}_A\,\mathbf{U}_R^H \tag{9}$$

where $\mathbf{U}_L$ and $\mathbf{U}_R$ are unitary matrices of rank L_X and L_h respectively, $\mathbf{\Sigma}_A$ is a $L_X \times L_h$ matrix whose off-diagonal elements are all zero and diagonal elements are singular values of $\mathbf{A}$. Expand (7) with (9) and simplify, we have

$$\mathbf{D} = \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H = \mathbf{U}_L\begin{pmatrix}\mathbf{I}_{L_h} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{pmatrix}\mathbf{U}_L^H. \tag{10}$$

Substituting (10) into (6), the objective function becomes

$$\tilde{\Lambda}(\epsilon) = \left\|\begin{pmatrix}\mathbf{I}_{L_h} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{pmatrix}\mathbf{U}_L^H\ (\mathbf{y}\odot\mathbf{F}(-\epsilon))\right\|^2 \tag{11}$$

where $\odot$ denotes vector inner product.

According to (11), we plot an implementation of the cost function in Figure 2, which gives more insights into the behavior of universal MLE. The estimator compensates the frequency offset of its observation $\mathbf{y}$ according to the given ϵ value, and removes the correlation in the compensated sequence by aligning it to the column vectors of $\mathbf{U}_L$. The goal of the estimator is to find the value of ϵ that maximizes the energy on the first L_h basis vectors of the space defined by $\mathbf{U}_L$, which is determined by the preamble symbol and channel length. It can be shown that in shorter channels the objective function is more selective, and thus has better performance. The extreme case is when $L_h = L_X = N$, the objective function loses all its selectivity, the estimator will be invalid.

C. Performance Analysis

The Cramer-Rao lower bound (CRLB) for joint channel and CFO estimation is derived in Appendix II of [10]. Under specific channel realization $\mathbf{h}$, the bound reads

$$\mathbf{CRB} = \frac{N\sigma^2}{8\pi^2}\left(\mathbf{h}^H\mathbf{A}^H\mathbf{M}\,\mathbf{S}\,\mathbf{M}\mathbf{A}\mathbf{h}\right)^{-1} \tag{12}$$

where $\mathbf{S} \triangleq \mathbf{I}_N - \mathbf{D}$, $\mathbf{M} \triangleq \mathrm{diag}(0, 1, \ldots, N-1)$.

Because the CRLB in (12) depends on the channel, it is not suitable for the cases where the channel is not fixed. Using the method of [8], we modify (12) to compute the average bound over all channel realizations that have the same covariance matrix $\mathbf{C_{hh}} \triangleq \mathbf{E}\left\{\mathbf{h}\,\mathbf{h}^H\right\}$.

$$\begin{aligned}\mathbf{E_h}\left\{\mathbf{CRB}\right\} &\geq \frac{N\sigma^2}{8\pi^2\,\mathbf{E_h}\left\{\mathbf{h}^H\mathbf{A}^H\mathbf{M}\,\mathbf{S}\,\mathbf{M}\mathbf{A}\mathbf{h}\right\}} \\ &= \frac{N\sigma^2}{8\pi^2\,\mathbf{tr}\left(\mathbf{A}^H\mathbf{M}\,\mathbf{S}\,\mathbf{M}\,\mathbf{A}\mathbf{C_{hh}}\right)} \\ &\triangleq \mathbf{MCRB}\end{aligned} \tag{13}$$

where the first inequality follows Jensen's inequality and the convexity of function $1/x$ for $x > 0$. Actually (13) gives the modified Cramer-Rao lower bound [8] of CFO estimation for joint CFO and channel estimators, which is also the lower bound for the proposed CFO MLE.

D. Optimality of Conventional Methods

We can show the conventional methods proposed by [1] and [2] are equivalent to the optimal universal MLE if and only if

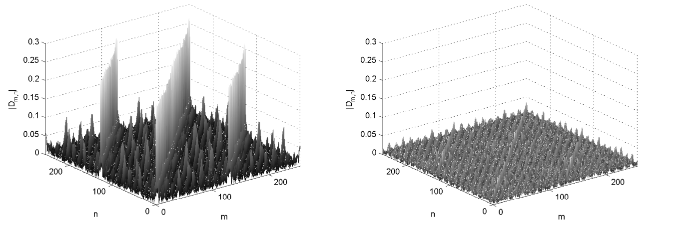
$$\mathbf{D} = \frac{L_X}{N}\begin{pmatrix}\mathbf{I}_{L_X} & \cdots & \mathbf{I}_{L_X} \\ \vdots & \ddots & \vdots \\ \mathbf{I}_{L_X} & \cdots & \mathbf{I}_{L_X}\end{pmatrix}_{N\times N}. \tag{14}$$

This implies the length of channel equals the number of used subcarriers in the preamble because $L_h = \mathrm{rank}(\mathbf{D}) = L_X$. From the expression of $\mathbf{D}$ in (14), we know N/L_X must be an integer, which implies there are no guard intervals at two ends of frequency spectrum.

We find the above mentioned conditions are violated in practical OFDM systems and the methods in [1] and [2] are no longer optimal. Figure 3 plots $|\mathbf{D}|$ matrices for IEEE 802.16 preamble symbol in $N/4$ and $N/32$ long channels. It is shown that the shorter the channel is, the bigger the difference is between $|\mathbf{D}|$ and that in (14), which leads to an increasing gap on performance between the proposed MLE and the conventional methods in [1] and [2]. This prediction is confirmed by our numerical results presented in Section V.

IV. Low-Complexity Approximation

The universal MLE has optimal performance, however, the global maximum search in (8) requires prohibitive computational complexity to realize. For example, if a mean square error (MSE) at 10^{-4} is targeted, we do brute force search at step length $L_s = 10^{-2}$ in the range $(-L_w, L_w)$, it takes $(200L_w)$ searches to generate one estimate. We therefore develop a low-complexity scheme to approximate the universal MLE performance. To be more specific, we cascade a rough CFO estimator with the proposed MLE to convert the global maximization problem to local maxima finding so that the number of searches can be reduced significantly.



(a) $L_h = N/4$ (b) $L_h = N/32$

Fig. 3. Matrix $|\mathbf{D}|$ for IEEE 802.16 preamble symbol in different channels

A. The Method

A diagram of the searching procedures is shown in Figure 4, and the algorithm can be described as follows.

1) **Rough Fractional CFO Estimation**.
Taking advantage of the repetition structure, a low complexity CFO estimator is able to provide a rough fractional CFO estimate $\epsilon_f \in (-\frac{K}{2}, \frac{K}{2})$, where K is the number of repeating sub-symbols in the preamble that the rough estimator takes into account.
2) **Integer CFO Search**.
In a set of integers $\mathcal{M} \triangleq \{m | -L_w \leq mK + \epsilon_f \leq L_w\}$, the integer CFO ϵ_i is defined by
$$\epsilon_i \triangleq \arg\max_{m\in\mathcal{M}} \tilde{\Lambda}(m\,K + \epsilon_f). \tag{15}$$
The integer CFO searching is illustrated in Figure 4 with circles for $\mathcal{M} = \{0, \pm 1, \pm 2\}$ and $\epsilon_i = 1$ case.
3) **Fractional CFO Refinement**.
To refine the estimate, do bisection search of the local maxima of $\tilde{\Lambda}(\epsilon)$ closest to $(\epsilon_f + \epsilon_i\,K)$ as shown in Figure 4 with crosses.

The first-stage fractional CFO estimator does not need to be accurate, because the value will be refined in step 3); it does not need a wide estimation range either, because step 2) can expand the range easily. In this case, low-complexity estimators like Moose's method [1] are preferable. Although originally that method only works for two repeating sub-symbols, when there are more than two repeating parts, one still can use Moose's method on every two consecutive repeating sub-symbols and take the mean of the estimates. Another option is to work on the longest two repeating sequences in the preamble, and set $K = 2$. Certainly, when higher complexity is affordable, the methods of [2], [11], [12] or their approximation can be considered.

The proposed integer CFO searching procedure is an extension of Schmidl's method [3] to more than two repeating sub-symbol case. When the number of repeating sub-symbol is two, the proposed method exploits the channel length knowledge and thus has better performance.

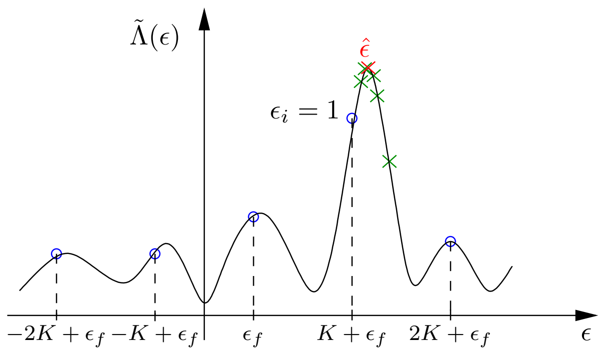


Fig. 4. Diagram of the searching procedures of an approximated MLE that covers estimation range $(-2K-1, 2K+1)$ subcarrier spacing with search window $\mathcal{M} = \{-2, -1, 0, 1, 2\}$

It takes $(2\lceil L_w/K \rceil - 1)$ searches to find the integer CFO ϵ_i, and $\lceil \log_2(K/L_s) \rceil$ searches to refine the fractional part. Totally, the number of searches is much less than $(2L_w/L_s)$ as the brute force method requires. For instance, if $K = 2$ and $L_s = 10^{-2}$, it takes $(L_w + 8)$ searches instead of $(200L_w)$ as we calculated earlier. Moreover, to reduce L_s by half, the approximated MLE only needs one extra search, whereas the brute force search method has to double its complexity to realize that.

B. Performance Analysis

First, we assume the integer CFO ϵ_i given by step 2) is correct and analyze the fractional CFO estimation performance. Because $\epsilon_f \in [-K/2, K/2]$, the bisection search starting point $(\epsilon_i K + \epsilon_f)$ is almost surely on the main lobe of $\tilde{\Lambda}(\epsilon)$. In this case, the bisection search must be able to find the true maximum and achieve MLE performance.

Second, we analyze the acceptable accuracy of the rough CFO estimator by an example. We approximate the rough CFO estimation error by a zero-mean Gaussian random variable whose variance is equal to the CRLB given in [2]

$$\mathrm{CRLB}_{\mathrm{Cnv}} = \mathrm{SNR}^{-1} \cdot \frac{3}{2\pi^2} \cdot \frac{1}{N} \left(1 + \frac{1}{K^2 - 1}\right). \tag{16}$$

For example, if Moose's method is used and the target SNR level is 0 dB, the rough CFO error has variance $2/(\pi^2 N)$, and it has a probability as high as 99.96% to fall in the range $[-0.07, 0.07]$. We know the repetition structure makes the distance between every two maxima of the objective function roughly 2, so there is virtually no performance loss for the approximated MLE to work at that SNR level.

V. Numerical Results

We use the preamble symbol defined in IEEE 802.16 standard [6] for simulations. The system uses $N = 256$ subcarriers, and the array of used subcarrier indices is

$$\alpha = [-100, -98, \cdots, -2, 2, \cdots, 98, 100]^T.$$

As all the odd subcarriers in the preamble are unused, the time-domain preamble consists of two repeating sub-symbols. Additionally, two guard intervals consist of 27 and 28 null subcarriers are allocated at two ends of the frequency spectrum. The standard also specifies the cyclic prefix can be $N/32$, $N/16$, $N/8$, or $N/4$ long.

All the simulations have assumed the channel to be as long as the cyclic prefix and have an exponential power delay profile with the power of k^{th} path equal to $e^{-k/5}$. The phases of each paths are modeled by independent random variables uniformly distributed in $[0, 2\pi)$.

A. Fractional CFO Estimation Performance

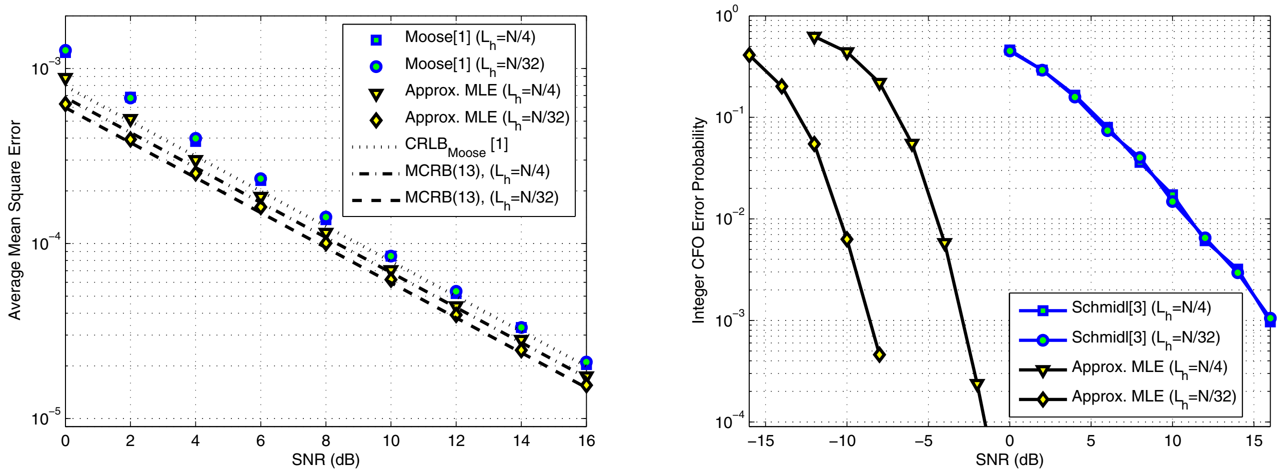
We evaluate fractional CFO estimation performance by mean square error (MSE). With perfect integer CFO knowledge, the approximated MLE uses Moose's method to provide a rough CFO estimate ϵ_f, and searches the local maxima closest to ϵ_f. To study the impact of channel length, we simulate in both $\frac{N}{4}$ and $\frac{N}{32}$ long channels. Figure 5(a) compares the MSE performance of the approximated MLE and that of Moose's method [1], where the CRLBs for conventional and proposed methods are also plotted as performance benchmarks. The true CFO is 0.48 subcarrier spacing, and every point in the figure is an average over 10^4 OFDM packets.

It is shown that the approximated MLE achieves the CRLBs we derived in Section III-C, and has better performance in shorter channels. Although Moose's method also approaches the $\mathrm{CRLB}_{\mathrm{Cnv}}$ given by (16), it does not benefit from the shorter channel and has 1-2dB performance loss comparing to the proposed estimator.

B. Integer CFO Estimation Performance

When the integer CFO estimate is different from the true value, we count integer CFO error by one, and compute the error probability as the ratio of the errors to the total number of tests. Figure 5(b) plots the integer CFO error probability of the approximated MLE and that of Schmidl's method [3]. In the simulations, both of the estimators are provided with perfect fractional CFO knowledge, which is 0.48. To support an estimation range of $(-11, 11)$ subcarrier spacing, the integer CFO search window is set to $\mathcal{M} = \{0, \pm 1, \pm 2, \ldots, \pm 5\}$. Because smaller integer CFO errors are more likely to happen than larger ones, we let the true integer CFO be 0, which corresponds to the worst case. At high SNR, most of the integer CFO errors are ± 2, so the error probability will be almost the same for all true integer CFO values within the estimation range except for ± 10, which are two ends of the search window. For these two values, the error probability can be reduced to half because either $(10 + 2)$ or $(-10 - 2)$ will be out of the search window and cause no errors.

It is shown in Figure 5(b) that the approximated MLE takes advantage of shorter channels, which gives 5.5 dB gain for $N/32$ long channel over $N/4$ ones. And, more than 12 dB gain is observed for the proposed MLE over Schmidl's method. The numerical results match our performance analysis in Section IV-B.



(a) Fractional CFO mean square error performance.

(b) Integer CFO error probability.

Fig. 5. Numerical results of the approximated CFO MLE in IEEE 802.16 system. True CFO is 0.48 subcarrier spacing.

VI. Conclusion

Since the well-known MLEs proposed in [1], [2] are no long optimal for realistic OFDM systems like IEEE 802.16, we derive a universal maximum likelihood CFO estimator to exploit all the available information about the preamble symbol and channel. We propose to cascade the universal MLE with a rough CFO estimator so that virtually optimal MLE performance can be achieved with relatively low complexity.

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