

# Dynamic Scheduling for a Turbo CDMA Receiver using EXIT Charts

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**Abstract**—In this paper we utilize extrinsic information transfer (EXIT) chart analysis in a coded direct-sequence code-division multiple-access (DS-CDMA) multiuser receiver to dynamically derive the optimal decoding schedule in the unequal user power level case. Conventional receivers generally follow a predetermined decoding schedule. However, decoding delay and complexity can be significantly reduced while maintaining BER performance through optimisation of the decoding schedule. We show dynamic scheduling, that is deriving the optimal schedule on-line after each iteration of the receiver, is a more flexible approach which improves the BER performance in comparison to the static (off-line) optimised schedule for similar complexity savings over the conventional receiver.

## I. INTRODUCTION

Power consumption and delay are important practical considerations for the design of an iterative multiuser detector (IMUD) receiver. Therefore, the optimal receiver decoding schedule is of interest. A conventional IMUD receiver in a multiuser code-division multiple access (CDMA) system follows a fixed decoding schedule, which is often less efficient (higher power consumption and longer delay) than is possible. The authors in [1] describe a technique for deriving the optimal schedule for decoding of an arbitrary number of concatenated codes. This was extended to the unequal power CDMA receiver in [2] where the optimal schedule was derived for a given system load. The schedule was derived off-line (static) at the convergence threshold and used for all signal-to-noise ratio (SNR) values as suggested in [1].

The scheduling algorithm works most effectively when the actual decoding trajectory matching the extrinsic information transfer (EXIT) chart predictions. While EXIT chart analysis is quite accurate in general, simulated trajectories do not always match exactly (see [3]) due to the limited interleaver (information block) length and system size (number of users). Furthermore, in a random additive white Gaussian noise (AWGN) channel, the noise variance varies from block to block and bursty errors occur such that the limited interleaver and the forward error correction (FEC) code are not able to stop the error propagation. As a result, in some blocks simulated trajectories deviate from the EXIT curves, which represents the statistical average of the information exchange property. We extend the scheduling algorithm to dynamically

derive the optimal schedule after every activation of the IC in order to compensate for these deviations from the predicted decoding trajectory, that is, on each iteration of the receiver we adjust the decoding schedule from the estimated position of the decoding trajectory.

We use the effective EXIT charts for an IMUD receiver [4], [5] in an unequal power system and extend the work in [2] through adding several features to the Viterbi search algorithm to enable *dynamic* derivation of the schedule which achieves a target (otherwise lowest possible) bit error rate (BER) with the lowest decoding complexity.

We show that our proposed scheduling algorithm results in a more consistent BER performance. Using static (as in [2]) scheduling, while the average BER may be under the target it is possible that burst errors will occur and some blocks will have high BER. That is, not all blocks achieve the target BER. Using our dynamic scheduling BER performance is improved for those blocks which fail using the static schedule.

## II. SYSTEM DESCRIPTION

We consider a turbo-coded CDMA system in which there are  $L$  groups with  $\mathbf{K} = [K_1, K_2, \dots, K_L]$  users at transmit power  $\mathbf{P} = [P_1, P_2, \dots, P_L]$ . Each user generates binary information sequences which are turbo encoded, interleaved and mapped onto BPSK symbols before being spread by unique direct sequence spreaders. The work in this paper is independent of the code used. In all simulations in this paper we use a 3GPP compliant turbo code (generator polynomial  $(G_r, G) = (015, 013)$ ) which is common for all users. The information block length is 3856 bits [6].

The IMUD receiver consists of an interference canceller (IC) and  $K^s = \sum_{i=1}^L K_i$  turbo decoders (TDs). See [2] for a description of the receiver and [7] for a good description of the turbo decoder.

The interference canceller takes channel values  $Y$  and a *priori* input  $A_k^{\text{IC}}$  (from each of the  $K^s$  users  $k = 1, 2, \dots, K$ ) and outputs extrinsic information (on the coded bits for each user)  $E_k^{\text{IC}}$  which is de-interleaved and becomes the *a priori* input  $A_k^{\text{TD}}$  to the TD for user  $k$ . On the first iteration of the receiver the *a priori* input to the interference canceller is zero. Each of the  $K^s$  TDs outputs extrinsic information (on the

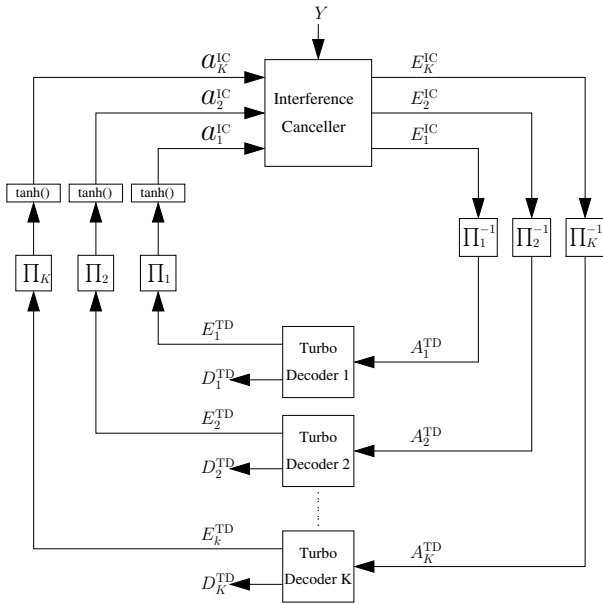


Fig. 1. IMUD receiver with interference canceller and turbo decoders.

coded bits)  $E_k^{\text{TD}}$  and a *posteriori* output (on the information bits)  $D_k^{\text{TD}}$ .  $E_k^{\text{TD}}$  is interleaved and converted to soft bits  $A_k^{\text{IC}} = \tanh(E_k^{\text{TD}}/2)$ . Hard decisions are made on  $D_k^{\text{TD}}$ .

### III. EXIT CHART ANALYSIS

In an unequal power CDMA system we group the users according to their power level. We assume all users within a power group are essentially identical and we therefore consider each group as a (virtual) single user. For convergence analysis, the EXIT charts are adjusted [4], [3], to reflect the behavior of the system under the unequal power conditions.

For the interference canceller, the *effective* EXIT function is [2]

$$I_{E,\text{eff}}^{\text{IC}} = f_{\text{mud}}(I_{A,\text{eff}}^{\text{IC}}, E_b/N_0) = J\left(\sqrt{\frac{4}{\left(1 - T^{-1}\left(I_{A,\text{eff}}^{\text{IC}}\right)\right)^{\frac{K_{\text{eff}}-1}{N}} + \frac{N_0}{2RP_{\text{ref}}}}}\right) \quad (1)$$

where  $J(\cdot)$  is the  $J$  function from [7],  $I_{A,\text{eff}}^{\text{IC}}$  is the effective prior mutual information for the IC,  $K_{\text{eff}} = \frac{1}{P_{\text{ref}}} \sum_{k=1}^L K_k P_k$  is the effective number of users,  $P_{\text{ref}}$  is some arbitrary reference power level,  $N$  is the processing gain,  $R$  is the code rate,  $N_0$  is the noise spectral density and  $T(\cdot)$  is the transfer function from [5] which describes mutual information  $I$  as a function of fidelity  $M$ ,

$$I = T(M) \approx 0.74M + 0.26M^2. \quad (2)$$

$I_{E,\text{eff}}^{\text{IC}}$  is calculated online from the TD outputs using [5], [4]

$$I_{E,\text{eff}}^{\text{IC}} = J\left(\frac{4}{1 - \frac{1}{L} \sum_{k=1}^L T^{-1}\left(I_{E,k}^{\text{IC}}\right)}\right) \quad (3)$$

where  $I_{E,k}^{\text{IC}}$  is estimated using [8]

$$I_{E,k}^{\text{IC}} = 1 - 2\mathbb{E}\left\{\frac{\log_2\left(1 + e^{-E_k^{\text{IC}}}\right)}{1 + e^{-E_k^{\text{IC}}}}\right\}. \quad (4)$$

We generate the EXIT chart for the TD $_k$ ,  $I_{E,k}^{\text{TD}} = f_{\text{dec}}(I_{A,\text{eff}}^{\text{TD}})$ , using Monte Carlo simulation. The *effective* EXIT function for group  $k$  is then

$$I_{E,k}^{\text{TD}} = f_{\text{dec}}\left(J\left(\sqrt{\frac{P_k}{P_{\text{ref}}}} J^{-1}\left(I_{A,\text{eff}}^{\text{TD}}\right)\right)\right), \quad (5)$$

where  $I_{A,\text{eff}}^{\text{TD}} = I_{E,\text{eff}}^{\text{IC}}$  is the effective prior mutual information for the TDs. We estimate  $I_{E,k}^{\text{TD}}$  and  $I_{D,k}^{\text{TD}}$  online using [8]

$$I_{\Lambda,k}^{\text{TD}} = 1 - 2\mathbb{E}\left\{\frac{\log_2\left(1 + e^{-\Lambda_{\text{TD}}}\right)}{1 + e^{-\Lambda_{\text{TD}}}}\right\}. \quad (6)$$

where  $\Lambda$  is  $E$  or  $D$ . The *effective* mutual information for the extrinsic output of the  $K$  TDs is calculated as [2]

$$I_{E,\text{eff}}^{\text{TD}} = 0.74 \left[1 - \sum_{k=1}^L \alpha_k^* \left(2.42 - \sqrt{2.03 + \frac{I_{E,k}^{\text{TD}}}{0.26}}\right)\right] + 0.26 \left[1 - \sum_{k=1}^L \alpha_k^* \left(2.42 - \sqrt{2.03 + \frac{I_{E,k}^{\text{TD}}}{0.26}}\right)\right]^2. \quad (7)$$

Now using (5) and (7) we express the *effective* TD EXIT chart as

$$I_{E,\text{eff}}^{\text{TD}} = f_{\text{dec}}\left(I_{A,\text{eff}}^{\text{TD}}\right). \quad (8)$$

Note that we derive the EXIT chart of the TD for  $i \in (1, 2, \dots, i_{\text{max}})$  iterations.

### IV. OPTIMAL SCHEDULING

We use an optimisation algorithm adapted for unequal power CDMA to optimise the decoding schedule such that the decoding complexity and delay (total number of TD iterations) are minimized while BER performance is maintained. The algorithm is modified from the algorithm proposed in [2] to account for the arbitrary starting point ( $I_{A,\text{eff}}^{\text{IC}} \neq 0$ ) and is summarised below. See [2] for a description of the decoding trellis.

#### A. Notation

Let  $m$  denote trellis transition. Each group is permitted  $i \in (1, 2, \dots, i_{\text{max}})$  iterations. Paths entering state  $n$  are defined as  $\mathbf{p}_m = (p_1, p_2, \dots, p_m)$  where  $p_j \in \{1, 2, \dots, i_{\text{max}}L + 1\}$  for  $1 \leq j \leq m-1$  and  $p_m = n$ . The metric for the corresponding path is defined as

$$\mathbf{v} = (\hat{P}_{b,1}, \dots, \hat{P}_{b,L}, C^{\text{IC}}, C^{\text{TD}}, I_{E,\text{eff}}^{\text{IC}}, I_{E,\text{eff}}^{\text{TD}}, I_{E,1}^{\text{TD}}, \dots, I_{E,L}^{\text{TD}}) \quad (9)$$

where complexity  $C^{\text{IC}}$  is the number of receiver iterations (IC activations) and  $C^{\text{TD}}$  is the total number of TD iterations. Complexity is updated as

$$C_m^{\text{IC}} = C_{m-1}^{\text{IC}} + 1 \quad \text{for an IC activation} \quad (10)$$

$$C_m^{\text{TD}} = C_{m-1}^{\text{TD}} + \begin{cases} i & \text{for a TD activation} \\ 0 & \text{otherwise,} \end{cases} \quad (11)$$

$$(12)$$

where  $i$  is the number of TD iterations. The receiver is permitted  $i^r \in \{1, 2, \dots, i_{\text{max}}^r\}$  iterations.

Note that the complexity metric is two-dimensional in contrast to one-dimension in [1]. This is due to our constraint on  $i^r$ .

Let  $I_{D,k}$  denote the mutual information of the *a posteriori* output from TD group  $k$ . It can be calculated as

$$I_{D,k} = J \left( \sqrt{J^{-1} \left( I_{A(s),k}^{\text{TD}} \right)^2 + J^{-1} \left( I_{E(s),k}^{\text{TD}} \right)^2} \right) \quad (13)$$

where  $A(s)$  and  $E(s)$  denote the *a priori* and extrinsic mutual information of the systematic bits, respectively. The expression in (13) can be used to estimate the BER of Group  $k$  as [7]

$$\hat{P}_{b,k} = Q(J^{-1}(I_{D,k})/2), \quad (14)$$

which are the  $L$  first elements in (9).

Define  $\mathcal{P}_m$  and  $\mathcal{V}_m$  as the sets of surviving paths and metrics respectively; and  $\mathcal{P}_{m,n} \subseteq \mathcal{P}_m$  and  $\mathcal{V}_{m,n} \subseteq \mathcal{V}_m$  as the sets of paths and metrics ending at state  $n$  after  $m$  trellis transitions. Define path  $\mathbf{p}^*$  with metric  $\mathbf{v}^*$  as the current (at transition  $m$ ) optimal path.

Define the metric initialisation function  $f_{\text{init}}$  where  $I_{E,\text{eff}}^{\text{IC}}$  is updated using (4) and (3),  $I_{E,k}^{\text{TD}}$  and  $I_{D,k}^{\text{TD}}$  using (6) and  $I_{E,\text{eff}}^{\text{TD}}$  using (7). This is done on-line after activation of the IC using the current  $E_k^{\text{IC}}$ ,  $E_k^{\text{TD}}$  and  $D_k^{\text{TD}}$ .

Define also the metric update function  $f_n$  for each state  $n$  [1], which updates all  $2L + 3$  elements in  $\mathbf{v}$  for all paths entering state  $n$  using (1), (5) (for  $i$  TD iterations), (7), (14) and (12).

Define domination as in [1], where metric  $\mathbf{v}$  dominates  $\mathbf{v}'$  if and only if the extrinsic mutual information  $v_q$  are higher than  $v'_q$  for  $q = L+2, L+3, \dots, 2L+3$ , respectively, and the complexity  $v_{L+1}$  is less than or equal to  $v'_{L+1}$ . Define target BER  $P_{\text{target}}$  as the desired BER of each group of users.

### B. Algorithm

The algorithm is divided into 2 parts - an off-line initialisation and the on-line Viterbi search. The initialisation component is as follows

- 1) Derive the EXIT chart for the load/power/SNR configuration of interest using the results from Section III (note that  $I_E = f_{\text{dec}}(I_A)$  must be generated using Monte Carlo simulation)
- 2) Determine the convergence point  $I_D^*$ , the intersection of the TD EXIT curve with the interference canceller curve
- 3) Calculate the convergence BER  $P^* = Q(J^{-1}(I_D^*)/2)$

The Viterbi search algorithm is as follows

- 1) Let  $m = 1$ . Initialize path set to contain only one path  $\mathcal{P}_m = \{(1)\}$  and corresponding metric set  $\mathcal{V}_m = \{f_{\text{init}}\}$ . Initialize  $\mathbf{p}^* = 1$  and  $v_{L+1}^* = \infty$ .
- 2)  $m = m + 1$ . For each state  $n'$  extend each path  $p'_{m-1}$  ending in state  $n'$  along the trellis defined transition  $n' \rightarrow n$ , producing the new path  $\mathbf{p}_m$  in  $\mathcal{P}_{m,n}$ , and update the metric in  $\mathcal{V}_{m,n}$  using  $\mathbf{v} = f_n(\mathbf{v}')$ .
- 3) Remove all paths with complexity greater than or equal to that of the current optimal path  $\mathbf{p}^*$ .
- 4) Define a set of metrics  $\mathcal{V}^*$  for paths that have reached the target BER ( $v_q \leq P_{\text{target}}, \forall q = 1, 2, \dots, L$ ), the convergence point  $I_D^*$  or  $i_{\text{max}}^r$  receiver iterations. If there are multiple paths in  $\mathcal{V}^*$  replace the candidate path  $\mathbf{p}^*$  with the path of the lowest complexity.
- 5) For each state, eliminate dominated metrics and their corresponding paths. If  $P^* < P_{\text{target}}$  eliminate paths in  $\mathcal{V}^*$  with any  $(\hat{P}_{b,1}, \hat{P}_{b,2}, \dots, \hat{P}_{b,L}) > P_{\text{target}}$ .
- 6) If no paths remain in  $\mathcal{V}_m$  the candidate path  $\mathbf{p}^*$  is the optimal path. Otherwise go to step 2.

## V. SIMULATION RESULTS AND DISCUSSION

A turbo coded unequal power CDMA system was simulated with  $K = [20, 20]$ ,  $P = [1, 2]$  and spreading factor  $N = [40]$ . We set the full decoding schedule as 4 receiver iterations where both groups run 6 TD iterations. The static schedule is the optimal schedule as derived in [2] at the convergence threshold  $E_b/N_0 = 1.15\text{dB}$  (where  $E_b = P_1$ ),

$$\text{IC} \rightarrow \text{TD}_{1,1} \rightarrow \text{TD}_{2,6} \rightarrow \text{IC} \rightarrow \text{TD}_{1,5} \quad (15)$$

$$\rightarrow \text{IC} \rightarrow \text{TD}_{1,6} \rightarrow \text{IC} \rightarrow \text{TD}_{1,6} \rightarrow \text{TD}_{2,1} \quad (16)$$

Unless specified otherwise, all BER values are the system average, calculated as

$$\hat{P}_b = \frac{1}{K} \sum_{k=1}^L K_k \hat{P}_{b,k}. \quad (17)$$

BER performance is plotted versus SNR in Fig. 2, where the full schedule (crosses), static schedule (triangles) and the dynamic schedule (circles) are shown. We see that BER performance of the dynamic schedule is very similar to that for the full decoding schedule. Fig. 3 shows the average total number of TD iterations used (per block), we see that dynamic scheduling gains a significant reduction in complexity. We also see that our dynamic scheduling algorithm has a small performance gain over static scheduling. For low  $E_b/N_0$  this comes at a cost as dynamic scheduling has higher complexity. However, above the convergence threshold (approx. 1.15dB), in the operating region of the system, dynamic scheduling has a complexity saving over static scheduling which grows with SNR. This is the advantage of dynamic scheduling, the ability to compensate for differences in the decoding trajectory from EXIT chart analysis allows the schedule to be adapted. More iterations are allocated when required and less are used when possible, the result is a more consistent BER performance. This is highlighted in Fig. 4 and Fig. 5 which give some insight

into distribution of packet errors (packet being an information block of 3856 bits). Fig. 4 shows the number of errors (for all users) per packet for randomly selected simulated packets at 1.1dB (below the convergence threshold). We see that some packets are virtually error free, however some packets (such as packets 6 and 12) have a large number of errors due to bad channel conditions. The full decoding schedule shows the performance the receiver can achieve and we see that the static schedule suffers significantly degraded performance, while the dynamic is very close to the full schedule. Fig. 5 shows a histogram of packet errors at 1.2dB, (above the convergence threshold) for 1000 randomly simulated packets. Again we see the full decoding schedule and our dynamic schedule reduce the number of errors in almost all packets to zero, while the static schedule has a greater number of packets with errors. This indicates the inflexibility of the statically derived schedule and the advantage of our dynamic schedule which can adapt decoding to suit the noise level in the received data packet.

One point to note is the complexity of the scheduling algorithm in comparison to the complexity savings realized. It is possible the number of surviving paths in the algorithm grows exponentially, however this has not been observed [1], [2] in practice. However, with a large number of groups and a large number of TD iterations ( $i$ ) the number of states and surviving paths can grow quite large. Remembering that even 1 TD iteration requires 2 activations of the BCJR algorithm and is as such high in complexity, the savings in general outweigh the cost. If, however, complexity of the algorithm is an issue measures such as

- reducing the number of survivor paths as in [9]
- limiting the number of survivor paths
- truncating the number of possible TD iterations  $i$  to some subset of  $i$
- running scheduling algorithm every  $x^{\text{th}}$  receiver iteration where  $x > 1$

can be explored.

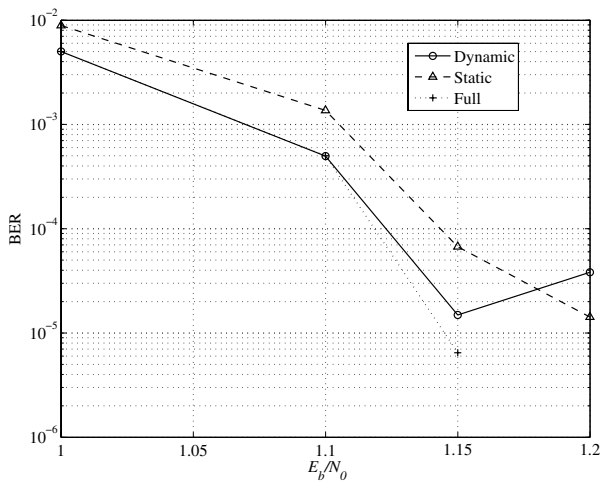


Fig. 2. BER performance of unequal power CDMA system for IMUD receiver following the optimal and static decoding (from Section V) schedules.

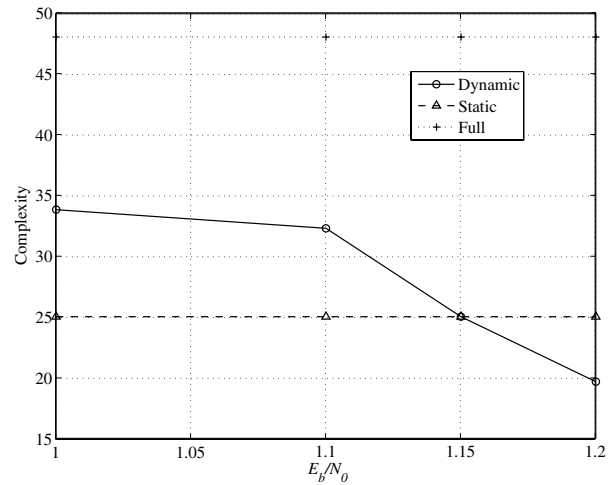


Fig. 3. Complexity of unequal power CDMA system for IMUD receiver following the optimal and static decoding (from Section V) schedules.

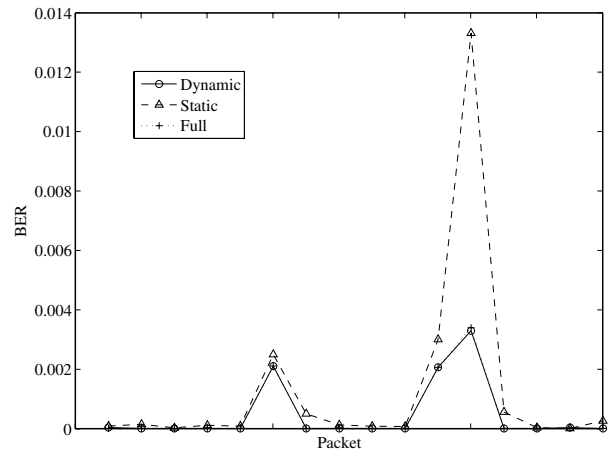


Fig. 4. Number of Errors per Packet (random simulated packets) for Dynamic, Static and Full Decoding Schedules at 1.1dB.

## VI. CONCLUSION

We utilised EXIT chart analysis of unequal power turbo-coded CDMA to dynamically derive the optimal decoding schedule. The *effective* EXIT functions for FEC decoders and an interference canceller from [3] were used which enabled analysis of the system as in the equal power case. We modified the algorithm in [2] to enable an arbitrary start point in the trellis search and facilitate dynamic scheduling. We showed through simulation that dynamic scheduling has a small performance gain over static scheduling and achieves similar BER performance as a conventional receiver. Furthermore we showed our proposed dynamic scheduling algorithm produces more consistent BER performance with similarly significant complexity savings over the conventional receiver as static scheduling.

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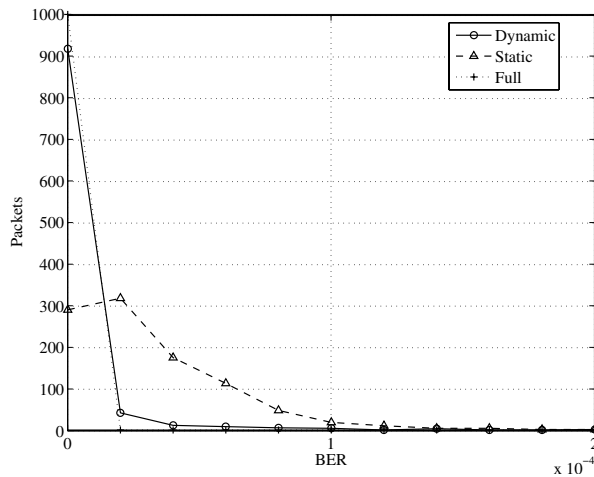


Fig. 5. Histogram of Per Packet Errors for Dynamic, Static and Full Decoding Schedules at 1.2dB.

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