# Low-Complexity Partitioned-Spreading CDMA System with Multistage MMSE Reception

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***Abstract*—**

**In this paper, we propose a low-complexity multistage filter for the partitioned-spreading CDMA (PS-CDMA). The proposed method embodies a MMSE filter that operates on the soft output of interference cancellation. Extrinsic information on the partitioned symbol is exchanged between the MMSE filter and the partitioned-spreading APP detector to improve the system performance. It is shown via variance evolution the signal-to-noise ratio of this novel technique achieves that of the single-user channel even for heavily loaded systems. It also outperforms IDMA by a fraction of dB to a few dBs, for a variety of system loads.The complexity of the proposed system is much lower that of its counterparts for conventional CDMA, which makes it a viable alternative for future 3G or beyond 3G systems.**

## I. INTRODUCTION

Code-division multiple-access (CDMA) is a communication system where a number of users can simultaneously access the common channel. Different transmissions are distinguished from each other by the user-specific spreading sequences. In reality, the random access nature of the wireless mobile network introduces cross-correlations between users, and multiuser detection [3], [4], [5], [13] has been used to improve the system performance, at the cost of increasing the complexity at the receiver.

Recently, a new access technique named *interleave-division multiple-access* (IDMA) [7], [8], [9], [10] has been proposed. It is shown that IDMA outperforms conventional CDMA with low-cost iterative soft-cancellation. IDMA requires chip-level interleavers to break up the correlation among spreading sequences. In addition, the receiver of IDMA is run at the chip rate, a working frequency that is typically difficult to achieve in reality. Schlegel and Kempter propose the partitioned-spreading CDMA (PS-CDMA) [1], [2]. In retrospect, PS-CDMA avoids the problem associated with large interleaving functions and high receiving frequency by partitioning the original CDMA spread signal into a few portions and transmitting the partitioned sub-symbols in random order. It has been shown in [1] that significant improvement can be achieved on both connectivity and system throughput by PS-CDMA in a random packet multiple-access network, compared to CDMA.

In this paper, we study the possibility of applying advanced signal processing technique to PS-CDMA. A multistage MMSE demodulator is developed to enhance the system performance. It is illustrated via variance evolution (VE) analysis that this MMSE-based reception outperforms its IC-based counterpart over stages. BER performance of the proposed system is then compared to that of the PS-CDMA systems in [1], [2] and IDMA [7] for a variety of system loads. In all cases the new design achieves a power gain ranging from a fraction of dB to a few dBs, with the largest gain being accomplished at heavy loads. Moreover, the computational load of the MMSE receiver for PS-CDMA is shown to be at the same order of magnitude of that for IDMA, which makes it feasible for practical implementation.

The rest of the paper is organized as follows. PS-CDMA system is reviewed in Section II. A multistage MMSE demodulator is presented in Section III. The performance of the new system is analyzed in Section IV via variance evolution, a technique suitable for tracking the dynamics of large-scaled multiple-access systems. In particular, explicit variance evolution function is derived for the equal-power channel as the worst case scenario. The BER performance and complexity of the new system are presented together with those for IDMA in Section V. Conclusions are outlined in Section VI.

## II. PS-CDMA SYSTEM MODEL

Figure 1 shows the transmitter diagram of a generic PS-CDMA system [1], [2]. $K$ users independently generate data stream $d_k$, which are then spread with user-specific sequences. We assume $d_k$ is BPSK modulated, while extension to higher modulations is straight-forward. The signal at the output of spreader for $k$-th user is given by[1]

$$s_k(t) \quad = \quad \sum_{l=0}^{L-1} \sqrt{P}_k d_{k,l} a_{k,l}(t - lT), \tag{1}$$

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[1] Here we consider a synchronous system, for the sake of brevity.

where $d_{k,l}$ is the $l$-th symbol of user $k$, $P_k$ is the receive power of user $k$, $T$ is the time duration of one symbol, and $L$ is the number of symbols per block. In (1), $a_{k.l}$ is the spreading sequences of $k$-th user at symbol $l$, given by

$$a_{k,l}(t) \quad = \quad \sum_{n=0}^{N-1} \frac{1}{\sqrt{N}} a_{k,l,n} g(t - nT_c) \qquad (2)$$

where $a_{k,l,n} \in \{-1, +1\}$ are random chips that are chosen independently, $g(t)$ is the normalized waveform that is supported on the interval $[0, T_c]$, and $T_c = T/N$ is the chip period.

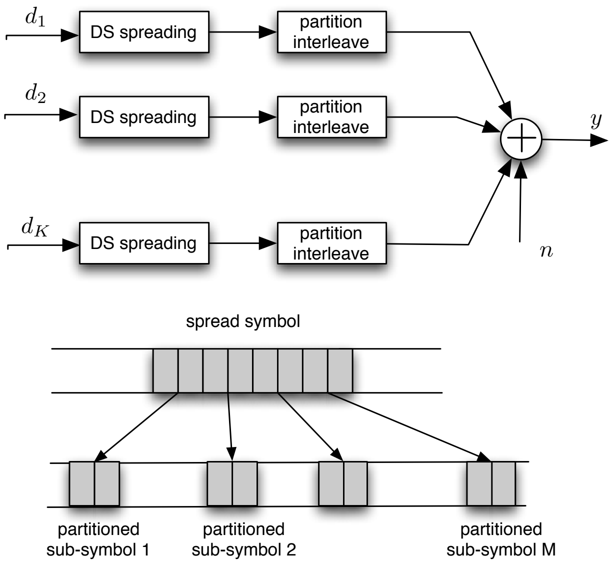

Fig. 1. Transmitter diagram for a generic PS-CDMA channel.

In Figure 1, each spread CDMA symbol is evenly partitioned into $M$ segments.[2] Those partitioned sub-symbols are then interleaved and transmitted over a common channel. The size of the interleaver for each user is $ML$, which is substantially smaller than that of IDMA for a moderate $M$. The receive signal of PS-CDMA is

$$\begin{aligned} y(t) = \sum_{k=1}^{K} x_k(t) + n(t) = \sum_{k=1}^{K} \sqrt{\frac{P_k}{M}} \sum_{l=0}^{L-1} d_{k,l} \\ \sum_{m=0}^{M-1} h_{k,l,m}(t - \pi(lM + m)MT_c) + n(t). \qquad (3) \end{aligned}$$

In (3), $h_{k,l,m}$ is the normalized spreading sequence of user $k$ at symbol $l$, for partition $m$, given by

$$h_{k,l,m}(t) = \sum_{n=0}^{N/M-1} \sqrt{\frac{M}{N}} a_{k,l,n+mN/M}\, g(t - nT_c), \qquad (4)$$

and $\pi(j)$ is the interleaving function that breaks up the correlations between the partitioned portions that belong to the same spread symbol. The $M$ partitioned sub-symbols constitute a rate-$1/M$ repetition codeword of the original symbol. As a result, the PS-CDMA channel can be considered as a serial concatenation of repetition code and the CDMA channel of dimension $N/M$, with interleaving function in between. Here $\pi()$ serves the purpose analogous to that used for turbo codes, i. e., to improve time diversity and generate a long code from a short block code.

## III. MULTISTAGE MMSE FILTER

In this section, we propose a multistage MMSE filter to resolve the PS-CDMA signal in (3). After perfect sampling, the $j$-th PS-CDMA symbol can be expressed as

$$\mathbf{y}_j \quad = \quad \sum_{k=1}^{K} \sqrt{\frac{P_k}{M}} \mathbf{a}_{j_k,m_k} d_{j_k} + \mathbf{n}_j, \qquad (5)$$

where $j_k = \lceil \pi_k^{-1}(j)/M \rceil$, $m_k = \pi_k^{-1}(j) - \lfloor \pi_k^{-1}(j)/M \rfloor M$, $\mathbf{a}_{j,k}$ denotes the normalized spreading chip for $m_k$-partition of $j_k$-th symbol for user $k$, given by

$$\begin{aligned} \mathbf{a}_{j_k,m_k} = \sqrt{\frac{M}{N}} [a_{k,j_k,(m_k-1)N/M+1}, \\ a_{k,j_k,(m_k-1)N/M+2}, \ldots, a_{k,j_k,m_k N/M}]^T. \qquad (6) \end{aligned}$$

Without causing confusion in further analysis, we ignore the time index $j$, and (5) becomes

$$\mathbf{y} \quad = \quad \sum_{k=1}^{M} \sqrt{\frac{P_k}{M}} \mathbf{a}_{k,m} d_k + \mathbf{n}. \qquad (7)$$

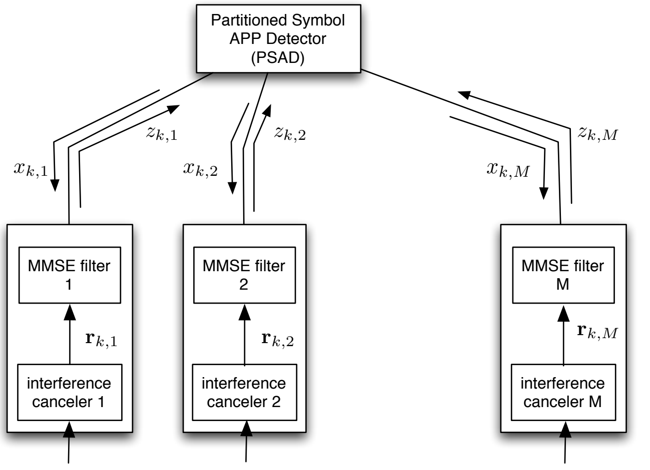

Fig. 2. Multistage MMSE demodulator diagram for $M$-partitioned PS-CDMA system.

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[2] For operating ease, we assume $M$ divides $N$.

Figure 2 shows the multistage MMSE demodulator for a PS-CDMA symbol. At each stage, the multiple-access interference is first estimated and cancelled, given by user $k$ as

$$\begin{aligned}\mathbf{r}_{k,m} = \mathbf{y} - \sum_{i \neq k} \sqrt{P_i/M}\mathbf{a}_{i,m}\hat{d}_i &= \sqrt{P_k/M}\mathbf{a}_{k,m}d_k \\ &+ \sum_{i \neq k} \sqrt{P_i/M}\mathbf{a}_{i,m}(d_i - \hat{d}_i) + \mathbf{n}, \qquad (8)\end{aligned}$$

where $\hat{d}_k = \sum_d p^{(A)}(d_k = d)d$ is the soft estimate for $d_k$ based on the *a priori* information. A linear filter is applied to $\mathbf{r}_{k,m}$ to further improve the system performance. We adopt the soft-input soft-out (SISO) MMSE filter analogous to that in [5], given by

$$\begin{aligned}\mathbf{w}_{k,m} &= \min_{\mathbf{w}} E[(d_k - z'_{k,m})^2] \\ &= \frac{\frac{P_k}{M}\mathbf{K}^{-1}_{k,m}\mathbf{a}_{k,m}}{1 + \frac{P_k}{M}\mathbf{a}^T_{k,m}\mathbf{K}^{-1}_{k,m}\mathbf{a}_{k,m}}. \qquad (9)\end{aligned}$$

In (9), $\mathbf{K}_{k,m} = \mathbf{A}_{-k,m}\mathbf{V}_{-k}\mathbf{A}^T_{-k,m} + \sigma^2\mathbf{I}$, $\mathbf{A}_{-k,m} = [\mathbf{a}_{1,m}, \mathbf{a}_{2,m}, \ldots, \mathbf{a}_{k-1,m}, \mathbf{a}_{k+1,m}, \ldots, \mathbf{a}_{K,m}]^2$, $V_{-k} = \mathrm{diag}\left[\frac{P_1}{M}\sigma^2_{d,1}, \ldots, \frac{P_{k-1}}{M}\sigma^2_{d,k-1}, \frac{P_{k+1}}{M}\sigma^2_{d,k+1}, \ldots, \frac{P_K}{M}\sigma^2_{d,K}\right]$ with $\sigma^2_{d,k} = E[(d_k - \hat{d}_k)^2]$ being the symbol error variance for user $k$.

$z'_{k,m} = \mathbf{w}^T_{k,m}\mathbf{r}_{k,m}$ is the soft output of the MMSE filter, whose log-likelihood ratio (LLR) is generated as

$$z_{k,m} = 2\frac{\mu(z'_{k,m})}{\sigma^2(z'_{k,m})}z'_{k,m}, \qquad (10)$$

with $\mu(z'_{k,m})$ and $\sigma^2(z'_{k,m})$ are the mean and variance of $z'_{k,m}$, respectively.

The partitioned-spreading APP detector (PSAD) collects the LLRs of sub-symbols to form the extrinsic LLRs for portion $m$, using the *a posteriori* probability (APP) decoding principle

$$x_{k,m} = \sum_{i \neq m} z_{k,i}, \qquad (11)$$

which are then fed back to the corresponding MMSE filter for next stage. The exchange of extrinsic information between RND and MMSE modules improves the reliability of LLR pair $(\mathbf{z}_{k,m}, x_{k,m})$. APP result $u_k = \sum_{m=1}^{M} z_{k,m}$ of $d_k$ is obtained after a certain number of stages.

## IV. VARIANCE EVOLUTION ANALYSIS

The PS-CDMA system can be deemed as the concatenation of repetition codes (rate-$1/M$) and a conventional CDMA channel. The dimension (spreading gain) of the effective CDMA channel for the partitioned sub-symbol is $N/M$, and the signal-to-noise ratio of PS-CDMA channel decreases by a factor of $M$, as shown in (3). Hence the performance of the multiuser detectors of PS-CDMA is different from that of their CDMA counterparts [4], [5], in particular, it depends on the number of partitions $M$.

### A. *Variance Evolution Function*

We investigate the performance of the MMSE demodulator over stages. This is accomplished by studying the dynamics of the input-output variance relationship of the detecting modules. It can be inferred from (10) that $z_{k,m}$ is Gaussian consistent, that is, $z_{k,m} \sim \mathcal{N}(\frac{2}{\sigma^2_{z,k}}, \frac{4}{\sigma^2_{z,k}})$, with $\sigma^2_{z,k}$ being the normalized variance of $z_{k,m}$. Extending the results by Tse and Hanly [6] for large-scaled systems, the noise variance of PSAD input $z_{k,m}$ is obtained as

$$\begin{aligned}\sigma^2_{z,k} &= \frac{\sigma^2(z'_{k,m})}{\mu^2(z'_{k,m})} \\ &= \left(\sigma^2 + \sum_{l=1}^{J} \frac{K_l}{N} \frac{P_l\sigma^2_{d,l}\frac{P_k}{M}\sigma^2_{z,k}}{\frac{P_l}{M}\sigma^2_{d,l} + \frac{P_k}{M}\sigma^2_{z,k}}\right)\frac{M}{P_k}. \qquad (12)\end{aligned}$$

The extrinsic LLR of PSAD output $x_{k,m}$ is also Gaussian consistent, with its statistical properties fully represented by the normalized variance

$$\begin{aligned}\sigma^2_{x,k} = \frac{1}{M-1}\sigma^2_{z,k} = \frac{M}{(M-1)P_k} \\ \left(\sigma^2 + \sum_{l=1}^{J} \frac{K_l}{N} \frac{P_l\sigma^2_{d,l}(M-1)P_k\sigma^2_{x,k}}{P_l\sigma^2_{d,l} + (M-1)P_k\sigma^2_{x,k}}\right). \qquad (13)\end{aligned}$$

A normalized variance measure is defined as

$$\bar{\sigma}^2_x = \frac{P_k\bar{\sigma}^2_{x,k}}{\bar{P}} = \frac{M}{M-1}\left(\frac{1}{2\gamma} + \frac{\sum K_l}{N}\bar{\sigma}^2_d\right), \qquad (14)$$

where $\bar{P} = \sum_{l=1}^{J} \frac{K_l}{\sum K_l}P_l$ is the average power, $\bar{\sigma}^2_d = \sum_{l=1}^{J} \frac{K_l}{\sum_j K_j}\left(\frac{P_l\sigma^2_{d,l}(M-1)\bar{\sigma}^2_x}{P_l\sigma^2_{d,l} + (M-1)\bar{P}\bar{\sigma}^2_x}\right)$ is the effective symbol error variance. $\bar{\sigma}^2_x$ is independent of the individual receive power, hence the user index $k$ is dropped in (14).

### B. *Effect of $M$ on MMSE Performance*

From (13), it can be derived that the normalized variance $\bar{\sigma}^2_x$ is closely linked with the number of partitions $M$ in two places. One is the scaling factor $M/(M-1)$ that monotonically decreases with $M$. This is because that PSAD increases the reliability of the extrinsic LLRs by $(M-1)$-fold by combining $(M-1)$ incoming messages. The other is the *effective interference* [6] from PS-CDMA users in $j$-th power group to the target user in $k$-th group, given by

$$\sigma^2_{\mathrm{eff},j} = \frac{P_j\sigma^2_{d,j}(M-1)P_k\sigma^2_{x,k}}{P_j\sigma^2_{d,j} + (M-1)P_k\sigma^2_{x,k}}, \qquad (15)$$

which, on the other hand, becomes larger with increasing $M$. This degeneration is due to that each sub-symbol of PS-CDMA has a dimension of $N/M$. Therefore, the advantage of MMSE filter in addition to interference cancellation is diminishing w.r.t. $M$. An extreme case is IDMA, where

MMSE filter is trivial, and renders the same performance as that of interference canceler.

By and large, SISO MMSE filter still provides significant improvement over the basic interference cancellation for general PS-CDMA, especially when $M$ is moderate. We show this in the following section.

### C. *Equal Power Case*

The variance evolution of PS-CDMA with an arbitrary power profile can be simplified by observing the variance $\bar{\sigma}_x^2$ of the equivalent equal power channel in (14), and the performance of individual power group can be easily scaled by $\sigma_{x,k}^2 = \bar{P}\bar{\sigma}_x^2/P_k$. We focus on the equal power case in this section, since it is the foundation to study general scenarios. In the equal power case, i.e., $P_k \equiv P$, (14) can be simplified to

$$\sigma_x^2 \;=\; \frac{M}{M-1}\left(\frac{1}{2\gamma} + \alpha\frac{\sigma_d^2(M-1)\sigma_x^2}{\sigma_d^2+(M-1)\sigma_x^2}\right), \qquad (16)$$

where $\gamma = P/(2\sigma^2)$ and $\alpha = K/N$. Re-arrange the equation, we can get

$$\frac{(M-1)^2}{M}\sigma_x^4 + (M-1)\left[(\frac{1}{M}-\alpha)\sigma_d^2 - \frac{1}{2\gamma}\right]\sigma_x^2 - \frac{\sigma_d^2}{2\gamma} = 0. \qquad (17)$$

The solution to (17) is given by

$$\sigma_x^2 = \frac{\phi(M) + \frac{M}{2\gamma} + (\alpha M - 1)\sigma_d^2}{2(M-1)}, \qquad (18)$$

where $\phi(M) = \sqrt{(\alpha M-1)^2\sigma_d^4 + \frac{\alpha M+1}{\gamma}\sigma_d^2 + \frac{M^2}{4\gamma^2}}$.

To investigate on the effect of the number of partitions $M$ on $\sigma_x^2$, we take the derivative of $\sigma_x^2$ in terms of $M$, as shown in (18). It can be seen that for over-loaded PS-CDMA, i. e. , $\alpha > 1$, $\partial\sigma_x^2/\partial M < 0$, which implies that the performance of multistage MMSE filter improves as $M$ increases.

Figures 3 and 4 show the output signal-to-noise ratio $\gamma(u_k)$ of multistage filters which are based on interference cancellation and MMSE filter, respectively. Figure 3 is for PS-CDMA with $K = 64$ users. It is shown that for PS-CDMA with small number of partitions, say, $M = 4$, the improvement of MMSE over IC is significant. However, this advantage diminishes as $M$ increases, due to the reason mentioned in previous section. Figure 4 is for PS-CDMA of $K = 96$ users, where the improvement by MMSE filter is more significant than that in less-loaded cases. It can be seen in both figures that the multistage MMSE filter performance is improving as $M$ increases, which verifies the result in (19).

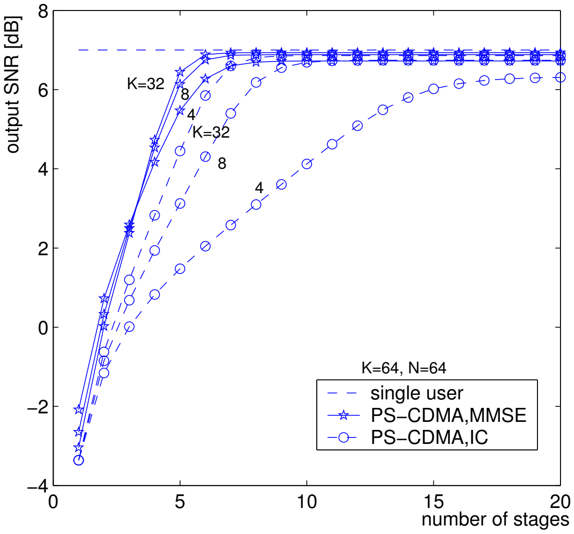

Fig. 3. Signal-to-noise ratio of PS-CDMA systems with $K = 64$, at $\overline{\gamma} = 7$ dB.

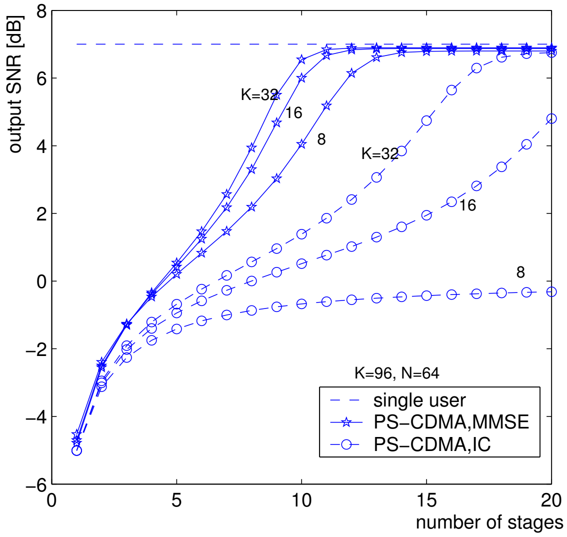

Fig. 4. Signal-to-noise ratio of PS-CDMA systems with $K = 96$ users at $\overline{\gamma} = 7$ dB.

## V. PERFORMANCE AND COMPLEXITY

### A. *BER performance*

In this section, we present the BER performance[3] of the multistage MMSE demodulator of an uncoded PS-CDMA system. The spreading gain is $N = 64$. Figures 5 and 6 are for $M = 8$ and 32, respectively. In each figure, BER curves for both MMSE-based and IC-based multistage filter are plotted for $K = 64, 96$, and 120, representing a system load ranging from fully loaded to almost doubly loaded. The

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[3] Raw data BER is obtained from the VE noise variance as $\mathrm{BER} = (1 - \mathrm{erf}(\sqrt{\gamma(u_k)}))/2$

$$\frac{\partial\sigma_x^2}{\partial M} = \frac{2(1-\alpha)(\alpha M-1)\sigma_d^4 - \frac{\alpha M+3}{\gamma}\sigma_d^2 - \frac{M}{2\gamma^2} - \left[\frac{1}{\gamma}+2(\alpha-1)\sigma_d^2\right]\phi(M)}{4(M-1)^2\phi(M)}. \tag{19}$$

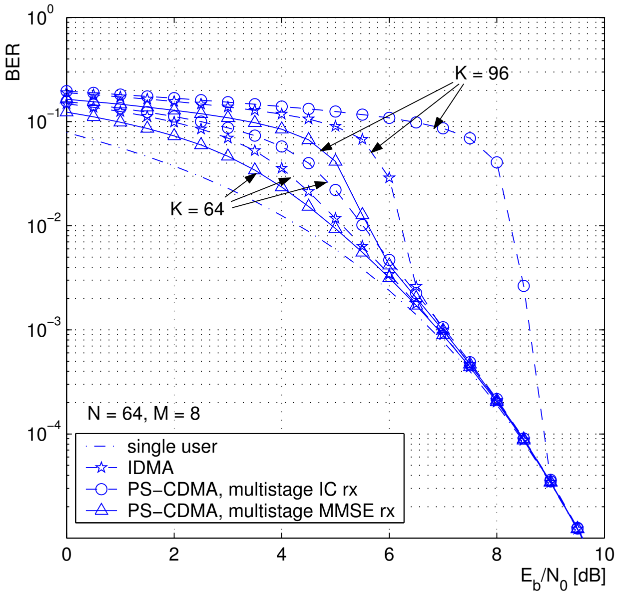

Fig. 5. BER performance for IDMA and uncoded PS-CDMA with $M = 8$ partitions.

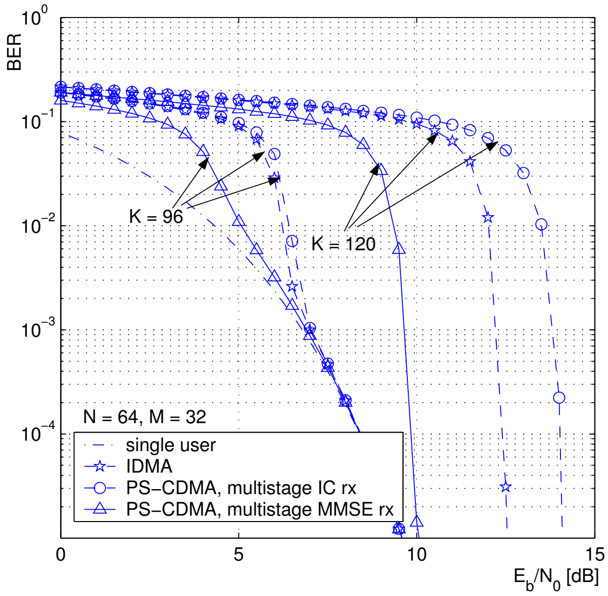

Fig. 6. BER performance for IDMA and uncoded PS-CDMA with $M = 32$ partitions.

curves are based on the demodulator output at 20-th stage. It is shown that PS-CDMA that is received with multistage IC demodulation [1], [2] has a BER performance inferior to that of IDMA [7]. However, the gap between the two narrows as the number of partitions $M$ increases. Multistage MMSE further improves the PS-CDMA performance and outperforms IDMA , especially when the system is over-loaded. It is shown that the proposed MMSE filter can achieve the single-user performance for $K = 120$ at SNR $\approx 10$ dB, while that for IC-based demodulation is 2.7 dB and 4.2 dB away for IDMA and PS-CDMA, respectively.

Power optimization techniques similar to that used for IDMA [7], [10] can be applied to PS-CDMA systems to drastically improve the system performance. However, it is beyond the scope of this paper, and will be studied in future research.

### B. Complexity

One major issue of implementing MMSE multiuser detector in conventional CDMA channel is its high complexity [5]. The exact evaluation of MMSE complexity is not straightforward, and depends on the specific technique used. The widely-accepted rule of thumb number is $O(N^3)$[4], which mainly comes from the computationally demanding inversion of the matrix $\mathbf{V} = \sum_{k=1}^{K} P_k\sigma_{d,k}^2\mathbf{a}_k\mathbf{a}_k^T + \sigma^2\mathbf{I}$. In PS-CDMA, each partitioned segment has a dimension of $N/M$, hence the MMSE filter for each partitioned sub-symbol is $O(N^3/M^3)$ per stage. Since the statistics of one PS-CDMA symbol is based on that of $M$ partitioned sub-symbols, $M$ MMSE filters are performed. Therefore, the proposed multistage filter has a complexity of $O(N^3/M^2)$, which is feasible for implementation even with a moderate $M$. A more detailed complexity comparison between IDMA and PS-CDMA detected by MMSE filter is listed in Table 1.

Figure 7 shows the complexity of both IDMA and MMSE-based multistage filters for PS-CDMA for a number of spreading gain $N$, with $K = 1.5N$ users in the cell. The number of partitions in PS-CDMA ranges from $M = N/8$ to $M = N/2$. It is shown that MMSE filter only requires a complexity that is 2 to 6 times of that for IDMA.[5]

Keep in mind that a $M$-partitioned PS-CDMA operates at slower rates as well as requires smaller interleavers compared to IDMA, which should also be taken into account when evaluating hardware complexity.

## VI. CONCLUSIONS

In this paper, we proposed an efficient multistage MMSE filter for PS-CDMA systems. The performance of this new

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[4] MMSE complexity is considered to be either $O(N^3)$ or $O(K^3)$, depending on specific implementations [14]. We adopt the former in this paper since it allows low-complexity implementation in PS-CDMA channels.

[5] We assume the complexity of inversion is $4N^3$ in this case.

Table 1. Complexity of MMSE-based multistage filter for PS-CDMA with $M$ partitions and IDMA.

| | number of multiplications | number of additions |
|---|---|---|
| IDMA | $6KN$ | $6KN$ |
| MMSE-based PS-CDMA | $\frac{1}{M^2}O(N^3)+\frac{3}{M}KN^2+9KN$ | $\frac{1}{M^2}O(N^3)+\frac{3}{M}KN^2+6KN$ |

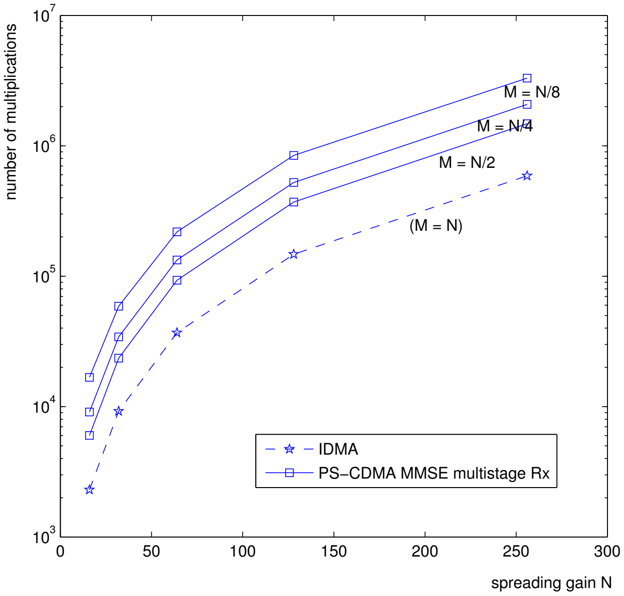

Fig. 7. Complexity in terms of number of multiplications for PS-CDMA multistage filters.

algorithm is noticeably better than that of IC-based filter for both PS-CDMA and IDMA systems. The new system allows smaller interleaving and slower processing rates at both transmit and receive ends. Analysis shows that the computational load of the proposed receiver is only a few times of that for IDMA. Since PS-CDMA assumes a similar modulation structure as that of IDMA, it can be considered as a more flexible alternative to IDMA [15] for 4G uplink physical layer.

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