MMSE Equalization for Serially Concatenated CPM over ISI Channels

Michael Anderson*, Mark C. Reed†, Gerard Borg*

*Research School of Physical Sciences and Engineering
Australian National University, Canberra, ACT, 0200
Email: {michael.anderson,gerard.borg}@anu.edu.au

†National ICT Australia, Australian National University
Canberra, ACT, 2612, Email: mark.reed@nicta.com.au

***Abstract*— It is well known that continuous phase modulation (CPM) serially concatenated with convolutional codes facilitates powerful error correction. CPM also has the advantage of being bandwidth efficient and compatible with non-linear amplifiers. The downfall of CPM systems is their poor performance over ISI channels. We propose a simple receiver consisting of a linear filter used to cancel ISI and improve the statistics for the iterative receiver that follows. We present results for the receiver based on three well known ISI channels and show that the performance is comparable to that of serially concatenated CPM (SCCPM) on an AWGN channel with no ISI.**

I. Introduction

Continuous phase modulation (CPM) is a commonly used modulation scheme in both commercial and military wireless systems due to its bandwidth efficiency and constant envelope [1]. When transmitted over frequency-selective channels, CPM signals will be subject to inter-symbol interference (ISI), which degrades the performance of the system. The optimum receiver for CPM signals transmitted over channels with memory is a matched filter bank to produce sufficient statistics, followed by MLSE or MAP (maximum a-posteriori) decoding. The number of states required for encoding in such receivers is exponential with the sum of the inherent CPM memory, and the memory of the channel.

Another method that has been used for the equalization of CPM is iterative interference cancellation [2]. This has been shown to be effective for some channels, but can fail for harsh ISI channels. This motivates us to explore other sub-optimal techniques. Designing linear filters under the minimum mean squared error (MMSE) criterion is a common way of removing ISI for linear modulation schemes. This technique was applied to CPM in [3] in order to simplify the receiver structure by minimising the partial response components of the signal. This was later extended in [4] to iterative decoding following the linear filter. Neither of these papers design the filter to equalize fading caused by ISI channels, which is the focus of this paper.

The remainder of this paper is arranged as follows. Section II describes the system model, including the CPM and channel models and the MMSE receiver. Section III presents simulation results for well known ISI channels with full and partial-response SCCPM schemes, and compares against SCCPM with no ISI. Finally, Section IV concludes the paper.

II. System Model

A. Continuous Phase Modulation

If the CPM modulation index, $h = K/P$, is rational and irreducible, then the system can be represented by a trellis. The CPM can then be described by the concatenation of a continuous-phase encoder (CPE) and a memoryless modulator (MM). In this case, the complex envelope of a CPM signal with symbol interval T and energy E is given by [5]

$$s(\tau + nT) = \sqrt{\frac{2E}{T}} e^{j\overline{\psi}(\tau+nT)} \tag{1}$$

where $\overline{\psi}$ is the tilted-phase and is given by

$$\overline{\psi}(\tau + nT) = \left[2\pi h \left[\sum_{i=0}^{n-L} u_i \text{mod} P\right] + 4\pi h \sum_{i=0}^{L-1} u_{n-i} q(\tau + iT) + W(\tau)\right] \text{mod} 2\pi \qquad 0 \le \tau < T \tag{2}$$

where $W(\tau)$ is a data-independent function. In (2), u_i is from an M-ary information set, $u_i \in \{0, 1, \ldots, M-1\}$, and $q(t)$ is the integral of a normalized frequency pulse that is non-zero for L symbol intervals and normalized to 1/2 for $t > LT$. When $L = 1$ the system is a full-response and when $L > 1$ it is partial-response.

B. Laurent decomposition of CPM

In [6], Laurent showed that CPM baseband signals can be represented as the sum of $Q = 2^{L-1}$ PAM pulses, i.e.,

$$s(t) = \sqrt{2E} \sum_{k=0}^{Q-1} \sum_{n} a_{k,n} g_k(t - nT) \tag{3}$$

In (3) the $a_{k,n}$ are referred to as “psuedosymbols” and $g_k(t)$ is a pulse shaping filter. When $h = 1/2$ the values of the psuedosymbols are in the set $\{\pm 1, \pm j\}$ and alternate between real and imaginary values. These symbols are functions of past and present data symbols such that they can be considered the output of a set of recursive encoders [4]. For the simple case of $h = 1/2$ and $L = 1$ the encoder is shown in Fig. 1. There is a separate encoder corresponding to each $g_k(t)$ in (3) having a

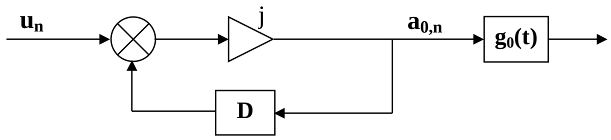

Fig. 1. Laurent encoder.

total of L memory elements between them. The pulse-shaping filters $g_k(t)$ are real and equal to the product of L time-shifted versions of a basic function $c(t)$, which is a function of $q(t)$, and defined in [6]. These filters have durations ranging from T to $(L+1)T$. The advantage of the Laurent decomposition of CPM is that the main PAM pulse, $g_0(t)$, contains most of the signal energy. This means that if the receiver is designed for this pulse alone, complexity is drastically reduced while performance is not.

C. Channel model

The channel model we use in this work is the discrete symbol-spaced tapped-delay-line (TDL). We assume herein that the channel is known and non-time-varying. The channel impulse response is then given by,

$$h(t) = \sum_{l=0}^{L_c-1} b_l \delta(t - lT) \tag{4}$$

where b_l are the symbol spaced taps and L_c is the memory of the channel. Results are presented for three channels from [1] with taps shown in Table I. Channels (a), (b) and (c) represent slight, moderate and severe ISI channels respectively.

Channel	Channel Coefficients
(a)	0.04,-0.05,0.07,-0.21,-0.5,0.72,0.36,0,0.21,0.03,0.07
(b)	0.407,0.815,0.407
(c)	0.227 0.46 0.688 0.46 0.227

TABLE I
PROAKIS CHANNELS

D. Receiver Model

In this section we derive the MMSE filter for a partial-response system with modulation index $h = 1/2$, which can be easily generalised to the full-response case. As in [3] we accurately estimate the CPM signal by the first and second Laurent pulses, i.e.

$$\begin{aligned} s(t) \approx& \sqrt{2E} \sum_{n=-\infty}^{\infty} a_{0,n} g_0(t - nT) \\ &+ \sqrt{2E} \sum_{n=-\infty}^{\infty} a_{1,n} g_1(t - nT) \end{aligned} \tag{5}$$

For the case of L=5 GMSK, for example, the percentages of energy in the remaining pulses are all in the order, or less than the order, of 10^{-2}. Substituting this approximation gives the expression for the received signal as

$$\begin{aligned} y(t) =& \sqrt{2E} \Bigg(\sum_{n=-\infty}^{\infty} a_{0,n} g_0(t - nT) \\ &+ \sum_{n=-\infty}^{\infty} a_{1,n} g_1(t - nT) \Bigg) \otimes h(t) + n(t) \end{aligned} \tag{6}$$

where $\otimes$ is the convolution operator and $n(t)$ is additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2$. This gives

$$\begin{aligned} y(t) =& \sqrt{2E} \Bigg(\sum_{n=-\infty}^{\infty} \sum_{l=0}^{L_c-1} a_{0,n} b_l g_0(t - (n+l)T) \\ &+ \sum_{n=-\infty}^{\infty} \sum_{l=0}^{L_c-1} a_{1,n} b_l g_1(t - (n+l)T) \Bigg) + n(t) \end{aligned} \tag{7}$$

In order to simplify the receiver as much as possible, we wish to only decode over the trellis resulting from the encoder shown in Fig.1, and ignore the statistics carried in the other psuedosymbols. Therefore, given that we are assuming channel information is available, we match to the $g_0(t)$ pulse corrupted by the channel to generate statistics for decoding. This gives

$$\Re \int_{-\infty}^{\infty} y(t) \Bigg(\sum_{k=-\infty}^{\infty} \sum_{l=0}^{L_c-1} a_{0,k}^* b_l g_0(t - (k+l)T) \Bigg) dt \tag{8}$$

$$= \Re \sum_{k=-\infty}^{\infty} a_{0,k}^* r_{0,k} \tag{9}$$

Where

$$r_{0,k} = \sum_{l=0}^{L_c-1} b_l \int_{-\infty}^{\infty} y(t) g_0(t - (k+l)T) dt \tag{10}$$

It can be seen that the statistic in (10) will be degraded by AWGN, ISI from neighbouring symbols and the channel, and interference from the PAM pulses which have not been considered in detection. We wish to design a filter under the MMSE criterion to counter these effects, minimising the error between $r_{0,k}$ and $a_{0,k}$ to increase the accuracy of the metric (9). From (10), we see that $r_{0,k}$ can be obtained from a matched filter $g_0(-t)$, a sampler acting at times $(k+l)T$, and a summer. The output of the linear filter at time kT corresponding to the input $r_{0,k}$ is then

$$z_k = \sum_{i=-N}^{N} c_i r_{0,k-2i} \tag{11}$$

such that the number of coefficients is $2N+1$. The filter in (11) is effectively a $2T$ tap spaced equalizer. By designing it this way, and recalling that $a_{0,n}$ alternates between real and imaginary values, we avoid having to equalize significant orthogonal components, improving performance. The filter coefficients can then be calculated using the orthogonality condition for

$a_{0,n}$ real, i.e. by forcing the error to be orthogonal to the sequence $r_{0,k}$

$$\mathbf{E}\big[\big(a_{0,2n} - \Re\{z_{2n}\}\big)\Re\{r_{0,2n-2m}\}\big] = 0 \qquad (12)$$

The orthogonality condition could also be invoked for $a_{0,n}$ imaginary to achieve the same result. Solving this equation results in

$$\sum_{i=-N}^{N} \gamma_{mi} c_i = \frac{1}{\sqrt{2E_b}} \sum_{j=0}^{L_c-1} \sum_{l=0}^{L_c-1} b_j b_l \int_{-\infty}^{\infty} g_0(\tau) g_0(\tau + (j-l+2m)T) d\tau \qquad -N \leq m \leq N \qquad (13)$$

where γ_{mi} is defined in (14) at the bottom of the page. This is a set of $2N+1$ linear equations that can be solved to obtain the filter coefficients c_i. It should be noted that this result relies on the fact that for a modulation index $h = 1/2$, $a_{0,n}$ and $a_{1,n}$ are orthogonal.

The system model that has been described is shown in Fig. 2. The SCCPM system operates as the serially concatenated

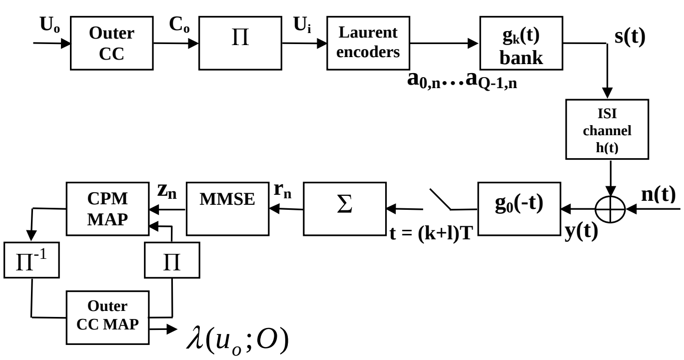

Fig. 2. System model.

convolutional code (SCCC) described in [7]. On the final iteration, a hard decision is made on the log-likelihood ratio $\lambda(u_o; O)$ to recover an estimate of the information symbols u_o. For the two soft-in soft-out (SISO) decoding modules, MAP decoding is performed over the trellis that results from the encoder in Fig. 1, where the state is defined as $S_n = a_{0,n-1}$. As such, a symbol-by-symbol metric is required rather than the metric (9). Slightly modifying the metric derived in [8] gives

$$\Pr\{S_n, z_n | S_{n+1}\} = \exp\left(\frac{2}{N_0}\sqrt{\frac{E}{T}}\Re\{a_{0,n}^{*} z_n\}\right) \qquad (15)$$

III. Results

A. Full-response system

The full-response system considered has parameters $M = 2$, $h = 1/2$ and $L = 1$ and uses the rectangular pulse shape. The outer code was a G[5,7] rate 1/2 non-recursive convolutional code. The log-MAP algorithm was used in both the CPM and outer CC decoders, with both trellises being fully terminated. The interleavers were psuedo-random. The block size used was 256 which corresponds to an interleaver size of 512. The number of filter taps was set to 11. Clearly for a full-response system, no performance loss is suffered from conditioning the decoder on the $a_{0,n}$ pseudosymbol only, and the linear filter will contain no $g_1(t)$ component. Results for the receiver of Fig. 2 for this system on channel (a), (b) and (c) are shown in Fig. 3. Also shown for comparison is the result for the same system in AWGN with no ISI. It can be

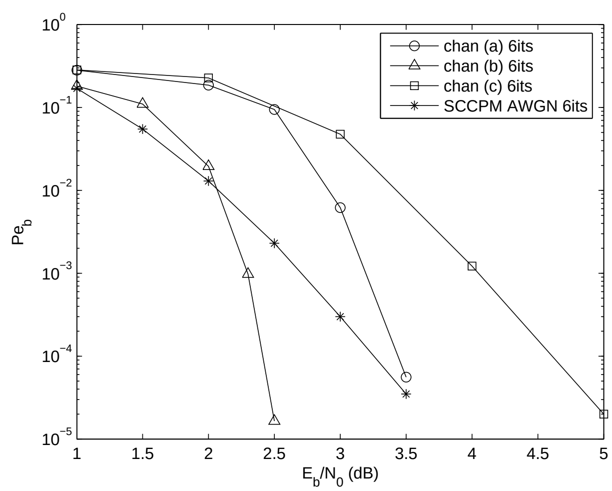

Fig. 3. Full-response result.

seen that with this receiver, convergence is obtained for all three channels. Channel (a) converges to AWGN performance at approximately 3.5 dB. Interestingly, channel (b) provides better performance than the AWGN case for an SNR greater than approximately 2.2 dB. It is feasible that the linear filter

$$\gamma_{mi} = \sum_{\mu=-\infty}^{\mu=\infty} \sum_{j=0}^{L_c-1} \sum_{j'=0}^{L_c-1} \sum_{l=0}^{L_c-1} \sum_{l'=0}^{L_c-1} b_j b_{j'} b_l b_{l'} \Bigg(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_0(\tau) g_0(\tau + (j-l+2\mu)T) g_0(\tau') g_0(\tau' + (j'-l+2(\mu-i+m))T)$$
$$+ g_1(\tau) g_0(\tau + (j-l+2\mu+1)T) g_1(\tau') g_0(\tau' + (j'-l'+2(\mu-i+m)+1)T) d\tau d\tau' \Bigg)$$
$$+ \frac{N_0}{2E_b} \sum_{j=0}^{L_c-1} \sum_{l=0}^{L_c-1} b_j b_l \int_{-\infty}^{\infty} g_0(\tau) g_0(\tau - (j-l+2(m-i))T) d\tau \qquad (14)$$

can provide this performance given the effect it has on the noise. However it should also be noted that detection of CPM signals using the Laurent decomposition has been found to be resilient to this particular ISI channel in previous work [9]. In this work, the channel (b) result was also better than the AWGN result for an interference cancellation type receiver. Channel (c), which is a severe channel, is only approximately 1.4 dB worse than AWGN performance at a bit error rate of 10^{-4}.

B. Partial-response system

The partial-response system considered was GMSK with parameters $M = 2$, $h = 1/2$ and $L = 4$. All other parameters were the same as the full-response case. Results for this system are shown in Fig. 4. In this case, the AWGN result is also for simplified decoding, i.e. only considering $a_{0,n}$ and no other psuedosymbols. It is interesting to compare the

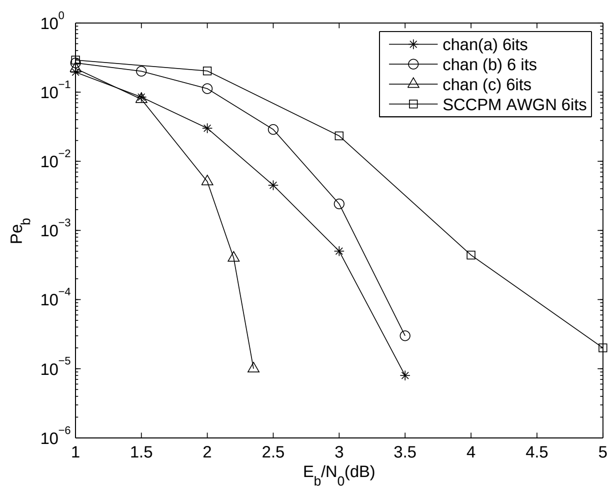

Fig. 4. partial-response result.

AWGN results from Figs. 3 and 4 given that MAP decoding is being performed over the same trellis. At low SNR, the full-response result is superior suggesting that interference from neighbouring $g_0(t)$ pulses dominate the partial-response performance. At high SNR, the partial-response result is better than the full-response result. This suggests that the memory of the partial-response system, i.e. the longer $g_0(t)$ pulse, provides a more reliable statistic from the matched filter. In terms of the three ISI channels, the results for the partial-response case are similar to those for full-response. Again the channel (b) result overtakes the AWGN result at a low SNR. The channel (a) and (c) results are approximately 0.2 dB and 1.3 dB worse than AWGN respectively at a bit error rate of 10^{-4}.

Complexity must be also considered when determining the performance of the receiver. Assuming channel information is available at the receiver, this depends largely on the rate of change of the channel as this will determine how often a $(2N + 1) \times (2N + 1)$ matrix inversion is required. However given that only 11 taps were used in these results, and the simplification of the MAP decoding, in general the complexity of the proposed receiver will be very low. This is particularly true for slow varying channels.

IV. Conclusions

A low-complexity, sub-optimal equalization technique has been proposed for CPM that involves a linear filter followed by a simplified turbo receiver structure. The system makes use of the Laurent decomposition of CPM signals and the MMSE criterion. Results for three well known ISI channels have been presented and shown the technique to be very effective. In all cases, AWGN performance has either almost been reached, or surpassed.

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