Iterative Multiuser Detection based on Monte Carlo Probabilistic Data Association

Zhenning Shi, Mark Reed
Wireless Signal Processing Program
National ICT Australia
Australian National University
Level 2, Northbourne Avenue
Braddon, ACT 2612, Australia
{zhenning.shi,mark.reed}@nicta.com.au

***Abstract*— Multiple-Access Interference (MAI) has been considered as a major performance-limiting factor in the new generation CDMA systems. Multiuser detection (MUD) methods have been proposed to mitigate the MAI from the co-channel users by incoporating the cross-correlation properties between users. Recently, two classes of emerging techniques, probabilistic data association (PDA) and Markov Chain Monte Carlo (MCMC) methods, have been applied to the multiuser detection. In this paper, we present a new method, named Monte Carlo PDA (MC-PDA), that incorporates the concepts of both to give a more reliable inference of the CDMA symbols by appropriately modelling and updating the MAI. The methodology is general and can be applied to other communication channels.**

I. Introduction

Multiuser Detection has attracted much attention during the last decade, and considered as a promising receiver for the next-generation wireless system. Iterative detection structure [3], [4], [5], [6], which applies the principle of the well-known turbo decoding to the joint detection of the CDMA channel, has been ststudiednd shown to achieve the single-user performance.

An iterative receiver breaks the task of joint decoding into two parts: a soft-input soft-output (SISO) multiuser detector that generates the *a posteriori* probability (APP), or its approximations, for each user, and the channel decoding bank. Extrinsic information on the CDMA signals is exchanged between the two processing cores and improves over iterations. A number of low-complexity iterative solutions [3], [6] have been proposed to reduce the interference from co-channel users and resolve the trtransmissionata. Recently, a prprobabilisticata association (PDA) algorithm has been proposed to address the detection in a variety of communication channels. In [12], Luo et. al. showed it can achieve near-capacity performance in a fully-loaded CDMA system. In [13], Pham et. al. extended it to a V-BLAST system and reported better performance for PDA. In [14], [15] Tan and Rasmussen developed an iterative multiuser receiver based on the PDA and showed that their new method outperforms linear SISO multiuser detectors via numerical examples. Yin et. al. employed the PDA for an equalizer to do iterative decoding in an ISI channel [16].

In this paper, we propose a new detector, named MC-PDA, that incorporates the concept of Monte Carlo Markov Chain (MCMC) to generate a number of *random copies* for PDA statistics by running over the underlying Markov process of the communication channel. By doing so, the Markov chain can travel across a number of importance samples (symbol candidates) driven by the dynamic statistics obtained by data association. The outputs are calculated by averaging over the conditional APP probabilities of those samples. We show via numerical examples that the MC-PDA achieves a considerable improvement over both the generic PDA detector and the MCMC methods through turbo iterations.

The paper is organized as follows. In Section II, the system model of the synchronous, random-spreading CDMA channel is described. The iterative receiver structure that performs joint decoding to the channel is reviewed. In Section III-A, a MCMC-type PDA algorithm named MC-PDA is presented. We show the conventional PDA and Gibbs sampler are special cases of MC-PDA, and the relation of the two is discussed in Section III-B. A reduced-complexity version of the MC-PDA is developed in Section III-C to make the cocomputationractable. In Section IV, we arguably show that the MC-PDA is a more flexible detector scheme than the conventional PDA and the Gibbs sampler, and results in a significant improvement on the system performance, via numerical examples. Conclusions are summarized in Section V.

II. System Model

We consider a synchronous K-user CDMA system embedded in an AWGN channel. Each user independently generates information data b_k, which are encoded with the FEC codes $\mathcal{C}$. The coded symbols d_k are then interleaved and spread with pepseudoandom signature sequences that are unique for each user, and passed to the CDMA channel. BPSK modulation is considered in this paper, but the extension to more complex modulation is straightforward. The received signal, after pulse-filtering and perfect sampling, can be expressed in a vector-form as

$$\mathbf{y} \quad = \quad \mathbf{Ad} + \mathbf{n}, \qquad (1)$$

where the channel observation $\mathbf{y}$ is a length-N vector, $\mathbf{A} = [\mathbf{a}_1, \ldots, \mathbf{a}_K]^T$ is a $N \times K$ matrix that contains the normalized spreading sequences of all users, $\mathbf{d}$ is a length-K vector of coded symbols, and $\mathbf{n}$ is the AWGN noise samples of variance $\frac{N_0}{2}$.

Figure 1 shows an illustrative diagram of the iterative detector for the CDMA channels. In the figure, $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ denote a posteriori outputs of the SISO multiuser detector and the channels decoders, which are defined for user k as

$$\lambda_1(d_k) = \ln\left(\frac{P(d_k=1|\mathbf{y},\boldsymbol{\lambda}_2^e)}{P(d_k=-1|\mathbf{y},\boldsymbol{\lambda}_2^e)}\right) \quad (2)$$

$$\lambda_2(d_k) = \ln\left(\frac{P(d_k=1|\lambda_1^e(d_k),\mathcal{C})}{P(d_k=-1|\lambda_1^e,\mathcal{C})}\right), \quad (3)$$

where $\boldsymbol{\lambda}_2^e = [\lambda_2^e(d_1),\ldots,\lambda_2^e(d_K)]^T$ are the extrinsic outputs of of K single-user channel decoders, and $\lambda_1^e(d_k)$ is the extrinsic information of the multiuser detector for user k.

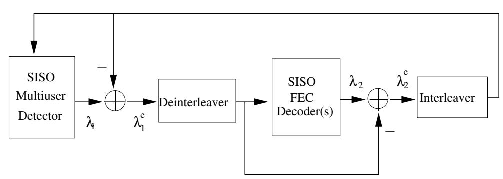

Fig. 1. Iterative receiver structure for a CDMA channel.

Apply the Bayesian rule to the APP value $P(d_k|\mathbf{y},\boldsymbol{\lambda}_2^e)$ in (2), we obtain

$$\begin{aligned} P(d_k|\mathbf{y},\boldsymbol{\lambda}_2^e) &= \sum_{\mathbf{d}_{-k}} P(d_k,\mathbf{d}_{-k}|\mathbf{y},\boldsymbol{\lambda}_2^e) \\ &\propto \sum_{\mathbf{d}_{-k}} P(\mathbf{y}|d_k,\mathbf{d}_{-k},\boldsymbol{\lambda}_2^e)P(\mathbf{d}_{-k},d_k|\boldsymbol{\lambda}_2^e), \quad (4)\end{aligned}$$

where $\mathbf{d}_{-k} = [d_1,\ldots,d_{k-1},d_{k+1},d_K]^T$ denotes all the data except for that of the target user. (4) includes the computation of 2^{K-1} terms, hence is intractable for even small systems. Low-complexity SISO multiuser detectors [3], [6] have been developed to provide an approximation to (4). Recently,two classes of emerging techniques, namely, the PDA and the MCMC, have been applied to the multiuser detection [12], [13], [14], [15], [7], [8], [9], [10], [11] and shown to outperform other suboptimal detectors. The basic processing core in both algorithms include interference cancellation and match filtering, hence it is very promising for implementation.

III. MCMC-TYPE PROBABILISTIC DATA ASSOCIATION

In this section, we present a novel probability data association tetechniquehat builds on the random samples generated by the Markov chain of the system under study. Hence we name the the new method as the Monte carlo PDA (MC-PDA). We show that the generic PDA and Gibbs Sampler, the two well-known algorithms, are the special cases of the more general MC-PDA. The intrinsic link and differences between the PDA and Gibbs sampler are then explored. A reduced-complexity MC-PDA is developed at the end of the section.

A. MC-PDA Algorithm

In decoding the CDMA channel in (1), APP obtains the inference on data d_k by summing $P(d_k,\mathbf{d}_{-k}|\mathbf{y},\boldsymbol{\lambda}_2^e)$ for all possible $\mathbf{d}_{-k}$ in (4), which has a complexity that is exponential in terms of the number of users. According to the principle of the importance sampling, APP in (4) can be approximated by summing over the $\mathbf{d}_{-k}$ that are important, i.e., with larger weights. An MCMC detector that serve this purpose calculates the APP by

$$P(d_k|\mathbf{y},\boldsymbol{\lambda}_2^e) \approx \frac{1}{N_s}\sum_{n=1}^{N_s} P(d_k,\mathbf{d}_{-k}^{(n)}|\mathbf{y}), \quad (5)$$

with $\mathbf{d}_{-k}^{(n)}$ being a random sample that is drawn from a distribution $\eta_k^{(n)}$ that is recursively computed in the Markov chain. In this paper, we extend this method by including the multi-tuple of the samples, rather than the explicit samples. By doing so, a more robust distribution can be estimated to perform future draws. The APP is estimated as

$$\begin{aligned} P(d_k=1|\mathbf{y},\boldsymbol{\lambda}_2^e) &\approx \frac{1}{N_s}\sum_{n=1}^{N_s} P(d_k,\{\mathbf{d}_i^{(n)}\}_{i\neq k}|\mathbf{y}) \\ &= \frac{1}{N_s}\sum_{n=1}^{N_s}\eta_k^{(n)}, \quad (6)\end{aligned}$$

where $\mathbf{d}_i^{(n)} = [d_{i,1},\ldots,d_{i,M}]^T$ is the vector containing M samples that are independently drawn from the distribution $\eta_i^{(n)}$. The key in the MC-PDA is to construct a reliable estimate on the interference term $\mathbf{I}_k = \sum_{i\neq k}\mathbf{a}_i d_i$ with sample multi-tuples $\{\mathbf{d}_i^{(n)}\}_{i\neq k}$. In classical MCMC methods, $\mathbf{I}_k$ is approximated with the sample $\mathbf{d}_{-k}^{(n)}$ as $\tilde{\mathbf{I}}_k = \sum_{i\neq k}\mathbf{a}_i d_i^{(n)}$. In the case that the wrong samples are drawn, this estimate becomes biased, which in turn affects the reliability of η_k. It typically costs the Markov chain a much longer burn-in period to converge to the stationary distribution, especially in a channel with high level of correlations and high SNRs. MC-PDA addresses this problem by independently drawing multiple samples $\mathbf{d}_i^{(n)}$ from η_i for each user. Therefore, the transition in the Markov chain is not between states $\mathbf{d}^{(n)}$, but between the multi-tuples of the states $\{\mathbf{d}_k^{(n)}\}_{k=1}^K$, whereby a much more reliable estimate of η_k can be achieved. Here we use the concept of the data association, which assumes the interference-plus-noise term $\mathbf{n}_k = \mathbf{I}_k + \mathbf{n}$ as a Gaussian source, i.e., $\mathbf{n}_k \sim \mathcal{N}(\boldsymbol{\mu}_k,\boldsymbol{\Sigma}_k)$, with the mean and variance can be approximated as

$$\begin{aligned} \boldsymbol{\mu}_k^{'} &= \sum_{i\neq k}\mathbf{a}_i\overline{d}_i \\ \boldsymbol{\Sigma}_k^{'} &= \sum_{i\neq k}\mathbf{a}_i\mathbf{a}_i^T\frac{\sum_{m=1}^M(d_{i,m}-\overline{d}_i)^2}{M-1} + \frac{N_0}{2}\mathbf{I}, \quad (7)\end{aligned}$$

where $\overline{d_i} = \sum_{m=1}^M d_{i,m}/M$ is the mean of M samples. Therefore, the sampling distribution for user k can be obtained as in (8).

$$\begin{aligned}\eta_k^{(n)} &= P(d_k = 1, \{\mathbf{d}_i\}_{i\neq k}|\mathbf{y}, \boldsymbol{\lambda}_2^e) \\ &\propto \exp\left(-\frac{1}{2}\left(\mathbf{y}-\mathbf{a}_k-\boldsymbol{\mu}_k^{'}\right)^T\left(\boldsymbol{\Sigma}_k^{'}\right)^{-1}\left(\mathbf{y}-\mathbf{a}_k-\boldsymbol{\mu}_k^{'}\right)+\lambda_2^e(d_k)\right) \\ &\propto \exp\left(2\mathbf{a}_k^T\left(\boldsymbol{\Sigma}_k^{'}\right)^{-1}(\mathbf{y}-\boldsymbol{\mu}_k^{'})+\lambda_2^e(d_k)\right) \end{aligned} \quad (8)$$

Now we outline the M-sample MC-PDA algorithm as follows

MC-PDA(M)

1 Initialization: set $\eta_k^{(0)} = 1/2, k = 1, \ldots, K$; draw M independent samples for each user $d_{i,m}^{(0)} \sim \eta_k^{(0)}$; set PDA sweep counter n=1.

2 FOR $k = 1$ TO K

Calculate the mean and the covariance for the interference as

$$\begin{aligned}\boldsymbol{\mu}_k^{'} &= \sum_{i=1}^{k-1}\mathbf{a}_i\overline{d}_i^{(n)} + \sum_{i=k+1}^{K}\mathbf{a}_i\overline{d}_i^{(n-1)} \\ \boldsymbol{\Sigma}_k^{'} &= \frac{N_0}{2}\mathbf{I} + \sum_{i=1}^{k-1}\mathbf{a}_i\mathbf{a}_i^T\frac{\sum_{m=1}^{M}(d_{i,m}^{(n)}-\overline{d}_i^{(n)})^2}{M-1} \\ &\quad + \sum_{i=k+1}^{K}\mathbf{a}_i\mathbf{a}_i^T\frac{\sum_{m=1}^{M}(d_{i,m}^{(n-1)}-\overline{d}_i^{(n-1)})^2}{M-1}, \end{aligned} \quad (9)$$

and update the sampling distribution by

$$\eta_k^{(n)} = \frac{\exp\left(2\mathbf{a}_k^T(\boldsymbol{\Sigma}_k^{'})^{-1}(\mathbf{y}-\boldsymbol{\mu}_k^{'})+\lambda_2^e(d_k)\right)}{1+\exp\left(2\mathbf{a}_k^T(\boldsymbol{\Sigma}_k^{'})^{-1}(\mathbf{y}-\boldsymbol{\mu}_k^{'})+\lambda_2^e(d_k)\right)}. \quad (10)$$

Draw M indepedent samples $[d_{i,1}^{(n)}, \ldots, d_{i,M}^{(n)}] \sim \eta_k^{(n)}$.

END FOR

3 Check the convergence on $\{\eta_k\}_{k=1}^K$ [1], if so, let

$$P(d_k = 1|\mathbf{y}, \boldsymbol{\lambda}_2^e) = \eta_k^{(n)}$$

for $k = 1, \ldots, K$, got to step 6.

4 If $n < N_s + N_b$, $n = n + 1$, go to step 2.

5 Calculate the APP

$$P(d_k = 1|\mathbf{y}, \boldsymbol{\lambda}_2^e) = \frac{1}{N_s}\sum_{n=N_b+1}^{N_b+N_s}\eta_k^{(n)} \quad (11)$$

for $k = 1, \ldots, K$.

6 Calculate the extrinsic LLR as

$$\lambda_1^e(d_k) = \ln\left(\frac{P(d_k = 1|\mathbf{y}, \boldsymbol{\lambda}_2^e)}{1-P(d_k = 1|\mathbf{y}, \boldsymbol{\lambda}_2^e)}\right) - \lambda_2^e(d_k) \quad (12)$$

for $k = 1, \ldots, K$.

[1] In simulations, we check if η_k agrees by one per cent in consecutive sweeps, i. e., $\left|\frac{\eta_k^{(n)}-\eta_k^{(n-1)}}{\eta_k^{(n-1)}}\right| < 1\%$; convergence is reached if the ininequal-ityolds for all users.

Equation (**??**) is obtained by normalizing the term in (8). The number of indindependentmples M is a parameter that determines the random nature of the algorithm. When M is large, the sampling multi-tuple represented by $\{\mathbf{d}_i\}_{i=1}^K$ contains numerous states. Hence the distribution η_k that is calcalculatedsed on the average statistics of these states will show a trend of convergence. Step 3 in the above algorithm is to serve the purpose of exporting the converged estimate of the APP. On the other hand, if M is small, MC-PDA tends toward a random machine like the MCMC methods. After a long enough burn-in period of N_b, the samples generated by the Markov process conform closely to its satstationarystribution, and the average in (11) converge to the true APP value as $N_s \to \infty$. To understand this point better, we look at two special MC-PDA methods:

PDA: MC-PDA(∞)

When the number of random samples $M \to \infty$, the estimates in (7) converge in probability to the true mean and covariance, i.e.,

$$\begin{aligned}\boldsymbol{\mu}_k^{'} &\xrightarrow{p} \boldsymbol{\mu}_k = \sum_{i\neq k}(2\eta_i - 1)\mathbf{a}_i \\ \boldsymbol{\Sigma}_k^{'} &\xrightarrow{p} \boldsymbol{\Sigma}_k = \sum_{i\neq k}4\eta_i(1-\eta_i)\mathbf{a}_i\mathbf{a}_i^T + \frac{N_0}{2}\mathbf{I}. \end{aligned} \quad (13)$$

Therefore, the updating value of the sampling distribution η_k solely depends on statistical values of other users, that is, $\eta_k = P(d_k = 1|\mathbf{y}, \boldsymbol{\lambda}_2^e, \{\eta_i\}_{i\neq k})$. This deterministic process is known as the proprobabilisticta association (PDA) [12], [13], [14], [15], [16]. It was shown by Yin et. al. [16] that the first sweep of the PDA is equivalent to the conditional MMSE filter [6], while the PDA may need more sweeps to converge to a more reliable fixed point, implying a better performance than that of the conditional MMSE filter at the end.

Gibbs sampler: MC-PDA(1))

If $M = 1$, then the statistics used for recursion on η_k becomes random by noting that the mean $\boldsymbol{\mu}_k^{'} = \sum_{i\neq k}\mathbf{a}_i d_i^{(n)}$ are linear superposition of the random sample, and the covariance $\boldsymbol{\Sigma}_{-k}^{'} = \frac{N_0}{2}\mathbf{I}$ reduces down to the noise variance since the interfering data is assumed to be $\mathbf{d}_{-k}^{(n)}$. As a result, $\eta_k^{(n)}$ will not converge to a certain value regardless how many sweeps are conducted. Instead, the sample $\mathbf{d}^{(n)}$ closely follows the target distribution as the Markov chain becomes stationary, and the average in (11) serves as a good estimate of the APP. The MC-PDA(1) is actually the Gibbs sampler, a MCMC method

$$\sigma_k^2 = \frac{N_0}{2} + \frac{1}{N}\left(\sum_{i=1}^{k-1}\frac{\sum_{m=1}^{M}(d_{i,m}^{(n)} - \overline{d}_i^{(n)})^2}{M-1}) + \sum_{i=k+1}^{K}\frac{\sum_{m=1}^{M}(d_{i,m}^{(n-1)} - \overline{d}_i^{(n-1)})^2}{M-1}\right) \quad (14)$$

that has recently become popular and widely used in various scientific explorations [1], [2], [7], [8], [9], [10], [11].

The numerical experiments show that if we compute the covariance of the MC-PDA(1) according to (13) rather than the noise variance, the performance is improved. We explain this phenomenon by arguing that (13) takes into account the variance of residual interference caused by the random samples, which in turn results in a more reliable sampling distribution η_k. To distinguish this treatment from the generic Gibbs sampler, we denote the method using (13) as the MC-PDA(1) and that using $\mathbf{\Sigma}_k = \frac{N_0}{2}\mathbf{I}$ as the Gibbs sampler.

B. Remarks

The MC-PDA methods with a finite parameter M provide a vast number of alternatives between the deterministic PDA and the random Gibbs sampler. Compared to the Gibbs sampler, (10) is based on the dynamic statistics of M random samples, hence it is not sensitive to the mistakes in the random draw as the Gibbs sampler. As a result, A more reliable distribution η_k is computed in MC-PDA for each step. It is also advantageous over the PDA in cases where the PDA converge to a *suboptimal* solution, switching it to the MC-PDA with finite M allows the generation of a number of *random copies* of the PDA results, with increasing reliability that is accumulated through the Markov chain. We verify this argument with numerical examples in the next section.

C. Reduced-Complexity MC-PDA

The MC-PDA detectors need to perform an inverse on the $N \times N$ covariance matrix $\mathbf{\Sigma}_k$, for each update of the probability η_k. Luo et. al. reduced the complexity to $O(KN^2)$ per sweep by applying the Sherman-Morrison-Woodbury formula to the inverse such that most of the computations can be shared among users [12]. Furthermore, Tan and Rasmussen observed that for large-scale systems, the diagonal elements of $\mathbf{\Sigma}_k$ is dominant [15], i.e.

$$\mathbf{\Sigma}_k \;\approx\; \mathrm{Diag}(\mathbf{\Sigma}_k) = \left(\frac{4}{N}\sum_{i\neq k}\eta_i(1-\eta_i) + \frac{N_0}{2}\right). \quad (15)$$

We take this assumption and simplify (10) as

$$\eta_k^{(n)} = \left[1 + \exp\left(-\frac{2}{\sigma_k^2}\mathbf{a}_k^T(\mathbf{y} - \boldsymbol{\mu}_k^{'}) - \lambda_2^e(d_k)\right)\right]^{-1}, \quad (16)$$

where σ_k^2 is the sum of the residual-interference and the noise, as shown in (14) at the top of this page.

IV. Simulation

In this section, we investigate the performance of the a number of MC-PDA detectors, in particular, the MC-PDA(∞), the MC-PDA(1), and the Gibbs sampler. The system in simulation is an overloaded CDMA channel, with the spreading gain $N = 10$. The rate-$\frac{1}{2}$ $(5,7)$ convolutional code is used for FEC. $L = 200$ information bits are transmitted per data frame. Reduced-complexity MC-PDA in (16) is used in simulation.

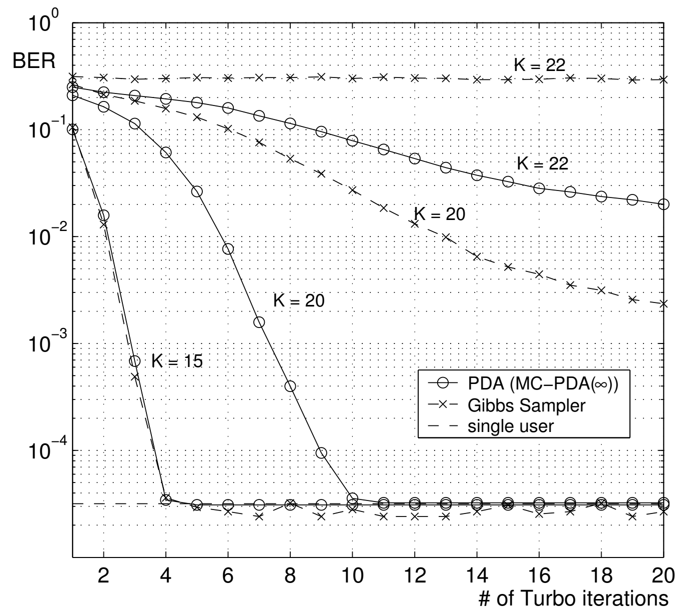

Fig. 2. BER comparison between the generic PDA and and Gibbs sampler Detectors.

Figure 2 demonstrates the BER performance of the PDA together with that of the Gibbs sampler ($N_s = 100$). It shows that both methods can achieve single user performance with a few iterations at $K = 15$. However, as K increases, which corresponds to a more correlated channel, the PDA achieves a much lower BER than the Gibbs sampler. This is due to the fact that the PDA updates the a posteriori sampling distribution η_k based on the average statistics in (13), while the Gibbs sampler uses just one random sample for that purpose. However, at $K = 22$, both detectors fail even with 20 turbo iterations, which means even the more robust PDA only converges to a suboptimal fixed point at this system load.

To address this problem, we propose switching the MC-PDA from the deterministic "PDA" mode to a random "MCMC" mode when the PDA reaches its convergence. In Figure 3, the SISO multiuser detector that switches from the PDA to MC-PDA(M) at the 11-th turbo iteration is reported. It shows that for $M = 1, 3, 10$, improvement is achieved through iterations, with the largest gain is obtained by setting $M = 1$.

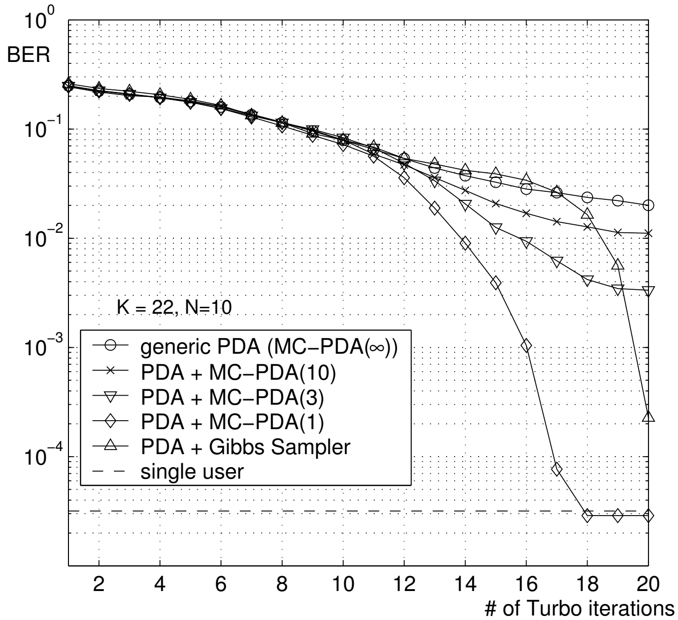

Fig. 3. BER comparison between MC-PDA and the generic Gibbs sampler.

Obviously, a random MCMC-type machine is prepreferred this case. The underlying reason is that when the PDA converges to a suboptimal fixed point, switching it to MC-PDA((1)) allows the detector to generate a number of "random copies" of the PDA results through the Markov chain. The relreliability of the APP estimate will be further improved as the Markov chain visits more states $\mathbf{d}^{(n)}$ that are close to the transmission data. The BER curve represented by the solid lines with upper triangles is for the detector which switches from PDA to the Gibbs sampler at 11-th iteration. Though the BER finally approaches the single user performance at 20-th iteration, it is inferior to that of the MC-PDA(1) due to the ignorance of the residual-interference in the random samples.

V. Conclusion

In this paper, we study the iterative detection for a FEC coded CDMA system. we propose a MCMC-type PDA algorithm, named MC-PDA(M), that runs the Markov process combines with the updating average statistics furnished by the random samples. We show that the two well-known algorithms, the PDA and the Gibbs sampler, are actually the special cases of the MC-PDA when $M \to \infty$ and $M = 1$, respectively. The relationship between the PDA, the Gibbs sampler and the more general MC-PDA is discussed. A reduced-complexity version of the MC-PDA is developed to make the receiver more tractable for implementation. Numerical examples show that the PDA is more robust than the Gibbs sampler, while switching from the PDA to a random MC-PDA(1) at certain stage can significantly improve the system performance. Hence a good algorithm is needed to monitor the dynamic performance of the detector to determine the turn-on of the MC-PDA. An adaptive receiver that serves this purpose will be developed to make the system feasible in practice. Thought the new method is developed for the synchronous CDMA channel, it can find the applications in other communication channels.

Acknowledgement

Z. Shi and M. C. Reed are with National ICT Australia and affiliated with the Australian National University. National ICT Australia is funded through the Australian Government's *Backing Australia's Ability* initiative and in part through the Australian Research Council.

References

[1] C.P. Robert and G. Casella, *Monte Carlo Statistical Methods.* Springer-Verlag, New York, 1999.

[2] J. Liu, *Monte Carlo Strategies in Scientific Computing*, Springer-Verlag, New York, 2001.

[3] P. Alexander, A. Grant and M. Reed, "Iterative Detection in Code-Division Multiple-Access with Error Control Coding", *European Transaction on Telecommunications, Special Issue on CDMA Techniques for Wireless Commmunications Systems*, vol. 9, pp. 419-426, Sep. - Oct. 1998.

[4] P. D. Alexander, M.C. Reed, J.A. Asenstorfer and C.B. Schlegel, "Iterative Multiuser Interference Reduction: Turbo CDMA", *IEEE Trans. Commun.*, vol. 47, no. 7, July 1999.

[5] M. Moher, "An Iterative Multiuser Decoder for Near-Capacity Communications", *IEEE Trans. Commun.*, vol. 47, pp. 870–880, July 1998.

[6] X. Wang and H.V. Poor, "Iterative (Turbo) Soft Interference Cancellation and Decoding for Coded CDMA", *IEEE Trans. Commun.*, vol. 47, no. 7, July 1999.

[7] V. Buchoux and O. Capp$\acute{e}$ and $\acute{E}$ Moulines, "Turbo Multiuser Detection for Coded DS-CDMA Systems: A Gibbs Sampling Approach," *Conference Record of the 34th Asilomar Conference on Signals, Systems and Computers*, pp. 1426-1430, November 2000.

[8] Y. Huang and P. Djuric, "Multiuser Detection of Synchronuous Code-Division Multiple-Access Signals by Perfect Sampling," *IEEE Trans. Signal Processing*, vol. 50, no. 7, pp. 1724-1734, July 2002.

[9] D. Guo and X. Wang, "Blind Detection in MIMO Systems via Sequential Monte Carlo," *IEEE J. Selected Area in comm.*, vol. 21, no. 3, pp. 464-473, April, 2003.

[10] Z. Shi, H. Zhu and B. Farhang-Boroujeny, "Markov Chain Monte Carlo Techniques in Iterative Detectors: a Novel Approach Based on Monte Carlo Integration," *IEEE Proceedings of Globecom*, pp. 325-329, November 2004.

[11] B. Farhang-Boroujeny, H. Zhu and Z. Shi, "Monte Carlo Markov Chain Techniques for CDMA and MIMO Communication Systems," *submitted to IEEE Transactions on Signal Processing*.

[12] J. Luo, K. Pattipati, P. Willett and F. Hasegawa, "Near-Optimal Multiuser Detection in Synchronous CDMA Using Probablistic Data Association," *IEEE Communications Letters*, pp. 361-363, September 2001.

[13] D. Pham, K. Pattipati, P. Willett, and J. Luo, "A Generalized Probabiblistic Data Association Detector for Multiple Antenna Systems," *IEEE Communications Letters*, pp. 205-207, April 2004.

[14] P. Tan, L. Rasmussen and J. Luo, "Iterative Multiuser Decoding Based on Probabilistic Data Association" *IEEE Proceedings of ISIT*, pp. 301-301, June 2003.

[15] P. Tan, L. Rasmussen, "Multiuser Detection Based on Reduced Complexity Probabilistic Data Association," *IEEE Proceedings of ISIT*, June 2004.

[16] Y. Yin, Y. Huang and J. Zhang, "Turbo Equalization using Probabiblitic Data Association," *IEEE Proceedings of Globecom* pp. 2535-2539, November 2004.