Iterative Switched-Decoding for Interleave-Division Multiple-Access Systems

Yi Hong
Institute for Telecommunications Research
University of South Australia
Mawson Lakes, SA 5095
yi.hong@unisa.edu.au

Lars K. Rasmussen
Institute for Telecommunications Research
University of South Australia
Mawson Lakes, SA 5095
lars.rasmussen@unisa.edu.au

***Abstract*— We consider an interleave-division multiple-access (IDMA) system with multiple users transmitting over a shared additive white Gaussian noise channel. At the transmitter, the information sequence of each user is encoded by a low rate code, corresponding to the concatenation of a convolutional code and a repetition code. We propose an iterative partial decoding scheme, where a receiver only decodes parts of the joint concatenated repetition-convolutional code at the first few iterations, before progressively including more and more code structure into the decoding process. By decoding only parts of the concatenated code, the resulting extrinsic information transfer (EXIT) curve for the partially decoded code is different from the EXIT curve for the fully decoded code. To demonstrate the general scheme, we focus on an iterative switched-decoding scheme in which a receiver only decodes the repetition code at a fixed initial number of iterations, and switches to decode the entire joint repetition and convolutional code. It is shown by simulation that the proposed scheme guarantees fast decoding convergence at low decoding complexity.**[1]

I. Introduction

Recently the concept of interleave-division multiple-access (IDMA) has been suggested as an alternative multiple-access scheme to code-division multiple-access (CDMA) [1, 2]. The key innovation in IDMA is the combined use of low-rate channel coding in place of spreading, and simple soft-input soft-output (SISO) iterative decoding techniques with low-complexity multiuser detection. The superposition of many users with low-rate codes creates an input distribution to the channel that quickly approaches a Gaussian distribution, and thus is capable of getting close to the Shannon capacity [3]. Due to the use of low-rate codes in place of spreading, user-specific interleavers are required to distinguish users from each other at the receiver. The use of user-specific interleavers as the unique user identification was first suggested in [4] for trellis-code multiple-access (TCMA).

The two key components in the IDMA innovation have been previously proposed for multiple-access systems. However, the innovation providing the strength in IDMA is exactly the combination of the two, which eluded previous contributions. Viterbi suggested already in [5] the use of low-rate codes and hard-decision decoding combined with successive interference cancellation. Also, the code-spread CDMA (CS-CDMA) system suggested in [6] makes use of a class of low-rate codes [7], hard-decision decoding and interference cancellation. The only difference between CS-CDMA and IDMA is the position of the user-interleaver within the transmitter structure, and the use of SISO iterative decoding strategies. Finally, chip-level multiuser detection within a SISO iterative multiuser decoder has been suggested in [8] for CDMA, and shown to perform better than similar bit-level approaches.

The low-complexity SISO multiuser detector suggested for IDMA [1, 2] has also been previously proposed for CDMA. In [9], MMSE-filtered parallel interference cancellation was suggested for iterative multiuser decoding, while the probabilistic data association detector within a SISO multiuser decoder was shown in [10] to be identical to the approach in [9]. The advantage of using this approach in an IDMA system is that due to low-rate coding and chip-level processing rather than spreading and bit-level processing, correlation between users is virtually eliminated[2]. Matrix inversion is therefore avoided, resulting in a low-complexity implementation that only grows linearly with the number of users. The price to pay is in terms of interleaver sizes, and thus in memory requirements.

The convergence performance of IDMA systems can be investigated through transfer function analysis. In [2], the effective signal-to-interference-noise ratio (SINR) is tracked through the iterative decoding algorithm. However, extrinsic information transfer (EXIT) chart analysis, first developed in [11], have been shown to give more accurate results. The approach has been applied to iterative multiuser decoding in [12], where the convergence behavior of coded CDMA systems are accurately predicted. EXIT chart analysis has also been applied in [13] for investigation of IDMA systems. In [13], EXIT charts were also used to motivate a complexity reduction in the iterative decoding scheme. The main idea is to allow some users to only decode the repetition code, while others decode the joint repetition and convolutional code. The scheme was suggested for *ad hoc* networks, where different receivers have different decoding capabilities. In our paper, we make a general assumption that all receivers in IDMA

[1]This works is supported by the Australian Research Council (ARC) under the ARC Discovery Project DP0663567 and Early Career Researcher Supportive Scheme, University of South Australia.

[2]For sufficiently large interleaver sizes, the correlation can be significantly reduced.

system have the same decoding capabilities. This holds for many uplink wireless communication scenarios such as mobile cellular networks.

Inspired by the approach in [13] we propose an iterative partial decoding scheme, where a receiver only decodes parts of the joint concatenated repetition-convolutional code at the first N' iterations, before progressively including more and more code structure into the decoding process. This is possible, since the two component codes in the concatenation are not separated by an interleaver. By decoding only parts of the concatenated code, the resulting EXIT curve for the partially decoded code is different than the fully decoded code. We can therefore obtain an entire class of EXIT curves, each representing the trajectory for a specific partial decoding scheme. Similar to the curve-fitting strategy proposed for code design in [14], we can now find an optimal sequence of partial decoding schemes for the iterative multiuser decoder to ensure high convergence speed at the lowest possible complexity.

To demonstrate the concept, in this paper we focus on an iterative switched-decoding (ISD) scheme in which a receiver only decodes the repetition code at the first N' iterations, and switches to decode the entire joint repetition and convolutional code. Let N_D denote the total number of iterations required of the ISD for convergence. An exhaustive search based on EXIT charts is conducted to find the preferable number of iteration N' and N_D, respectively. It is shown by simulation that the proposed decoding scheme with the found values of N' and N_D provides fast decoding convergence at low decoding complexity.

The paper is organised as follows. In Section II, the IDMA system model and the iterative multiuser decoder are described. EXIT chart analysis for IDMA is considered in Section III, while the design of the ISD scheme is detailed in Section IV. Simulation results are presented in Section V and concluding remarks are found in Section VI.

The following notation is used throughout the paper. Let $\mathbf{E}[\cdot]$ and Var$[\cdot]$ denote mean and variance of a random variable. Let I_a^{MUD} and I_a^{Dec} denote *a priori* mutual information for the multiuser decoder (MUD) and the single-user channel *a posteriori* probability (APP) decoders, respectively, while I_e^{MUD}, $I_e^{Dec_1}$, and $I_e^{Dec_2}$ denote output extrinsic mutual information from the MUD, the repetition-code decoder and the joint repetition-convolutional-code decoder, respectively. Also, we let $\max(a,b)$ represent the maximum of a and b. Finally, let N_{max} denote the maximum number of iterations allowed in the decoding process, and let N_F denote the number of iterations required for the decoding process to converge to single user performance at a given signal-to-noise ratio (SNR).

II. Transmitter and Iterative Receiver for IDMA

We consider a synchronous IDMA system with K users transmitting with equal power over an additive white Gaussian noise (AWGN) channel using BPSK modulation [1, 2]. System spectral efficiency is defined as $\alpha = K/S * R1$.

A. Transmitter

Fig. 1(a) shows the transmitter structure for user k, where $k = 1, \ldots, K$. The input data sequence $\mathbf{d}^{(k)}$ is encoded by a convolutional code with rate R_1, producing $\mathbf{b}^{(k)} = [b_k(1), \ldots, b_k(l), \ldots, b_k(L)]$. The coded sequence $\mathbf{b}^{(k)}$ is further encoded by a low rate code (repetition code) with rate $R_2 = 1/S$ (each bit is repeated S times), generating $\mathbf{c}^{(k)} = [c_k(1), \ldots, c_k(j), \ldots, c_k(J)]$, where $J = SL$. The coded sequence $\mathbf{c}^{(k)}$ is permutated by an interleaver Π_k, producing $\mathbf{x}^{(k)} = [x_k(1), \ldots, x_k(J)]$, where we denote the components of $\mathbf{x}^{(k)}$ as *chips*.

B. Iterative Receiver

At the receiver chip-matched filtering is performed, proving a sequence of chip-level observables,

$$y(j) = \sum_{k=1}^{K} h_k x_k(j) + n(j), \qquad j = 1, \ldots, J, \tag{1}$$

where h_k is the channel coefficient for the k-th user and $n(j)$ denotes the sample of an AWGN process with variance $\sigma^2 = N_0/2$. We here consider Gaussian channels, i.e., $h_k \equiv 1$.

In IDMA, users are distinguished by distinct interleavers, i.e., $\Pi_k \neq \Pi_{k'}$ for $k \neq k'$. Assuming the interleavers are generated independently and randomly, the adjacent chips in the sequence $\mathbf{x}^{(k)}$ are approximately uncorrelated. A simple version of the SISO parallel interference cancellation (PIC) MMSE MUD [9], is adopted at the receiver. This scheme is a special case of the one derived in [9], where spreading sequences have length 1 and the code rate is $R_1 R_2$. The receiver structure for user k is shown in Fig. 1(b). The MUD is concatenated with two APP decoders; one for each channel code. The multiple access constraints and code constraints are considered separately by the MUD and the two channel decoders.

1) SISO MUD: Following the derivations in [1, 2], the received signal in (1) can be rewritten as

$$\begin{aligned} y(j) &= h_k x_k(j) + \sum_{k' \neq k} h_{k'} x_{k'}(j) + n(j) \\ &= h_k x_k(j) + V_k(j), \end{aligned} \tag{2}$$

where

$$V_k(j) = \sum_{k' \neq k} h_{k'} x_{k'}(j) + n(j) \tag{3}$$

is the sum of the multiple access interference and noise terms of the received sample $y(j)$ relative to user k. For a large number of users K, it follows from the central limit theorem that $V_k(j)$ is approximately Gaussian distributed [1, 2].

Define

$$\mathbf{E}[y(j)] = \sum_{k=1}^{K} h_k \mathbf{E}[x_k(j)], \tag{4}$$

and

$$\text{Var}[y(j)] = \sum_{k=1}^{K} |h_k|^2 \text{Var}[x_k(j)] + \sigma^2. \tag{5}$$

It follows that $V_k(j)$ has mean

$$\mu_k(j) = \mathbf{E}[V_k(j)] = \mathbf{E}[y(j)] - h_k\mathbf{E}[x_k(j)], \quad (6)$$

and variance

$$\nu_k(j) = \text{Var}[V_k(j)] = \text{Var}[y(j)] - |h_k|^2\text{Var}[x_k(j)]. \quad (7)$$

The SISO MUD is employed to calculate extrinsic log-likelihood (LLR) of the chip $x_k(j)$ using [2, 9]

$$\begin{aligned} L_e^{MUD}(x_k(j)) &= \ln\frac{p\left(y(j)|x_k(j)=+1\right)}{p\left(y(j)|x_k(j)=-1\right)} \\ &= 2h_k\frac{y(j)-\mu_k(j)}{\nu_k(j)} = 2h_k\frac{z_k(j)}{\nu_k(j)}, \quad (8) \end{aligned}$$

where $z_k(j) = y(j) - \mu_k(j)$. The extrinsic LLRs $L_e^{MUD}(x_k(j)), \forall j$ are further deinterleaved and fed to the SISO channel decoders.

2) SISO Channel Decoders: At the receiver for user k, we employ two APP decoders; one for the decoding of the repetition code and one for the decoding of the convolutional code, respectively. Considering the decoding of the repetition code, recall that for the first convolutionally encoded bit $b_k(1)$, $c_k(j) = x_k\left(\Pi_k(j)\right) = b_k(1)$ for $j = 1, \ldots, S$. We can then compute the *a posteriori* LLR of $b_k(1)$ as

$$\begin{aligned} L_{\text{rep}}(b_k(1)) &= \sum_{j=1}^{S}\ln\frac{p\left(c_k(j)=+1|y(j)\right)}{p\left(c_k(j)=-1|y(j)\right)} \\ &= \sum_{j=1}^{S} L_e^{MUD}\left(x_k\left(\Pi_k(j)\right)\right). \quad (9) \end{aligned}$$

The APP decoder of the repetition code can be seen as a *despreading* operation, and in the same manner, we compute $L_{\text{rep}}(b_k(l)), l = 1, \ldots, L$. The *despread* LLRs are used as *a priori* information for the APP decoder of the convolutional code, which in turn computes the final *a posteriori* LLRs of $b_k(l)$, denoted by $L_{\text{con}}(b_k(l)), \forall l$.

For the first bit $b_k(1)$, the *a posteriori* LLR $L_{\text{con}}(b_k(1))$ is repeated S times. We then compute the extrinsic LLRs of the corresponding coded bits $c_k(j), j = 1, \ldots, S$, using

$$\begin{aligned} L_e^{Dec}\left(c_k(j)\right) &= L_e^{Dec}\left(x_k\left(\Pi_k(j)\right)\right) \\ &= L_{\text{con}}(b_k(1)) - L_e^{MUD}\left(x_k\left(\Pi_k(j)\right)\right) \quad (10) \end{aligned}$$

In the same manner, we compute all the extrinsic LLRs $L_e^{Dec}\left(c_k(j)\right), j = 1, \ldots, J$. The above extrinsic LLRs $L_e^{Dec}\left(c_k(j)\right), \forall j$ are further interleaved, producing extrinsic LLRs of the chips $x_k(j)$, denoted by $L_e^{Dec}\left(x_k(j)\right), \forall j$. The extrinsic LLRs are fed back to the MUD to update the *a priori* LLRs using [9]

$$\mathbf{E}[x_k(j)] = \tanh\left(L_e^{Dec}(x_k(j))/2\right), \quad (11)$$

and

$$\text{Var}[x_k(j)] = 1 - \left(\mathbf{E}[x_k(j)]\right)^2. \quad (12)$$

Note that at the first iteration, assuming equally likely chips, we have $\mathbf{E}[x_k(j)] = 0$ and $\text{Var}[x_k(j)] = 1$, $\forall j, k$. This completes one iteration of the joint SISO detection and decoding process, which we denote as the *full iterative decoding* (FID) scheme.

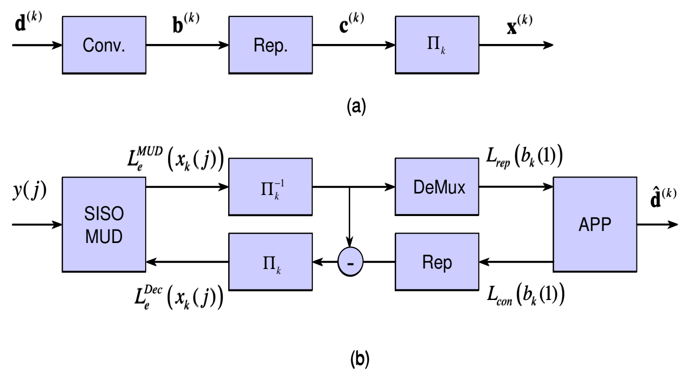

Fig. 1. The IDMA transmitter and receiver structures for user k.

3) Complexity of the FID scheme: The FID scheme has a complexity of $\Omega_F = K \times N_{max} \times (n_1 + n_2 + n_3)$ operations, where n_1, n_2, n_3 denote the complexity of MUD, APP decoding of the repetition code, and APP decoding of the convolutional code, respectively. The complexity of APP decoding of convolutional code is high, when compared to APP decoding of the repetition code and the SISO MUD, i.e. $n_3 \gg \max(n_1, n_2)$ [13]. Moreover, decoding complexity of IDMA increases linearly with K and N_{max}.

It is known that the number of iterations can be optimized to reduce decoding complexity. Since the number of iterations is related to decoding convergence speed, we first investigate decoding convergence behavior using EXIT chart.

III. EXIT Chart with Trajectory of IDMA

In the section, we show an example of an EXIT chart and the trajectory analysis of IDMA. Following that, we propose an ISD scheme, which guarantees fast decoding convergence, low decoding complexity and good system performance.

A. An Example of EXIT Chart/Trajectory Analysis of IDMA

Let γ^* denote the threshold E_b/N_0, at which the decoding process converges to single user performance. Also, let N_b denote the number of information input bits per user.

Example 1: We consider an IDMA system operating at $\gamma^* =$ 5 dB, with $K = 12$, $S = 8$, $\alpha = 0.75$, using the $R_1 = 1/2$, $(23, 35)_8$ convolutional code. Each user transmits $N_b = 128$ information bits, leading to interleaver sizes of 2048 chips. □

Fig. 2 depicts the extrinsic output (I_a^{MUD}, I_e^{MUD}), $(I_a^{Dec}, I_e^{Dec_1})$ and $(I_a^{Dec}, I_e^{Dec_2})$[3], respectively, where we make the following observations.

1) The tunnel between the extrinsic output I_e^{MUD} (solid line) and the extrinsic output $I_e^{Dec_2}$ (dash-dot line) is closed when $I_a^{MUD} \leq 0.434$.

[3]The computation of $(I_a^{MUD}, I_e^{MUD}), (I_a^{Dec}, I_e^{Dec_1}), (I_a^{Dec}, I_e^{Dec_2})$ is given in [11, 12], respectively.

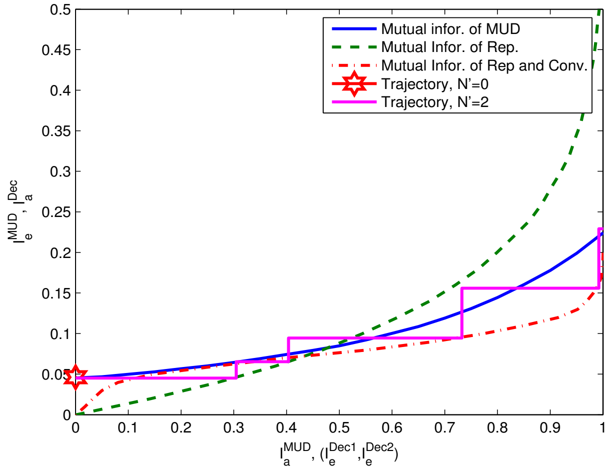

Fig. 2. EXIT chart with trajectory of IDMA, $K = 12, S = 8$, interleaver size of 2048 bits, $\gamma^* = 5$dB, $(23, 35)_8$ convolutional code with rate $R_1 = 1/2$.

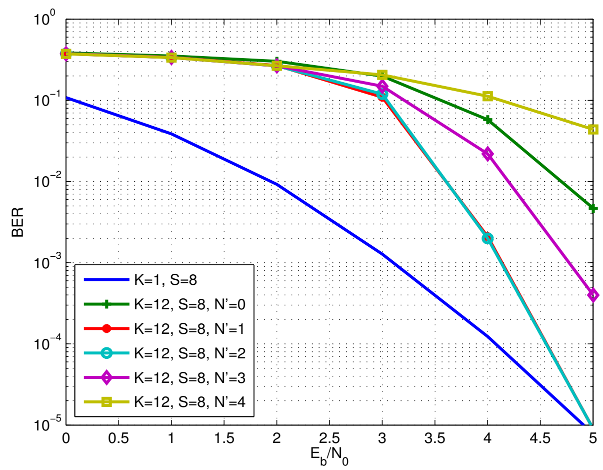

Fig. 3. BER performance of IDMA, $K = 12, S = 8$, interleaver size of 2048 bits, $(23, 35)_8$ convolutional code with rate $R_1 = 1/2$, 5 iterations.

2) When $0 \leq I_a^{MUD} \leq 0.434$ or $0 \leq I_a^{Dec} \leq 0.07244$, we observe $I_e^{Dec_1}$ (dash line) is always greater than I_e^{Dec2}. When $I_a^{Dec} \geq 0.07244$, we have $I_e^{Dec_1} \leq I_e^{Dec_2}$.
3) The extrinsic output I_e^{MUD} and $I_e^{Dec_1}$ cross each other when $I_a^{MUD} = 0.4775$.

From the first observation, we conclude that the decoding of the joint repetition and convolutional code gets stuck after a few iterations. The second observation confirms the fact that for low input mutual information, the repetition code decoded alone provides a higher output mutual information than the joint code. Since the multiple access interference is large at the first few iterations, the channel decoders receive very low input mutual information levels from the SISO MUD, and as a consequence the extrinsic output mutual information $I_e^{Dec_1}$ is greater than $I_e^{Dec_2}$ for a number of initial iterations at these low input mutual information levels. The third observation justifies to only decode the repetition code for a few initial iterations N' before switching to decode the joint repetition and convolutional code. Similar observations and conclusions were made in [13].

The idea of ISD is further supported by trajectory analysis. Fig. 2 depicts *average trajectories* of both the FID and the ISD schemes at $\gamma^* = 5$ dB, where 50 blocks of information bits per user are used in the simulation. It is shown that the trajectory of the FID scheme (denoted with stars in Fig. 2) gets stuck at the first iteration, which agrees with our analysis. This is in contrast to the ISD scheme, where for $N' = 2, N_{max} = 10$ we observe the corresponding staircase trajectory between the transfer curves of the MUD and the repetition-code decoder at iterations $n \leq N' = 2$, as well as the trajectory following the transfer curves of I_e^{MUD} and $I_e^{Dec_2}$ at iterations $n > N'$. In this case, we have convergence after 5 iterations. We also observe, however, that the trajectory of the ISD scheme does not match well with the transfer curve of the MUD immediately after the switch to the joint code trajectory. This is to some degree due to the relatively small interleaver size, but also something that needs further investigation.

IV. Design of the Proposed ISD Scheme

In order for the ISD scheme to be useful, suitable values of the number of iterations before switching, N', and the number of required iterations for convergence, such that the IDMA system under investigation achieves the fastest convergence at the lowest possible decoding complexity must be found off-line.

A. Design Rules of the ISD Scheme

Given a threshold γ^*, we conduct an exhaustive search to find the optimum choice of N' and N_D. The corresponding design rules are to

1) minimize the number of total iterations (N_D).
2) maximize the number of iterations of decoding only the repetition code (N').

The optimum value of N_D is the minimum number of iterations n for which the following condition is satisfied

$$I_{e,n}^{Dec_2} \leq I_{e,n-1}^{Dec_2}.$$

Let $\delta_n = I_{e,n}^{Dec_1} - I_{e,n-1}^{Dec_1}$. The optimum value of N' is the minimum number of iterations n such that

$$\delta_{n+1} < \epsilon_1^* < \delta_n$$

where $\delta_{n+1} \geq 0$ and $\epsilon_1^* \neq 0$ is a threshold to control the switching. The value of ϵ_1^* is non-zero, since larger steps in trajectory guarantee fast overall decoding convergence. We here choose ϵ_1^* according to experimental measures $(\delta_{N'}, \delta_{N'+1})$, as discussed in Section V.

B. Complexity of the ISD scheme

The ISD scheme has a maximum complexity of Ω_D operations, where

$$\begin{aligned}\Omega_D &= K \times N' \times (n_1 + n_2) \\ &+ K \times (N_D - N') \times (n_1 + n_2 + n_3)\end{aligned}$$

Hence, the proposed ISD scheme has a computational cost advantage quantified by

$$\eta = \frac{\Omega_F - \Omega_D}{\Omega_F} \approx \frac{N_F - N_D + N'}{N_F} \times 100\%$$

as compared to the FID scheme, provided $n_3 \gg n_1 + n_2$.

C. Further Results

Table I lists all the related parameters of Example 1 at a fixed threshold $\gamma^* = 5$ dB. It lists the number of users K, the repetition code length S, number of iterations N', N_F, N_D, computational cost advantage η, and bit error rate (BER) performance at $\gamma^* = 5$ dB, respectively. Note that in Example 1, the FID scheme does not converge. We let $N_F = N_{max} = 10$ to terminate iterations, where $N_{max} = 10$ is adopted in all examples. Table I also lists the measures $\delta_{N'} = I^{Dec_1}_{e,N'} - I^{Dec_1}_{e,N'-1}$, where $N' = 1, \ldots, 4$. The ISD scheme with $N' = 2, N_D = 5$ guarantees the lowest decoding complexity and the fastest decoding convergence.

Fig. 3 depicts the corresponding BER performance. Employing the ISD schemes with $N' = 2, N_D = 5$ optimized at γ^*, it is shown that the system achieves the best performance for all E_b/N_0.

V. Simulation Results

In this section, we consider IDMA systems with varying system spectral efficiency, repetition code lengths and interleaver sizes. For IDMA systems with spectral efficiency $\alpha > 1/2$ and $\alpha \leq 1/2$, the decoding process can converge at $\gamma^* = 5$ dB and 4 dB, respectively. The convolutional code $(23, 35)_8$ with rate $R_1 = 1/2$ is considered in all examples.

Tables I, II, III and IV list relevant parameters of the ISD scheme for IDMA systems with various spectral efficiency. We use 50 blocks of information bits per user in the computation of extrinsic mutual information. Three groups of examples are given for IDMA with various spectral efficiency.

Examples 2–4 (high spectral efficiency): Here we consider an IDMA system operating at $\gamma^* = 5$ dB, with spectral efficiency $\alpha = 0.75$, using the $R_1 = 1/2$, $(23, 35)_8$ convolutional code. Each user transmits $N_b = 512$ information bits. □

In Example 2, the IDMA system has $K = 24, S = 16$ and interleaver size of 16384 chips. In Table I, we observe that the ISD scheme with $N' = 2, N_D = 5$ is preferable in terms of decoding convergence behavior and decoding complexity. Further in Table I, we see that using FID decoding in Example 2 leads to convergence, while the system in Example 1 does not. This difference is due to the smaller interleaver size, which causes adjacent chips to be correlated. In Examples 3 and 4, IDMA systems with $K = 12, S = 8$ and $K = 6, S = 4$ and corresponding interleaver size of 8192 chips, 4096 chips, respectively, are investigated. Observing the results in Table I, we conclude that for spectral efficiency a $\alpha = 0.75$, the option of $N' = 2, N_D = 5$ is preferable for these examples. Note that in Table I we have the corresponding minimum $\delta_2 = 0.015$ and the maximum $\delta_3 = 0.006$.

Examples 5–8 (medium spectral efficiency): Here we consider an IDMA system operating at $\gamma^* = 4$ dB, with spectral efficiency $\alpha = 1/2$, using the $R_1 = 1/2$, $(23, 35)_8$ convolutional code. □

Ex.	K	S	N_F	N_D	N'	$\delta_{N'}$	η	BER	N_b
1	12	8	10	–	–	–	0%	4.7e-3	128
				5	1	0.08	60%	9e-6	
				5	2	0.02	70%	9e-6	
				8	3	0.006	50%	4e-4	
				8	4	0.0006	60%	4e-2	
2	24	16	5	–	–	–	0%	1e-5	512
				5	1	0.08	20%	1e-5	
				5	2	0.015	40%	1e-5	
				7	3	0.002	20%	1e-4	
				7	4	0.0004	40%	2.5e-2	
3	12	8	5	–	–	–	0%	1.1e-5	512
				5	1	0.08	20%	1.1e-5	
				5	2	0.02	40%	1.2e-5	
				7	3	0.006	20%	1.3e-4	
				7	4	0.0006	40%	3e-2	
4	6	4	5	–	–	–	0%	9.5e-6	512
				5	1	0.06	20%	9.9e-6	
				5	2	0.03	40%	9.7e-6	
				7	3	0.002	20%	4.5e-5	
				7	4	0.0002	40%	2.8e-2	

TABLE I
IDMA (Examples 1-4) with selected values of K, S, N_b and $\alpha = 0.75, \gamma^* = 5$dB, $(23, 35)_8$ convolutional code $R_1 = 1/2$.

Ex.	K	S	N_F	N_D	N'	$\delta_{N'}$	η	BER	N_b
5	8	8	4	–	–	–	0%	1.7e-4	128
				4	1	0.09	25%	1.7e-4	
				4	2	0.02	50%	1.5e-4	
				5	3	0.005	50%	7.7e-4	
6	16	16	4	–	–	–	0%	1.6e-4	512
				4	1	0.1	25%	1.6e-4	
				4	2	0.02	50%	1.6e-4	
				5	3	0.004	50%	9.9e-3	
7	8	8	4	–	–	–	0%	1.7e-4	512
				4	1	0.09	25%	1.7e-4	
				4	2	0.02	50%	1.7e-4	
				5	3	0.004	50%	8.9e-3	
8	4	4	4	–	–	–	0%	1.55e-4	512
				4	1	0.087	25%	1.4e-4	
				4	2	0.021	50%	1.3e-4	
				5	3	0.003	50%	8e-3	

TABLE II
IDMA (Examples 5-8) with selected values of K, S, N_b and $\alpha = 1/2, \gamma^* = 4$dB, $(23, 35)_8$ convolutional code $R_1 = 1/2$.

In Example 5, we consider a system with $K = 8, S = 8$, $N_b = 128$ bits, and interleaver size of 2048. In Examples 6, 7, and 8, we investigate system with $K = 16, S = 16$, $K = 8, S = 8$, and $K = 4, S = 4$, respectively. We assume $N_b = 512$ bits and the corresponding interleaver sizes of 16384 chips, 8192 chips and 4096 chips, respectively.

Table II lists the related parameters of Examples 5–8, and we conclude that the choice of $N' = 2, N_D = 5$ is prefereable in terms of BER performance and decoding complexity. Note that in Table II we have the minimum $\delta_2 = 0.015$ and the maximum $\delta_3 = 0.005$.

Examples 9–10 (low spectral efficiency): Here we consider an IDMA system operating at $\gamma^* = 4$ dB, with spectral effi-

Ex.	K	S	N_F	N_D	N'	$\delta_{N'}$	η	BER	N_b
9	8	16	4	–	–	–	0%	1.5e-4	128
				4	1	0.09	33%	1.5e-4	
				4	2	0.011	50%	1.4e-4	
				5	3	0.001	20%	6.4e-4	
10	8	16	4	–	–	–	0%	1.2e-4	512
				4	1	0.1	33%	1.2e-4	
				4	2	0.011	50%	1.2e-4	
				5	3	0.001	20%	6.8e-4	

TABLE III

IDMA (EXAMPLES 9-10) WITH SELECTED VALUES OF K, S, N_b AND $\alpha = 0.25$, $\gamma^* = 4$DB, $(23, 35)_8$ CONVOLUTIONAL CODE, $R_1 = 1/2$.

Ex.	K	S	N_F	N_D	N'	$\delta_{N'}$	η	BER	N_b
11	32	16	6	–	–	–	0%	1e-5	128
				6	1	0.06	17%	1.5e-5	
				6	2	0.014	33%	1.9e-5	
				8	3	0.001	17%	4.3e-3	
				9	4	0.0004	17%	8e-2	
				10	5	0.0001	17%	2e-1	

TABLE IV

IDMA (EXAMPLES 11) WITH SELECTED VALUES OF K, S, N_b AND $\alpha = 1$, $\gamma^* = 5$DB, $(23, 35)_8$ CONVOLUTIONAL CODE, $R_1 = 1/2$.

ciency $\alpha = 0.25$, using the $R_1 = 1/2$, $(23, 35)_8$ convolutional code. □

In Example 9, we study an IDMA system with $K = 8, S = 16$, $N_b = 128$ bits, and interleaver size of 4096 chips, respectively, while in Example 10, the IDMA system employs $K = 16, S = 16$, $N_b = 512$ bits, and interleaver size of 16384 chips. As seen in Table III, the FID scheme and ISD scheme with $N' \leq 2$ have the same BER performance, while the ISD scheme with $N' = 2, N_D = 4$ provides the lowest decoding complexity. We have the minimum $\delta_2 = 0.011$ and the maximum $\delta_3 = 0.001$ in Table III.

Examples 11 (very high spectral efficiency): Here we consider an IDMA system operating at $\gamma^* = 5$ dB, with spectral efficiency $\alpha = 1$, using the $R_1 = 1/2$, $(23, 35)_8$ convolutional code. □

In Example 11, we study an IDMA system with $K = 32, S = 16$, $N_b = 128$ bits, and interleaver size of 4096 chips. As shown in Table IV, the FID scheme and ISD scheme with $N' \leq 2$ have the same BER performance, while the ISD scheme with $N' = 2, N_D = 6$ provides the lowest decoding complexity. We have the minimum $\delta_2 = 0.014$ and the maximum $\delta_3 = 0.001$ in Table III.

In summary, we can make the following remarks.

1) For IDMA systems with high spectral efficiency ($\alpha = 0.75$), the FID scheme may not converge for small interleaver sizes. In contrast, for these such cases the ISD schemes achieve fast decoding convergence.
2) For IDMA systems with various spectral efficiency ($0 < \alpha \leq 1$), we can choose a value between the maximum $\delta_{N'+1}$ and the minimum $\delta_{N'}$ as threshold ϵ_1^*. In all examples, we have $0.006 < \epsilon_1^* < 0.011$. After choosing ϵ_1^*, the optimum values of N' and N_D can be easily obtained using the ISD scheme.

VI. CONCLUSIONS

In this paper, we consider IDMA systems with K simultaneous users transmitting over a common AWGN channel. We propose an iterative switched-decoding scheme, where at the first N' iterations, the receiver only decodes the repetition code. The receiver then switches to decoding the joint repetition and convolutional codes at the remaining $N_D - N'$ iterations. The values of N' and N_D can be found off-line through exhaustive search. It is shown by simulation that the proposed ISD decoding scheme with preferable values of N' and N_D is effective in improving convergence and reducing complexity for IDMA systems with various spectral efficiency.

REFERENCES

[1] L. Ping, "Interleave-division multiple access and chip-by-chip iterative multi-user detection," *IEEE Communication Magazine*, vol. 43, no. 6, pp. S19–S23, June 2005.

[2] L. Ping, L. Liu, K. Wu, and W. K. Leung, "Interleave Division Multiple-Access," *IEEE Trans. Wireless Communications*, vol. 5, no. 4, pp. 938–947, April 2006.

[3] P. A. Hoeher and H. Schoeneich, "Interleave-division multiple access from a multiuser theory point of view," *Int. Symp. Turbo Codes* (Munich, Germany), Apr. 2006.

[4] F. N. Brännström, T. M. Aulin and L. K. Rasmussen, "Iterative detectors for trellis code multiple access," *IEEE Trans. Commun.*, vol. 50, pp. 1478–1485, Sept. 2002.

[5] A. J. Viterbi, "Very low rate convolution codes for maximum theoretical performance of spread-spectrum multiple-access channels," *IEEE J. Sel. Areas Commun.*, Vol. 8, pp. 641–649, May 1990.

[6] P. K. Frenger, P. Orten, and T. Ottosson, "Code-spread CDMA with interference cancellation," *IEEE J. Sel. Areas Commun.*, Vol. 17, pp. 2090–2095, Dec. 1999.

[7] P. K. Frenger, P. Orten, and T. Ottosson, "Code-spread CDMA using maximum free distance low-rate convolutional codes," *IEEE Trans. Commun.*, Vol. 48, pp. 135–144, Jan. 2000.

[8] P. H. Tan and L. K. Rasmussen, "Belief Propagation for Coded Multiuser Detection," *IEEE Int. Symp. Inform. Theory* (Seattle, Washington, USA), July 2006.

[9] X. Wang and H. V. Poor, "Iterative (turbo) soft interference calcellation and decoding for coded CDMA," *IEEE Transactions on Communications*, vol. 47, pp. 1046–1061, July 1999.

[10] P. H. Tan and L. K. Rasmussen, "Nonlinear MMSE Multiuser Detection Based on Multivariate Gaussian Approximation," *IEEE Trans. Commun.*, Vol. 54, pp. 1427-1438, Aug. 2006.

[11] S. ten Brink, "Convergence behavior of iteratively decoded parallel concatenated codes," *IEEE Transactions on Information Theory,* vol. 49, pp. 1727–1737, Oct. 2001.

[12] K. Li and X. Wang, "EXIT chart analysis of turbo multiuser detection," *IEEE Transactions on Wireless Communications*, vol. 4, no. 1, pp. 300–311, Jan. 2005.

[13] K. Kusume and G. Bauch, "Some aspects of Interleave Division Multiple Access in Ad Hoc networks," *IEEE Turbo Symposium*, Munich, Germany, April 2006.

[14] S. ten Brink, G. Kramer, and A. Ashikhmin, "Design of low-density parity-check codes for modulation and detection," *IEEE Transactions on Communications*, vol. 52, no. 4, pp. 670–678, April 2004.