Wireless Communication Systems With Spatial Diversity: A Volumetric Model

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Abstract—This paper presents a novel physical-modeling approach to wireless systems with multiple antennas. The fundamental problem of modeling the communication channel is studied, where the channel consists of a finite spatial volume for transmitting, a finite spatial volume for reception, and an arbitrary set of reflective-scattering bodies. The number of communication modes (or degrees of freedom) for such a system is calculated, using the procedure developed. We present a simple model for multipath channels, which allows insight into the development of a correlated multiple-input multiple-output (MIMO) channel model. In particular, the model is independent of transmitter and receiver elements and relies on the physical parameters of the channel involved. Our work explains which physical parameters determine the channel model and its channel capacity.

Index Terms—Information rates, multipath channels, multiple-input multiple-output (MIMO) systems.

I. INTRODUCTION

MULTIPATH has been shown to improve capacity in wireless communications under the assumption that the channel model is a random matrix with independent identically distributed (i.i.d) elements [1], [2]. More specifically, the capacity of a multiantenna system with dense scattering has been shown to grow proportionally with the minimum of the numbers of transmitters and receivers [1], [2]. This is known as the linear-growth property of MIMO systems.

Much of the early work on MIMO wireless systems assumed that the channel model was a random matrix with i.i.d. element distribution [1], [2]. In [3], an experimental system was developed and tested for an indoor environment confirming the increase in channel capacity when multiple antennas are used. Recently several authors have addressed correlations in the fading environment [4], [5], antenna separation [6], and coupling between array elements [7]. A large amount of work has also focused on modeling of channels for multiantenna systems [8] and many authors have presented measurement campaigns addressing MIMO wireless channels. The diversity benefits of nonlinear-of-sight (NLOS) indoor channels have been investigated in the presence of dominant paths and polarization [9], and in outdoor environments [10]. In each case, greater or lesser improvements for channel capacity have been shown dependent upon the channel correlation. This leads to the natural question “Given an arbitrary volume, containing transmit elements, a second arbitrary volume containing receive elements and a group of scattering bodies, how many independent parallel channels are available?”

In order to address this question we must move away from standard point-defined multiple-antenna models to a functional-analysis viewpoint. The modeling of multiple-element arrays as collections of points is not appropriate for large numbers of closely placed elements: correlation matrices and mutual coupling hide the fact that antenna elements only sample a continuous spatial signal, in an analogous way to the sampling of a continuous-time signal. This has been examined in [11], where a random i.i.d. channel model was used, but the unbounded growth is rectified using a scaling parameter. It has been suggested that dense antenna arrays in the presence of scattering should also exhibit such a limit [12], however the authors’ focus was on the coupling of antenna elements rather than an intrinsic capacity of the volume-to-volume communication channel. Some work has considered exploiting the coupling effects of antenna elements [13] for multiple-antenna arrays. The work of [7] and [14] has also addressed this concept. Coupled with the theoretical work of [15] these results suggest that there are fundamental physical limits to the capacity growth of a given MIMO channel, independent of the number of antenna elements.

We may view antenna elements as sampling functions, as given in [16], and consider space as a continuous parameter for input and output signals. We shall not address sampling in this paper; it would suffice to say that different arrangements of antenna elements correspond to different choices of sampling functions in the same way that continuous-time signals may be sampled at varying intervals. We provide a modeling technique that allows the examination of fundamental properties of the channel: we may consider correlation as an artifact of the use of antenna elements, rather than a property associated with spatial diversity. Consequently, once the volumetric approach is adopted, we may examine how to best utilize the channel in terms of the continuous spatial modes produced. The case without scattering bodies has been examined in [17] and its references for optical communications in free space. It is interesting that the setting in [17] is general and the results are equally applicable to radio communications.
This paper provides a simple model for multipath scattering channels, which allows insight into the development of a volumetric MIMO channel model. In particular, we present a model that is independent of transmitter and receiver elements, and instead relies on the physical parameters of the channel involved. We show how to determine the number of communication modes (or degrees of freedom) for such a system.

This paper is arranged as follows. In Section II, we detail the wireless model for the two-body case (in free space) and provide a novel numerical approach in Section II-B for calculating the transfer-matrix elements for arbitrary bodies. In Section III, we generalize these results to include simple reflective-scattering bodies. In Section IV, examples are used to show how scattering bodies influence the channel model. We discuss the results in Section V and draw the conclusion in Section VI.

II. MODELING FOR TWO-VOLUME COMMUNICATIONS

Consider wireless communications between two arbitrary volumes as described in Fig. 1, where $V_T$ is the transmitting body and $V_R$ is the receiving body. The centers of the two bodies are separated by a distance $D$. It is assumed that the dimensions of the $V_T$ and $V_R$ are small compared to the distance $D$. We choose the coordinates $(x, y, z)$ such that the $z$-axis is along the direction of $D$ and the origin is the center of $V_T$. For simplicity, we consider single-frequency signals, and we assume that the channel is memoryless. The work in this paper may be generalized to the case where a finite frequency bandwidth is available.

Under the above constraints we may consider “signals” within $V_T$ and $V_R$ that are functions of space rather than time. The intuition of continuous-time channels [18, Ch. 8] may be applied with appropriate changes of variable. In particular, the channel between $V_T$ and $V_R$ may be represented in MIMO-matrix form using a matrix $\Gamma$. However, unlike standard wireless MIMO-matrix representations, where the matrix $H$ represents channel gains between pointlike antenna elements, the matrix $\Gamma$ represents channel gains between input signals and output signals that are functions defined over space. We may then consider exciting the channel with a particular choice of transmit signal, and measuring the output against a particular choice of receive signal.

It is shown in [17] that if the two bodies are hyperrectangles with sides parallel to the $x$, $y$, $z$ axes, there is a simple expression for $\Gamma$. More specifically, the eigenfunctions of the channel may be expressed in terms of a focusing function and a set of prolate spheroidal functions that are all easily computable. These eigenfunctions form a set of complete orthonormal functions in their respective volumes. Further, the corresponding eigenvalues have roughly a constant magnitude up to a critical number, after which the magnitudes are negligible. This critical number determines the number of communication modes between $V_T$ and $V_R$ and is given by

$$n_c = \frac{V_R V_T}{D^2 \lambda^2 (2\Delta z_T)(2\Delta z_R)}$$

(1)

where $V_T$ and $V_R$ are the volumes of the two bodies (with some abuse of notation), $2\Delta z_T$ and $2\Delta z_R$ are their thicknesses in the $z$ direction, and $\lambda$ is the signal wavelength. Consequently, the transfer function between $V_T$ and $V_R$ is given by a diagonal matrix of a finite dimension, with the diagonal elements being the nonnegligible eigenvalues.

A. Transfer Function

Let $\Psi(r_T)$ be a transmit signal at any point $r_T \in V_T$ and $\Phi(r_R)$ be the electrical potential of the received signal at any point $r_R \in V_R$. Then, we have

$$\Phi(r_R) = \int_{V_T} G(r_R, r_T) \Psi(r_T) d^3 r_T$$

(2)

where $G(\cdot, \cdot)$ is the retarded Green’s function defined by

$$G(r_R, r_T) = \frac{\exp(-i k |r_R - r_T|)}{4\pi |r_R - r_T|}$$

(3)

where $i = \sqrt{-1}$ and $k = 2\pi/\lambda$, where $\lambda$ is the wavelength of the signal. Let $\{\psi_{T_i}(r_T), i = 1, \ldots, N_T\}$ (respectively, $\{\phi_{R_j}(r_R), j = 1, \ldots, N_R\}$) be a finite set of orthonormal functions in $V_T$ (respectively, $V_R$). Then, given any $\psi_{T_i}(r_T)$, the received signal is given by

$$\Phi_i(r_R) = \int_{V_T} G(r_R, r_T) \psi_{T_i}(r_T) d^3 r_T.$$  

(4)

Define the projection of $\Phi_i(r_R)$ on $\phi_{R_j}(r_R)$ as follows:

$$\Gamma_{ji} = \int_{V_R} \int_{V_T} \phi_{R_j}^{*}(r_R) G(r_R, r_T) \psi_{T_i}(r_T) d^3 r_T d^3 r_R$$

(5)

then, we have

$$\Phi_i(r_R) = \sum_{j=1}^{N_R} \Gamma_{ji} \phi_{R_j}(r_R) + \delta \Phi_i(r_R)$$

(6)

where $\delta \Phi_i(r_R)$ is a residual term orthogonal to all $\phi_{R_j}(r_R)$. We may write any transmit signal as

$$\Psi(r_T) = \sum_{i=1}^{N_T} a_i \psi_{T_i}(r_T).$$

(7)
The received signal is

\[ \Phi(r_R) = \sum_{j=1}^{N_R} b_j \phi_{Rj}(r_R) + \delta\Phi(r_R) \quad (8) \]

where \( \delta\Phi(r_R) \) is a residual signal orthogonal to all \( \phi_{Rj}(r_R) \) and

\[ b_j = \sum_{i=1}^{N_T} a_i \Gamma_{ji}. \quad (9) \]

In a vector form, we have

\[
\begin{bmatrix}
  b_1 \\
  \vdots \\
  b_{N_R}
\end{bmatrix} = \begin{bmatrix}
  \Gamma_{11} & \cdots & \Gamma_{1N_T} \\
  \vdots & \ddots & \vdots \\
  \Gamma_{N_R1} & \cdots & \Gamma_{N_RN_T}
\end{bmatrix} \begin{bmatrix}
  a_1 \\
  \vdots \\
  a_{N_T}
\end{bmatrix}
\]

or in a compact form

\[ b = \Gamma a. \quad (11) \]

If the sets \( \{\psi_{Ti}(r_T), i = 1, 2, \ldots\} \) and \( \{\phi_{Rj}(r_R), j = 1, 2, \ldots\} \) are complete, then any transmit signal can be expressed as in (8) without the residual term \( \delta\Phi(r_R) \). In this case, (11) represents the true transfer function of the communication system, regardless of the choice of the basis functions \( \{\psi_{Ti}(r_T)\} \) and \( \{\phi_{Rj}(r_R)\} \).

Completeness of the function sets requires \( N_T \) and \( N_R \) to be infinite—and consequently, \( \Gamma \) is infinite dimensional. However, the channel between two finite volumes has a finite number of communication modes [17], [19]. Equivalently, the number of nonnegligible singular values of \( \Gamma \) is finite. If we choose the eigenfunctions of the channel \( \{\varphi_{Ti}(r_T)\}_{i=1}^{\infty} \) as our basis functions for the transmit signal, then the transfer matrix \( \Gamma \) is diagonal with \( \sigma_i \) as the entries, ordered from largest magnitude to smallest. Further, the number of communication modes \( n_c \) correspond to the number of nonnegligible singular values.

The functions \( \varphi_{Ti}(r_T) \) may be found through the analagous solution of an eigenvalue problem [17]

\[ \sigma_i^2 \varphi_{Ti}(r_T) = \int_{V_T} \int_{V_R} G^*(r_R, r_{T2})G(r_R, r_{T1}) \]

\[ \times \varphi_{Ti}(r_T) \alpha^3 \alpha^3 \alpha^3 \alpha^3 \alpha^3 \quad (12) \]

where \( \varphi_{Ti}(r_T) \) is the \( i \)th (normalized) eigenfunction and the set \( \{\varphi_{Ti}(r_T)\}_{i=1}^{\infty} \) is a complete orthonormal set. However, this problem has tractable analytical solutions in only a small number of carefully designed geometric situations. The difficulty lies in computing these eigenfunctions when we have arbitrary bodies \( V_T \) and \( V_R \).

Because the dimensions of the bodies are much smaller than the distance \( D \), we use a paraxial approximation [17] to simplify \( G(r_R, r_{T1}) \). We write the basis functions as [17]

\[ \psi_T(r_T) = F_T(r_T) \beta_T(r_T) \quad (13) \]

where \( \beta_{Ti}(\cdot) \) are new functions and \( F_T(\cdot) \) is the so-called focusing function defined by

\[ F_T(r_T) = \exp \left(-ik\left(x_T^2 \frac{2D}{2D} - y_T^2 \frac{2D}{2D}\right)\right) \quad (14) \]

so we may write (with some abuse of notation)

\[ \beta_{Ti}(r_T) = \beta_{Ti}(x_T, y_T) \quad (15) \]

and similarly for \( \beta_{Rj}(x_R, y_R) \). Each eigenfunction \( \psi_{Ti}(r_T) \) or \( \phi_{Rj}(r_R) \) is the product of a three-dimensional (3-D) focusing function and a two-dimensional (2-D) function. This is a crucial step towards numerical solutions for the transfer function between \( V_T \) and \( V_R \).

We wish to find a computationally efficient method for determining the channel eigenfunctions. We note that while direct numerical solution (12) for arbitrary volumes is not efficient, we may use the analytic results of [17] to provide a significant computational saving in our numerical solution. To motivate our numerical solutions, we first consider the case where both \( V_T \) and \( V_R \) are hyperrectangles parallel to the \( x, y, z \) axes. Suppose the dimensions of \( V_T \) are given by \( 2\Delta x_T, 2\Delta y_T \), and \( 2\Delta z_T \), and similarly for \( V_R \). This case is analyzed in [17] and the solution is given as

\[ \beta_{Ti}(x_T, y_T) = \frac{S_{0m}(c_x, \hat{x}_T)S_{0n}(c_y, \hat{y}_T)}{\sqrt{\Delta x T \Delta y T \Delta z T}} \quad (16) \]

for \( m, n = 0, 1, \ldots \), where

\[ c_x = \frac{k \Delta x T \Delta z R}{D} \quad (17) \]

and similarly for \( c_y \) and

\[ \hat{x}_T = \frac{x_T}{\Delta x T} \quad (18) \]

and similarly for \( \hat{y}_T \). The function \( S_{0m}(c, \xi) \) is the so-called \( (0, m) \) th angular prolate spheroidal function [20], [21] with eigenvalues \( v_m \) that are well studied. The eigenvalues obey the relation

\[ 1 > |v_1| > |v_2| > \cdots > 0. \quad (19) \]

Only a finite number of \( v_i \) have nonnegligible magnitudes and for fixed values of \( c \), the \( v_i \) fall off rapidly for \( i > 2c/\pi \) [21]. Further, functions \( \{S_{0m}(c, \xi)\}_{i=1}^{\infty} \) are complete and orthonormal over the unit interval. Computational methods for \( v_m \) and \( S_{0m}(c, \xi) \) are available in [20] and [22].

B. Numerical Solutions for Arbitrary Volumes

Consider now the (general) case where \( V_T \) and \( V_R \) have arbitrary shapes, i.e., they are no longer prisms aligned along the \( z \)-axis. In this case, the basis functions \( \beta_{Rj}(x_R, y_R) \) (or \( \beta_{Rj}(x_R, y_R) \)) cannot be separated (analytically) into \( x \) and \( y \) components of the form given by (16). We need to resort
to a numerical solution. We achieve this by modifying the
eigenfunctions of the prism case to avoid solving (12) directly.
1) Project $V_T$ along the $z$ direction to obtain a surface $S_T$ on
the $xy$ plane.
2) Define $S_T$ to be the smallest rectangle, with sides parallel
to the $xy$ axes, in the $xy$ plane that covers $S_T$.
3) The side lengths $2\Delta x_T$ and $2\Delta y_T$ are the lengths of $S_T$
in the $x$ and $y$ directions, respectively.
4) The process is repeated for $V_R$ to obtain $S_R$, $S_T$, $2\Delta x_R$, and
$2\Delta y_R$.

We now have two rectangular surfaces that may be used
to generate functions $\beta_T(x_T, y_T)$ and $\beta_R(x_R, y_R)$, substituting
the “volumes” $V_T = S_T \times (2\Delta z_T)$ and $V_R = S_R \times$.

The “new” functions $\beta_T(x_T, y_T)$ and $\beta_R(x_R, y_R)$ are complete
in both $S_T$ and $S_R$, although not necessarily orthogonal or
orthonormal. To see this, extend any function $f(x_T, y_T)$, defined over $S_T$, to a function $g(x_T, y_T)$, defined over the
whole rectangle $S_T$

$$g(x_T, y_T) = \begin{cases} f(x_T, y_T), & (x_T, y_T) \in S_T \subseteq S_T \\ 0, & \text{otherwise} \end{cases}$$

Then, $g(x_T, y_T)$ may be expressed as a linear combination of
$\{\beta_T(x_T, y_T)\}$ over $S_T$, since $\{\beta_T(x_T, y_T)\}$ is a complete
set in $S_T$, i.e., $g(x_T, y_T) = \sum_{\alpha} \alpha \beta_T(x_T, y_T)$. The same
combination $\{\alpha_1, \ldots\}$ still holds over $S_T$ because $S_T \subseteq S_T$. A
similar claim holds for $\beta_R(x_R, y_R)$.

The set $\{\beta_T(x_T, x_j)\}$ may be transformed to a complete and
orthonormal set $\{\tilde{\beta}_T(x_T, y_T)\}$ in $V_T$ using the Gram–Schmidt
process. The norm is taken over $V_T$, i.e.,

$$\tilde{\beta}_T(x_T, y_T) = \frac{\beta_T(x_T, y_T)}{\|\beta_T(x_T, y_T)\|_{V_T}}$$

where the volume integral in $V_T$ may be simplified to a scaled
surface integral on $S_T$

$$\|f(x_T, y_T)\|_{V_T}^2 = \int_{V_T} f^*(x_T, y_T) f(x_T, y_T) \ dx_T \ dy_T \ dz_T$$

$$\approx \int_{S_T} 2\Delta z_T f^*(x_T, y_T) \ f(x_T, y_T)$$

and $2\Delta z_T (x_T, y_T)$ is the “thickness” of $V_T$ in the $z$
direction at point $(x_T, y_T)$. We transform the set $\{\tilde{\beta}_R(x_R, y_R)\}$ to a
complete orthonormal set $\{\tilde{\beta}_R(x_R, y_R)\}$ in a similar way.

We now apply the paraxial approximation, which allows us to
write the 3D basis functions in terms of the 2D functions
$\beta_T(x_T, y_T)$ and a focusing function

$$\tilde{\psi}_T(r_T) = F_T(r_T) \tilde{\beta}_T(x_T, y_T)$$

and similarly

$$\tilde{\phi}_R(r_R) = F_R(r_R) \tilde{\beta}_R(x_R, y_R).$$

We approximate the Green’s function in terms of the transmit
and receive focusing functions $[17]$

$$G(r_T, r_R) \approx \frac{e^{-ikD}}{4\pi D} F_T(r_T) e^{ik(r_T + y_R y_T)} F_R(r_R).$$

It follows that

$$\Gamma_{ji} = \frac{\exp(-ikD)}{4\pi D} \int_{V_T} \tilde{\beta}_T(x_T, y_T) \ \exp \left( \frac{ik}{D} (x_T y_T + y_R y_T) \right)$$

$$\times \tilde{\beta}_R(x_R, y_R) \ dx_T \ dy_T \ dz_T \ dx_R \ dy_R \ dz_R.$$
with equality if and only if the sets are complete. Under the assumption that the dimensions of \( V_T \) and \( V_R \) are sufficiently small compared to the separating distance \( D \), (29) becomes

\[
\sum_{i,j} |\Gamma_{ji}|^2 \leq \frac{V_T V_R}{(4\pi D)^2}
\]

(30)

\[
= \sum_{i}^{N} \sum_{j}^{N} |\Gamma_{ji}|^2 + \delta_E, \quad \delta_E > 0. \quad (31)
\]

From (31), if \( \delta_E \) is sufficiently\(^1 \) small, then the finite estimation of \( \Gamma \) is sufficiently accurate. If \( \delta_E \) is too large (the sum of \( |\Gamma_{ji}|^2 \) is too small), then we increase the number of basis functions and compute a larger dimension \( \Gamma \). By the orthogonality of the basis functions, we may simply append the additional terms onto the matrix \( \Gamma \). As a rule of thumb, we choose the dimension of the truncated \( \Gamma \) to be \( \bar{n}_e \times \bar{n}_e \), where \( \bar{n}_e \) is the number of communication modes for \((\bar{S}_T, \bar{S}_R)\). Recall from (1)

\[
\bar{n}_e = \frac{\bar{S}_R \cdot \bar{S}_T}{D^2 \lambda^2}.
\]

(32)

We then increase the dimension of \( \Gamma \), if necessary, according to (31).

In general, the dimension of \( \Gamma \) as obtained above is larger than the number of communication modes for \((V_T, V_R)\). This is because \( \psi_{V_T} (r_T) \) and \( \phi_{V_R} (r_R) \) are not necessarily the eigenfunctions for \((V_T, V_R)\). In order to obtain a minimal representation of \( \Gamma \), we perform a singular-value decomposition, i.e., we form

\[
\Gamma = U^* \Lambda V
\]

(33)

where \( U \) and \( V \) are unitary matrices and \( \Lambda \) is a diagonal matrix containing the singular values of \( \Gamma \) in descending-magnitude order. We may generate the optimal-diagonalizing-basis sets by application of \( V \) and \( U \)

\[
\psi_{opt}^{Ti} (r_T) = \sum_{j} \psi_{Tj} (r_T) V_{i,j}
\]

(34)

\[
\phi_{opt}^{Ri} (r_R) = \sum_{j} \phi_{Rj} (r_R) U_{i,j}
\]

(35)

which means that the basis functions are now the eigenfunctions of the channel. After the decomposition (33), we may have several negligible singular values. These may be discarded to provide a reduced-transfer function. The number of remaining terms corresponds to the number of communication modes for the system.

The unitary matrices \( U \) and \( V \) may be interpreted as optimal energy distributions for the (suboptimal) basis functions used, i.e., if we were forced to transmit using signals \( \hat{\psi}_T (r_T) \) and forced to receive using \( \hat{\psi}_T (r_T) \), then \( U \) and \( V \) would provide “steering” for our signals to ensure that the appropriate channel modes were activated.

\(^1\)For simulations, we chose \( \delta_E < 1 \times 10^{-8} \) as a threshold.

Fig. 2. Two-volume arrangement, with absorbing plane \( P_s \) and finite hole \( S(s, n) \). Size of \( S(s, n) \) determines scatterer size.

III. MODELING FOR VOLUME COMMUNICATIONS WITH SCATTERING BODIES

A. An Imaginary Exercise

Consider the two volume communication system in Fig. 2, where we have placed an infinitely large absorbing plane \( P_s \) at point \( s \) between \( V_T \) and \( V_R \). The normal direction \( n \) of \( P_s \) is arbitrary. The plane is fully energy absorbing except for a hole \( S(s, n) \) in the middle, i.e., the only means for any signal to reach \( V_R \) from \( V_T \) is through \( S(s, n) \). We ask “how large must this hole be so that communication between the two volumes is not affected?”

The answer to this question is as follows: If \( \lambda \) is sufficiently small, then \( S(s, n) \) is the minimum area containing all the intersection points of \( P_s \) and \( \ell (r_T, r_R) \), where \( \ell (r_T, r_R) \) is a line linking \( r_T \in V_T \) and \( r_R \in V_R \). If \( \lambda \) is not small compared with \( S(s, n) \), then \( S(s, n) \) must be enlarged by several wavelengths so that fringing and diffractions on the edges are negligible. We denote \( S(s, n) \) as the viewing area (with respect to \( P_s \)). If we replace the finite “hole” with a “scatterer,” we have the smallest size scatterer, allowing all possible communication modes between \( V_T \) and \( V_R \). Smaller scatterers will only allow partial communication.

B. Single Scatterer

Now we consider the scenario where there is a single scatterer \( S \), as depicted in Fig. 3. We assume that \( S \) is a (purely) reflective plane, i.e., there is no penetration of the electromagnetic field, using a similar model to that of [23]. This simplification ignores the specular effects of scattering through clouds of small particles. However, modeling scatterers as pure reflectors becomes exact as the total path length, via a single scatterer, becomes large with respect to the signal wavelength [24].

The reflective nature of \( S \) means that the reflection angle \( \theta_i \) is the same as the incident angle \( \theta_i \). The reflection involves a loss that is represented by the following gain, cf. [25]

\[
\eta(\theta_i) = \exp \left\{ \left( \frac{\pi}{\lambda} \sigma_h \cos \theta_i \right)^2 \right\} J_0 \left\{ 8 \left( \frac{\pi}{\lambda} \sigma_h \cos \theta_i \right)^2 \right\}
\]

(36)

where \( \sigma_h \) is the standard deviation of the surface height, and \( J_0(\cdot) \) is the modified Bessel function of order zero. Note that for a perfectly flat plane, \( \sigma_h = 0 \) and \( \eta(\theta_i) = 1 \). The normal angle \( n \) of the scatterer must be such that the reflected signals
are directed at \( V_1 \), so that signal transmission from \( V_T \) to \( V_R \) is possible.

To determine the field within \( V_R \), we mirror-image the body \( V_R \) with respect to the scatterer to generate a virtual body \( V'_R \) (see Fig. 3). The scatterer \( S \) may be viewed as the “hole” through which communications between \( V_T \) and \( V'_R \) occurs. The area of \( S \) must be larger than the corresponding viewing area between \( V_T \) and \( V'_R \), in order for \( V_T \) to fully communicate with \( V_R \). Once this condition holds, the transfer function between \( V_T \) and \( V_R \) can be computed using the result in Section II-B. The algorithm becomes as follows.

**Algorithm 1: Scatter channel model**

1. The effective propagation distance is the total path length 
\[ D_s = D_1 + D_2. \]
2. The mirror-image body \( V'_R \)—the rotated mirror-imaged copy of \( V_R \)—is substituted for \( V_R \).
3. Using the propagation distance \( D_s \) and the receive body \( V'_R \), the process of Section II-B is applied.
4. The computed eigenfunctions for \( V'_R \) are rotated and mirror-imaged back so that they become functions for \( V_R \).

Note that the same result can be obtained by mirror-imaging \( V_T \)—to generate \( V'_T \)—due to the symmetry between transmit and receive volumes. Let us summarize the assumptions we use for the scatterer.

- **A1** The normal direction \( n \) of \( S \) is such that the reflected signals from \( V_T \) reach \( V_R \).
- **A2** Each scatterer \( S \) is a (small) finite-size purely reflective plane with loss given by (36).
- **A3** The distance \( D_1 \) from \( V_T \) to \( S \) is sufficiently large compared to the size of \( S \) that for any point in \( V_T \), the incident angle across the whole of \( S \) is roughly constant. A similar condition holds for \( D_2 \).
- **A4** Each scatterer is “local” to either \( V_T \) or \( V_R \), i.e., either \( D_1 \ll D \) or \( D_2 \ll D \).

We note that it is possible to relax these assumptions, at the expense of increasing the model complexity. We make the following observations.

- **Obs1** A scatterer that is not appropriately aligned is equivalent to a smaller (appropriately aligned) scattering object.

- **Obs2** A scatterer \( S \) that is smaller than the necessary viewing area\(^2\) reduces the communication strength for the path from \( V_T \) to \( V_R \) via \( S \). Modeling is possible by considering only the “visible” part of \( V_R \), i.e., by considering transmission between \( V_T \) and a smaller \( V'_R \). From (29), the reduction in effective volume reduces the number of communication modes for the link via this path.

- **Obs3** A scatterer \( S \) that is “too close”\(^3\) that A3 does not hold may be considered as a collection of several smaller scattering bodies \( \{S_1, S_2, \ldots \} \) such that A3 holds for each.

- **Obs4** Any scatterer \( S \) that is not local to either volume is ineffective for transmission.

To see Obs4 intuitively, when \( S \) is distant from both \( V_T \) and \( V_R \), the added propagation distance makes the received signal significantly weaker. Further, the number of communications modes available (1) through a link via \( S \) is very small, due to the large propagation length independent of transmission power. Consequently, the link via \( S \) appears as a low-power link with a few degrees of freedom. This implies that a distant scatterer will not contribute new orthogonal modes. Although such a scatterer may improve the signal power received, the contribution is dominated by any local scattering.

The consequence of Obs4 is that we only need to consider scatterers that are local to \( V_T \) and/or \( V_R \). A simple example of this approach is the well known ring-scatter model [26], [27]. An NLOS path (via a scatterer) may be comparable to the line-of-sight (LOS) path, provided that assumptions A1–A4 are satisfied and the reflection loss (36) is not significant.

**C. Multiple Scatterers per Path**

The model described above may be easily extended to the case where multiple reflections occur along a single path from \( V_T \) to \( V_R \). For a given scatterer \( S \), we may consider a virtual free-space transmission path \( P_S \) from \( V_T \) to \( V_R \). The communication modes are given by Algorithm 1.

For a path containing more than one scatterer, we simply repeat the mirror imaging of \( V'_R \) for each scattering body along the path. In this way, a virtual path is generated that incorporates the effect of each scattering body—concatenating the reflections and losses from each body—along the way. Having “unwound” the scattering path—performing a reflection for each scatterer and incrementing the virtual path length—the communication modes are calculated using Algorithm 1.

**D. Multiple Paths**

Consider the scenario in Fig. 4 with scatterers \( S^{(k)} \), \( k = 1, \ldots, K \), in accordance with assumptions A1–A4. Each scatterer provides a communication path and the transfer function \( \Gamma^{(k)} \) for each path can be computed using the method in Section III-B. The true value of \( \Gamma \) cannot be obtained by directly summing the various \( \Gamma^{(k)} \) transfer functions, because each \( \Gamma^{(k)} \) uses different eigenfunctions.

\(^2\)Either due to misalignment or small physical size.

\(^3\)Relatively close but not local.
To obtain a correct $\Gamma$, we denote by $\{\psi_{T_i}^{(k)}(r_T), i = 1, 2, \ldots\}$ and $\{\phi_{R_j}^{(k)}(r_R), j = 1, 2, \ldots\}$ the two sets of eigenfunctions corresponding to $\Gamma^{(k)}$ and let the dimension of $\Gamma^{(k)}$ be $N_c^{(k)} \times N_c^{(k)}$. Then, we have the following mapping:

$$
\left\{\psi_{T_i}^{(k)}(r_T)\right\} \xrightarrow{\Gamma^{(k)}} \left\{\phi_{R_j}^{(k)}(r_R)\right\}.
$$

(37)

Let $\{\psi_{T_i}(r_T)\}$ and $\{\phi_{R_j}(r_R)\}$ be any sets of complete orthonormal basis functions for $V_T$ and $V_R$, respectively. In particular, they may be the basis functions corresponding to the direct transmission path. We can project each $\psi_{T_i}^{(k)}(r_T)$ onto $\psi_{T_j}(r_T)$, $i = 1, 2 \ldots$ to obtain the following:

$$
\psi_{T_i}^{(k)}(r_T) = \sum_{j=1}^{\infty} \Gamma_{ij}^{(k)} \psi_{T_j}(r_T)
$$

(38)

where $\Gamma_{ij}^{(k)}$ are projection coefficients. In order to perform the projection, we first note that both the eigenfunctions $\psi_{T_i}^{(k)}(r_T)$ and the focusing function of the rotated-mirror-imaged volumes must be project back to a common reference.

Denote $F(x, y, z)$ as the original focusing function and $F^{(k)}(x, y, z)$ as the focusing function for path $k$—where we have already carried out the rotation and mirror-imaging—we may write

$$
\Gamma_{ij}^{(k)} = \int_{V_T} \psi_{T_i}^{(k)}(x_T, y_T) F^{(k)}(x, y, 0) F^{-1}(x, y, 0)
\cdot \psi_{T_j}(x_T, y_T) dx \, dy \, dz.
$$

(39)

In vector form, we have

$$
\psi_{T_i}^{(k)}(r_T) = \Gamma_{ij}^{(k)} \psi_{T_j}(r_T)
$$

(40)

for some matrix $\Gamma_{ij}^{(k)}$ with $N_c^{(k)}$ columns and infinitely many rows. For numerical calculations, $\Gamma_{ij}^{(k)}$ is truncated in similar form as $\Gamma$. Similarly, each function $\phi_{R_j}(r_R)$ may be projected onto $\phi_{R_j}^{(k)}(r_R)$, $j = 1, 2, \ldots$ to obtain

$$
\phi_{R_j}(r_R) = \Gamma_{ij}^{(k)} \phi_{R_j}(r_R) + \delta^{(k)}(r_R)
$$

(41)

for some matrix $\Gamma_{ij}^{(k)}$ with $N_c^{(k)}$ rows and infinitely many columns, where $\delta^{(k)}(r_R)$ is the residual term orthogonal to all $\phi_{R_j}^{(k)}(r_R)$. The elements of $\Gamma_{ij}^{(k)}$ and $\Gamma_{ij}^{(k)}$ may be computed numerically.

With the above projections, we can express the mappings from $\psi_{T}(r_T)$ to $\phi_{R}(r_R)$ through $S^{(k)}$ as follows:

$$
\psi_{T}(r_T) \xrightarrow{\Gamma_{ij}^{(k)}} \psi_{T}^{(k)}(r_T) \xrightarrow{\Gamma^{(k)}} \phi_{R}^{(k)}(r_R) \xrightarrow{\Gamma_{ij}^{(k)}} \phi_{R}(r_R).
$$

(42)

Thus, the overall transfer function is given by

$$
\Gamma = \sum_{k=1}^{K} \Gamma_{ij}^{(k)} \Gamma^{(k)} \Gamma_{ij}^{(k)}.
$$

(43)

The only difficulty is that there is no sum rule to determine the numerical accuracy of the computation, i.e., (29) is invalid for transmission in scattering environments. This implies that $\Gamma_{ij}^{(k)}$ and $\Gamma^{(k)}$ must have large dimension to ensure numerical accuracy. Once $\Gamma$ is obtained, the singular-value decomposition in (33) can be performed to obtain the eigenfunctions, singular values, and number of communication modes.

We may easily include an LOS path. Denote its transfer function and the corresponding projection matrices by $\Gamma^{(0)}$, $\Gamma_{ij}^{(0)}$, and $\Gamma_{ij}^{(0)}$, respectively. Then, (43) is modified to

$$
\Gamma = \sum_{k=0}^{K} \Gamma_{ij}^{(k)} \Gamma^{(k)} \Gamma_{ij}^{(k)}.
$$

(44)

Using (44), the direct transmission path is simply one of many possible transmission paths, each with their respective transfer-matrix and basis functions.

IV. SIMULATION

We compare a direct (nonscattering) transmission with various scattering environments. We consider a 3-GHz transmission frequency giving $\lambda = 0.1$ m. The volumes $V_T$ and $V_R$ are hyperrectangles. $V_T$ has side lengths of $15\lambda \times 9\lambda \times 9\lambda$ and $V_R$ has side lengths of $18\lambda \times 9\lambda \times 9\lambda$. The volumes are separated by $D = 10$ m. For this arrangement, the number of communications modes for direct transmission (1) is $N_c \approx 2.6$. We have used [28] for the numerical computation of (16).

The arrangement has been deliberately chosen to ensure a small nonnegligible number of direct transmission modes to emphasize the effect of scattering. Physically, such a model may correspond to placing antenna elements within two office filing cabinets, and placing the cabinets within adjacent offices. Under this arrangement, we may ask "how many well-connected parallel channels exist between the filing cabinets?"

Examining this question provides insight into the effect of scattering upon the wireless channel. The physical channel is of less importance than the concept of the continuous modes.

Fig. 5 shows the squared singular values for the different scattering environments. The squared singular values provide the channel gains for the effective parallel channels from $V_T$.
to $V_R$. The singular values are not normalized, which allows comparison among the various channel scenarios.

The direct-transmission case is shown by the solid line. The singular values $\sigma_k$ drop sharply beyond $k = 3$. This corresponds to the value of $N_c$ for this arrangement. The first singular value $\sigma_1 = 0.0058$ represents the channel gain, and gives an indication of distance (and other losses).

Using the same physical arrangement, we introduced scattering, in both an NLOS and LOS case. Scatterers were modeled as reflective planes with a gain (36), $\eta = 1$. Each scatterer was placed in a random location, between $V_T$ and $V_R$, with a random orientation. Monte Carlo simulations were then used, to generate average-case results.

The NLOS transmission is shown by the solid line with squares. In this situation, transmission was possible only via the reflective scatterers, where $K = 20$ scatterers were used. Physically, this corresponds to using an absorbing region between $V_T$ and $V_R$ that prevents any direct transmission. It can be seen that the channel gain $\sigma_1 = 0.0058$ is similar to the direct-transmission case. As additional scatterers are introduced, this gain tends to increase—as the total signal power received at $V_R$ increases with each additional path. Comparing the NLOS case with the direct-transmission case, scattering has provided approximately double the number of equal-strength communications modes. This can be seen by comparing the value of $k$ for which the direct-case singular values $\sigma_{\text{direct}}$ and $\sigma_{\text{NLOS}}$ fall below some threshold (say $1 \times 10^{-3}$). This corresponds to a significant improvement in the number of parallel channels. Equally, the relative magnitude of the singular values in the NLOS case is much smaller, which may be interpreted as a less correlated channel than the direct-transmission case.

This case may be compared with the single-scatterer NLOS channel, where a scatterer was placed randomly at a distance from both $V_T$ and $V_R$. As can be seen, the channel gain in this case is significantly reduced. The pinhole channel suffers from two reductions in singular values.

1) The scatterer is at an angle to the volumes, so the number of well-connected modes is reduced (although there may be several poorly connected modes).

2) The distance from $V_T$ to $V_R$ is larger than the direct-transmission path length, so the gain is also reduced.

The LOS case is shown by the circled line. Here, communication occurred via both dense scattering ($K = 20$) and direct transmission. The channel gain $\sigma_1 = 0.0068$, and this can be seen in the same way as the channel-gain improvement in the NLOS case. Each scatterer provides an additional communication path, and consequently, additional signal power at the receiver. This is precisely the effect seen when using a parabolic-dish antenna around a radio transmitter: the presence of the dish improves the power gain of the point-to-point channel. Comparing with NLOS, we see a significant improvement in the number and strengths of effective communication modes as expected.

In Fig. 6, we have plotted the (normalized) magnitude of the first and second communication modes for a direct-transmission case $\Gamma_d$, pinhole NLOS case, $\Gamma_n$ and LOS case, and $\Gamma_1$ between the hyperrectangles $V_T$ and $V_R$ described in Section IV. The pinhole NLOS case was generated by placing a single scatterer along the diagonal of $V_T$, and the LOS case generated by combining both the direct and pinhole NLOS channels. It was assumed that the gain of the scatterer (36), was $\eta = 0.5$. The LOS channel and the direct channel are related by

$$\Gamma_1 = \Gamma_d + \frac{1}{2} \Gamma \gamma_n \Gamma \phi.$$ \hspace{1cm} (45)

We note that the low-order modes of the LOS scenario are similar to those of the direct (nonscattering) scenario, while the NLOS modes are significantly different.

V. DISCUSSION

Let us consider two volumes in free space, i.e., the direct-transmission case. We have a set of basis functions, which may be calculated as described in Section II. Although there are infinitely many orthonormal functions in the sets $\psi_{Ti}(r_T)$ and $\phi_{Ri}(r_R)$, only a finite number $n_c$ have nonnegligible connection strengths, corresponding to the well-connected modes.

For every scatterer $S^{(k)}$, a new pair of basis sets $\psi_{Ti}^{(k)}(r_T)$ and $\phi_{Ri}^{(k)}(r_R)$ are generated. These are not necessarily the same as those of the direct path. Some of the basis functions $\psi_{Ti}^{(k)}(r_T)$ (and $\phi_{Ri}^{(k)}(r_R)$) have components that are parallel to the well-connected direct-path modes and some have components that are orthogonal to all of the well-connected direct-path modes. Further, if the path length via $S^{(k)}$ is approximately equal to the direct-path length $D$, and the sizes of $V_T$ and $V_R$ are unchanged, each scatterer path has the same sum of path gains as the direct case (ignoring reflection loss) given by the sum rule.

The components of $\psi_{Ti}^{(k)}(r_T)$ and $\phi_{Ri}^{(k)}(r_R)$ that are orthogonal to existing eigenfunctions produce additional (orthogonal) modes of communication. These orthogonal modes correspond to additional parallel-communication channels that are unavailable for direct transmission. The remaining components (parallel to existing eigenfunctions) contribute to the gain of
the corresponding modes. The gain contribution may be either constructive or destructive, depending on phases. A particular example of this is the parabolic dish: the various pieces of the dish may be considered as reflective scatterers. No new modes of communication are created. However, the power gain of the channel is markedly improved over a free-space LOS case.

It is clear that we cannot increase the number of communication modes indefinitely because there is a finite area that can be illuminated by scattering—each scatterer shadows those behind it. The theoretical limit is reached when scatterers essentially act as a lens. As such, scattering provides improvements in channel capacity by:

1) increasing the gain of particular communication modes by constructive interference; and

2) increasing the number of well-connected communication modes.

VI. CONCLUSION

We have presented a numerical approach for the computation of the number and strength of communication modes between two arbitrary volumes. We have extended these numerical computations to account for transmission in the presence of reflective-scattering bodies. We have shown that multipath scattering may be modeled by iteratively considering each scatterer as a particular instance in a general multipath field. This has been used to provide an insight into the physical channel characteristics necessary to achieve "rich" scattering. We have shown that the continuous channel may be considered as the limiting case for densely placed receiver and transmitter elements.

We have shown that reflective scattering can improve channel capacity through a combination of two methods: increasing channel gains, and increasing the number of communication modes. The first case is predominant in the “pinhole” channel,
where scatterers are placed closely together. In this case, the number of communication modes are dominated by the direct transmission. In the second case, so-called “dense” scattering, where scatterers are placed randomly, provides additional orthogonal communication modes over those already present in direct transmission. Simulation results have been used to show the improvement scatterers provide to the low-order singular values of the transfer matrix, and the conditions under which scattering can increase the number of communication modes.

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**REFERENCES**


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